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ACTUARIAL SOCIETY OF AMERICA

"The work of science is to substitute facts for appearances and demonstrations for impressions."—RUSKIN.

TRANSACTIONS

1912

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OCTOBER, 1912.

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NOTATION

Life Symbols.

As Revised October 11th, 1911.

Resolved, that in the presentation of papers the symbols of the Text-Book of the Institute of Actuaries, with the additions noted below, be followed when convenient, and that, in any event, the use of such symbols in other senses be avoided; all letters used as symbols to be printed in italics except "a" when used to denote an annuity of 1 first payment immediate, *i. e.* $(1+a)$; that N_x and S_x be used in accordance with the practice of the Institute of Actuaries, *i. e.*,

$$N_x = N_{x-1} = D_x + D_{x+1} + \cdots$$
$$S_x = S_{x-1} = N_x + N_{x+1} + \cdots$$

Additional symbols

$$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}.$$

$$u_x = \frac{l_x}{l_{x+1}}(1+i) = \frac{D_x}{D_{x+1}}.$$

Disability Symbols.

As Adopted May 16th, 1912.

l_x^{aa} = the number of active lives at age x.
l_x^{ii} = the number of invalid lives at age x,

then $l_x^{aa} + l_x^{ii} = l_x$.

l_x^{i} = the survivors at age x in a mortality table based on invalid lives (not to be confused with l_x^{ii} amongst whom there are additions each year from those just disabled).

d_x^{aa} = the number dying as active lives between ages x and $x+1$.

d_x^{ii} = the number dying as invalid lives between ages x and $x+1$.

i_x = the number of active lives becoming invalid between ages x and $x+1$.

p_x^a = the probability of an active life aged x being alive one year hence, whether then active or invalid.

q_x^a = the probability of an active life aged x dying within a year, whether still active or after becoming invalid,

$$\text{then } p_x^a + q_x^a = 1.$$

p_x^i = the probability of an invalid life aged x being alive one year hence.

q_x^i = the probability of an invalid life aged x dying within a year,

$$\text{then } p_x^i + p_x^i = 1.$$

p_x^{aa} = the probability of an active life aged x being alive and active one year hence.

p_x^{ai} = the probability of an active life aged x being alive but invalid one year hence.

q_x^{aa} = the probability of an active life aged x dying while still active within a year.

q_x^{ai} = the probability of an active life aged x becoming invalid and dying within a year.

r_x = the probability of an active life aged x becoming invalid within a year,

then $p_x^{aa} + p_x^{ai} = p_x^a$; $q_x^{aa} + q_x^{ai} = q_x^a$; $p_x^{ai} + q_x^{ai} = r_x$; $p_x^{aa} + q_x^{aa} = 1 - r_x$; $p_x^{aa} + p_x^{ai} + q_x^{aa} + q_x^{ai} = 1$.

r_x' = the absolute annual rate of invalidity,

$$\text{then } r_x' = \frac{i_x}{l_x^{aa} - \frac{1}{2}d_x^{aa}} = \frac{r_x}{1 - \frac{1}{2}q_x^{aa}}.$$

$D_x^{aa} = v^x l_x^{aa}$.

$D_x^{ii} = v^x l_x^{ii}$.

$D_x^i = v^x l_x^i$.

a_x^{ai} = value of an annuity payable at the end of each year

provided an active life now aged x is then alive but invalid.

a_x^a = value of an annuity-due on an active life payable during survival, active or invalid.

a_x^{aa} = value of an annuity-due on an active life payable during activity.

a_x^i = value of an annuity-due on an invalid life.

${}_n|\mathrm{a}_{xt}^{a(\overline{t}\overline{n}|)}$ = value of a deferred temporary annuity payable at the beginning of each of t years deferred n years, provided the disability occur during the n years.

$\mathrm{a}_{x:\overline{n}|:\overline{y-x}|}^{aa}$ = value of an annuity-due on an active life for n years or for $y-x$ years, whichever is the shorter, during activity.

Vol. XIII, Part I. No. 47.

TRANSACTIONS

MAY 16TH AND 17TH, 1912.

Address of the President, Archibald A. Welch.

Extended Insurance.

In my address at the fall meeting in 1910 I endeavored to set forth the value of presenting to this Society the results of investigations into mortality among all sorts and conditions of men and contracts, explaining that changes in contracts and social conditions might effect changes in the mortality experienced under any class of policies, and that we as a body must have "up to date" data for proof of even the oldest tenets of our profession.

In order that I may show my willingness to practice what I preach I am placing before the members of this Society the results of the experience of the Phoenix Mutual Life Insurance Company under its extended insurance, and while the amount of data involved is not sufficient to warrant a formal paper, and while, on account of this limited amount of data, the investigation has been confined to amounts of insurance alone and not extended to policies or lives, still I believe the experience here given will be of interest.

The company first introduced the extended insurance feature into its contracts in 1891, and since no policy could lapse and come under such provision until after three annual premiums had been paid it was not until 1894 that we met with any experience whatever, and as our experience closes with the anniversary of the policies in 1910, we have but sixteen years as the longest period of duration.

It should be borne in mind that the extended insurance feature was the "automatic feature" of the policy: i. e., on lapse of a contract the extended insurance went automatically into effect. Also, during the early years of the experience, in case of death occurring within the first three years of the extended insurance period the unpaid premiums under the policy were deducted. Later this period was reduced, until, at the present time, no premiums whatever are deducted in event of death occurring under the extended insurance provision.

TABLE I.

Summary by Years from Original Date of Issue.

Extended in Year.	Actual Loss.	Expected Loss.	Percentage Actual to Expected.
3	$ 11,000	$ 3,180	
4	119,974	99,911	
5	21,568	49,944	
Total 3–5	152,542	153,035	100
6	30,883	30,609	
7	22,660	15,436	
8	6,812	10,697	
9	11,365	9,257	
10	5,773	3,994	
Total 6–10	77,493	69,993	111
11	4,000	4,300	
12	2,000	2,518	
13		2,451	
14		1,956	
15		205	
Total 11–15	6,000	11,430	53
16	2,000	329	
17		169	
18		453	
19		225	
Total 16–19	2,000	1,176	170
Grand total	238,035	235,634	101

In Table I will be found the actual and expected losses under all extended insurance grouped by the *years of lapse of the original policies.* That is, the extended insurance granted in exchange for policies which lapsed in the third year of their duration was classed together, as was also such insurance granted in exchange for policies which lapsed at the end of every other year of duration of the

original contract; this table therefore shows the actual and expected losses (by the American Experience Table of Mortality) during the whole period of extension under each of these groups.

A glance at this table shows that the company has, during the sixteen years under which this insurance has been in force, suffered a loss ratio of 101 per cent. of the expected, and, when it is known that on all the policies issued during the period since 1891 covered by this investigation the ratio of actual to expected deaths was less than 70 per cent. by amounts, we have at the outset forced upon us the fact that for some reason or other the experience under this class of policies is quite different from that under the ordinary contracts of the company.

The second fact to be noted is that, whereas the loss ratio on all extended insurance granted in exchange for policies lapsing in the 3rd, 4th and 5th years was only 100 per cent. of the expected, on extended insurance granted under policies lapsing from the 6th to the 10th years inclusive the losses were 111 per cent. of the expected. The experience on policies lapsing after the 10th year is so meagre that no lessons can be drawn from the results of this class.

We might expect this relative mortality because the older a policy is when it lapses the further away from medical selection it is, and therefore a higher mortality might be expected.

At the same time, too, the older a policy is when it lapses, the older the insured is, and it has been found in most experiences that however much below the American Table the whole experience may be the actual mortality comes nearer to the expected mortality at the older ages than at the younger.

The experience here seems identical with that of the Mutual Benefit which was presented to the Society by Mr. Rhodes in a paper appearing in Volume X of the *Transactions*.

In Table II the experience has been classified by the *years of duration of the extended insurance*. That is, we have taken the experience during the first year of the extended insurance by itself, as we also have taken the experience of each other year of duration of this class of insurance.

In examining this table we are immediately struck by the abnormal losses during the first year of the extension. Out of a total amount of actual losses in the whole experience of $238,035 nearly one-half occurred during the first year of the extended insurance, giving a ratio of actual to expected losses of 187 per cent.

TABLE II.

SUMMARY BY YEARS EXPOSED UNDER EXTENDED INSURANCE.

Year of Extension.	Actual Loss.	Expected Loss.	Percentage Actual to Expected.
1	$114,177	$61,184	187
2	41,683	45,239	92
3	20,175	34,975	58
4	12,000	27,768	43
5	9,000	21,671	42
Total 1–5	197,035	190,837	103
6	3,500	14,128	
7	14,000	10,112	
8	8,500	7,103	
9	2,000	5,082	
10	13,000	4,043	
Total 6–10	41,000	40,468	101
11		2,053	
12		1,218	
13		554	
14		206	
15		66	
Total 11–15		4,097	
Grand total	238,035	235,402	101

As this amount seemed entirely abnormal a further analysis of the data was made, with the result that it was found that of the $114,177 of actual losses which occurred during this first year of existence of the extended insurance, $72,200 were represented by losses under five policies.

While it is possible that the death of so many holders of large policies during the first year of existence was entirely accidental and does not represent in any way a normal rate, still the fact of this heavy loss during the first year should not be ignored entirely, especially since the second year's exposures show a loss of 92 per cent. of the expected.

It is not inconceivable that there will be a greater "selection" against the company among large policies under this extended insurance provision than under small policies. A man with $50,000 or $100,000 of insurance with large premiums falling due, who knows that his death will occur within a certain period more than covered by the extension endorsed in his policy, will be more liable to make the direct choice when those premiums fall due than will one whose premiums are much less in amount.

Then, too, if the holder of a large policy is actually at home in his last illness when the premiums fall due, a confidential clerk or partner who might have charge of his money affairs during his absence from the office might make this same selection against the company already referred to, when such responsibility would not be taken by the office associates of a man insured for a comparatively small amount.

It is noteworthy that the experience of the Phoenix Mutual in this respect is identical with that of the Mutual Benefit which was previously alluded to, where the experience during the first year was greater than it was at practically any other period.

In justice to these tables the fact should be noted that in all probability credit has not been given for all the exposures to which this experience would rightly be entitled. For instance; the premiums on many policies are not promptly paid, but are accepted by the company one, two, three or even a greater number of months after they are due. Should a death occur during this period of delinquency it is charged to the extended insurance account, while it has been impossible to credit this account with the exposure which has actually resulted while these premiums were in delinquency and before the policies were revived on account of the acceptance of the premiums by the company. But even if allowance is made for certain modification of the actual figures for the reason just given, still we are forced to the conclusion that the policyholder does, both consciously and unconsciously, exercise this privilege of extension to the loss of the company: consciously when he deliberately refrains from paying a premium on a policy that he knows will become a death claim within a few months; unconsciously when through forgetfulness he is slow in paying his premium, giving it to the company with interest at a later date if he remains in good health and saving it for his estate if during the period of his delinquency he is overtaken by a fatal illness.

TABLE III.

Summary by Original Age of Issue.

Original Age of Issue.	Actual Loss.	Expected Loss.	Percentage Actual to Expected.
15–24	$ 12,773	$27,169	47
25–34	45,565	78,324	58
35–44	110,710	83,219	133
45–54	61,987	37,711	164
55–69	7,000	9,942	70

Table III shows the results under extended insurance where the data have been classified by the age of the insured at the time he took out the original contract. Here it will be noted that this extension privilege is most expensive for the company when attached to policies that are issued to men who are over 40 years of age. Here again we find the same results as obtained in the Mutual Benefit experience.

It might be inferred from the above that the experience given has proven that the adoption of this provision was a very expensive one for the company. The reply to this is that the provision has undoubtedly resulted in some loss to the company. I firmly believe, however, that if the policies had contained an automatic paid-up provision rather than an automatic extended term provision, in many cases the premiums that were unpaid would have been paid to the company and the same claims would have arisen against the running policies that resulted under the extended insurance option. In such cases as this all that the company would save would be the premiums that were paid under the policy with the automatic paid-up provision and which were not paid under the policy that lapsed and was extended. While I believe that a company which has an automatic extended insurance provision in its policies will in the end suffer a greater financial loss than one that has the automatic paid-up provision endorsed on its contracts, still I do not believe the loss will be enough greater to cause any anxiety to the company. Furthermore, the inclusion of this privilege of extended insurance in the contract may be looked upon as a selling feature, and the resulting additional cost to the company as a well invested "advertising" premium.

But over and above all, for a mutual company especially, this extended insurance provision may be looked upon as a service to the policyholder for which the members as a whole would be glad to pay extra if necessary, for it is "service" in these days which a company should strive to offer under all its contracts.

On the other hand, however, the experience of the Phoenix Mutual Life Insurance Company under its extended insurance provision emphasizes the conclusion which Mr. Rhodes drew in the paper already alluded to; the excessive mortality resulting under this class of policies does not seem in any way to warrant the payment of dividends on this class of insurance. On the contrary, it would seem to justify a larger surrender charge where a surrender

value is taken in the form of extended insurance purchased at net rates.

These two investigations into the mortality under extended insurance should be only the first of many similar investigations. It would be of great interest to know the mortality that may be expected under this provision where it is not automatic but elective. Also, how the mortality under these extended insurance provisions compares with the mortality experienced under paid-up policies for reduced amounts. These and many other questions offer broad opportunities for individual investigations of the mortality experiences of our companies which should tempt the Fellows and Associates of this Society from their routine tasks.

Survivorship and Deferred Survivorship Annuities.

SOME POINTS RAISED BY RECENT RULINGS AND LEGISLATION.

BY

HERBERT N. SHEPPARD.

Though survivorship annuities do not appear to appeal to the insuring public, continuous instalment policies have for some years past been pushed with success by enterprising life insurance companies and agents. The position taken by the New York Insurance Department, when the law compelling the standardization of life insurance policies became effective, in refusing to approve of the issuance by domestic companies of such contracts, suggested the issuance of deferred survivorship annuities as subsidiary contracts, which, being taken in conjunction with standard policies, gave all the benefits of the original continuous instalment policies.

Though the Department has receded from this position, so that continuous instalment policies may now be issued in the State of New York, attention was drawn to deferred survivorship annuities as independent contracts, and, as a result, the following lines have been written dealing with some questions that arise in connection with them and with survivorship annuities, which form they take when the period of deferment, instead of being a specified number of years, is zero.

The legal status of both survivorship and deferred survivorship annuities does not at present seem to be well defined. The position taken by the Insurance Commissioner of the State of Massachusetts, as set forth in pages XXIII–XXV, inclusive, of Part II of the Fifty-fourth Annual Report of his Department, is very clearly expressed and the present writer feels that it is the logical one. To quote his own words; "The supplemental contract is, in my opinion, clearly a contract of life insurance and not the purchase of an annuity, for the reason that the right to receive the deferred survivorship annuity becomes fixed upon the death of the assured without the further payment of a premium"; and, further on, "I

am confirmed in this belief by the language of the court in the case of Commonwealth *vs.* Wetherbee, 105 Mass. 149, 160, where it is said: 'A contract of insurance is an agreement by which one party, for a consideration . . . promises to make a certain payment of money upon the destruction or injury of something in which the other party has an interest . . . in life insurance it is the life of a person . . . neither the times and amounts of payments by the assured, nor the modes of estimating or securing the payment of the sum to be paid by the insurer affect the question whether the agreement between them is a contract of insurance.'" The same line of argument applies to survivorship annuities, which will be first considered as being the simpler case.

SURVIVORSHIP ANNUITIES.

As a survivorship annuity becomes a claim by the death of the insured, the amount of the claim being the value of an immediate annuity on the life of the annuitant at his or her age at the end of the policy year in which the death of the insured takes place, the formula for the single premium is the sum, for all values of t from $t=1$ until one of the lives reaches the end of the mortality table, of

$$v^t \cdot \frac{d_{x+t-1}}{l_x} \cdot \frac{l_{y+t}}{l_y} \cdot \mathrm{a}_{y+t}.$$

$$\text{Therefore the single premium} = \Sigma_{t=1}^{t=\infty} \frac{C_{x+t-1}}{D_x} \cdot \frac{l_{y+t}}{l_y} \cdot \mathrm{a}_{y+t}$$

where x is the life insured and y the annuitant. Since, after the period of childhood is passed, both l_{y+t}/l_y and a_{y+t} diminish as t increases, this formula brings out clearly that a survivorship annuity contract is one for a continually decreasing amount of insurance. The formula also shows that different tables may with propriety be used for the insured and the annuitant.

If premiums be payable during the joint lifetime of the insured and beneficiary, limited to n years, the net annual premium is obtained by dividing the above given single premium by the temporary annuity for n years on the two joint lives x and y.

The question of the choice of tables becomes a practical one if we view a survivorship annuity as a contract of insurance, when it becomes at once necessary to decide upon the method of calculation of the net premium and mortality profits required by Sec-

tion 97 of the New York law limiting the expenses of the company, and of the maximum surrender charge allowable under Section 88 dealing with the surrender value of lapsed or forfeited policies. The Superintendent of Insurance, acting under the power conferred on him by the legislature, has declared Mr. McClintock's Tables to be the standard tables for the valuation of annuities. There is no doubt that the ordinary annuities on one or more lives purchased by single premiums come strictly under this head. But Section 84 also says that annuities deferred ten or more years and written in connection with life or term insurance shall be valued on the same mortality table as that according to which the premiums were computed, with interest at a rate not greater than three and one half per centum. This would indicate that where a strong selection can be exercised against the company Mr. McClintock's Tables should be used, while greater latitude is allowed if, as in the case of deferred survivorship annuities, selection may be considered to be absent or of small effect. This leaves survivorship annuities in a doubtful position, but if they are to be considered as insurance contracts, while at the same time the companies wish to protect themselves against adverse selection, the most scientific course seems to the writer to be the choice of the American Experience Table for the insured and Mr. McClintock's Table for the beneficiary.

A company which calculates premiums, reserves, surrender values, etc., under all its insurance contracts on the basis of the American Experience Table at 3 per cent., may wish to bring survivorship annuities into line with them, and, at the same time, when such an annuity becomes a claim by the death of the insured, may wish to be able to put up the reserve by Mr. McClintock's Table at $3\frac{1}{2}$ per cent. without disturbing its surplus. In this case the alternative formula to the one above given will not apply.

This can readily be seen by referring to the Text Book of the Institute of Actuaries, Chap. XIV, Sec. 18, where the truth of the equation

$$\Sigma v^t \frac{d_{x+t-1}}{l_x} \cdot \frac{l_{y+t}}{l_y} \mathrm{a}_{y+t} = a_y - a_{xy}$$

is dependent upon one rate of interest being assumed from the issuance of the contract to the end of the life of the annuitant. It would be reasonable for a company which issues participating poli-

cies of insurance on the 3 per cent. basis and non-participating life annuities based on $3\frac{1}{2}$ per cent., to write survivorship annuity contracts with rates based on the combination above referred to, with participation in profits only during the joint lifetime of the insured and the beneficiary. The following table shows the level net premium payable during the joint lifetime of the insured and beneficiary per $100 per annum of annuity for such a contract for certain combinations of ages of the insured and the beneficiary, and is calculated by means of the "Nugget Formula" (see J. I. A., Vol. XXVII, p. 154), according to which

$$\bar{a}_{xy} = \frac{1}{D_x l_y} \cdot n\{Au_0 + Bu_n + Cu_{3n} + Du_{5n}\},$$

where $$u_k = D_{x+k} l_{y+k},$$

and

$$\bar{A} \text{ in this case} = \frac{1}{D_x l_y} \cdot n\{Au'_0 + Bu'_n + Cu'_{3n} + Du'_{5n}\},$$

where $$u'_k = D_{x+k} l_{y+k} \mu_{x+k} a_{y+k},$$

and A, B, C and D are constants.

D_{x+k} was based on American Experience 3 per cent., μ_{x+k} on Mr. Hunter's Makehamized American Table, a_{y+k} on Mr. McClintock's Female Annuitants' Table at $3\frac{1}{2}$ per cent. interest.

The net premium for the contract was taken as:

$$\bar{A} \div (\bar{a} + \tfrac{1}{2})\,(1.03)^{\frac{1}{2}}.$$

Age of Insured.	Age of Beneficiary.				
	20	30	40	50	60
25	$23.10	$18.59	$14.42	$10.96	$ 8.03
35	33.47	26.91	20.33	14.61	10.11
45	52.45	43.58	33.62	23.85	15.75
55	87.93	75.88	61.21	45.21	30.17

A comparison of these net premiums with the gross premiums charged by one of the principal companies publishing a complete set of rates, and using 3 per cent. interest for the valuation of its insurance contracts, shows a loading represented by a percentage of the net ranging from only 6 per cent. or 7 per cent. for age 20 of the beneficiary to 3 per cent. or 4 per cent. for the combination

of ages under which most of the contracts would probably be written. It seems, therefore, to the writer, as if the rates under this form of contract could in some cases be raised with advantage, even if the contracts are issued on the non-participating plan.

Having decided what mortality table to use, the next point is the calculation of reserves and mortality profits. It is here that we meet with the problem of negative reserves. It will be seen that negative reserves are tabulated in Mr. Dawson's recent book, "Survivorship Annuity Tables" based on the Makehamized American Table for the insured life and Danish Female Survivorship Annuitants for the life of the beneficiary. The publication of negative reserves, in the writer's opinion, is susceptible to criticism, when we come to examine closely what negative reserves really mean.

The most general expression for the formula connecting two consecutive policy values may be written

$$ {}_nV_x + {}_{n+1}\pi_x = v({}_{n+1}q_{x+n} \cdot {}_{n+1}S_x + {}_{n+1}p_{x+n} \cdot {}_{n+1}V_x), $$

where ${}_{n+1}\pi_x$ is the net premium paid at the beginning of, and ${}_{n+1}S_x$ the sum insured during the $(n+1)$th year.

If, therefore, ${}_nV_x + {}_{n+1}\pi_x$ is less than $v{}_{n+1}q_{x+n} \cdot {}_{n+1}S_x$, then ${}_{n+1}V_x$ is negative, or, in other words, if the initial reserve is insufficient to cover the risk for the current policy year, a negative terminal reserve is the result.

Assuming the rate of interest constant, negative reserves would only occur when the mortality is rapidly diminishing, as at very young ages (and these are excluded by practical and legal considerations), or, as in the case before us, where the sum assured is rapidly diminishing, so that the level net premium which is obtained by averaging the risk is insufficient to cover it in the early years. The criticism that the writer has to make is, that, if the initial reserve is insufficient to cover the risk of that policy year, the mean reserve is understated when, as is commonly the practice, one half of the initial reserve is taken. This is only correct when the initial reserve is just sufficient to cover the risk, the resulting terminal reserve being zero. Such a case seems to the writer to be one where it is very difficult to reconcile theory with practice. Theory would tell us either to charge decreasing office premiums, for which the corresponding net premiums would either just cover the risk from year to year (thus making the policy a renewable term contract for a decreasing insurance) or a level premium with

an increasing loading. The latter would be inadvisable on account of the heavy initial expense incurred under the contract, while the necessity for the former would be difficult to explain to agents and prospective applicants, as the benefit, though really a decreasing one, appears to be fixed in value.

In any case the office premium charged in the first year should not be less than the net premium for the first year's risk. The formula for this is:

$$v\frac{d_x}{l_x}\cdot\frac{l_{y+1}}{l_y}a_{y+1};$$

and the values are given for comparison with the level net premiums above shown, the same assumption of mortality and interest as before being made, as follows:

Age of Insured.	Age of Beneficiary.				
	20	30	40	50	60
25	$17.77	$16.40	$14.58	$12.25	$ 9.50
35	19.71	18.20	16.17	13.58	10.54
45	24.59	22.71	20.18	16.95	13.15
55	40.91	37.78	33.57	28.20	21.87

This table shows that where the age of the insured is 25, the level net premium is insufficient to cover the first year's risk if the beneficiary is about 40 years of age, when the age of insured increases to 35 the corresponding age of the beneficiary is about 58; if the age of the insured be 45, the age of the beneficiary must be about 70 for the level premium to be insufficient to cover the first year's risk, etc.

SELECT AND ULTIMATE VALUATION.

An interesting problem is to calculate the mortality gains allowed in accordance with Section 97 of the New York law. In the case of a survivorship annuity these occur only on account of the insured life.

The writer has always found that questions dealing with the select and ultimate method of valuation can best be treated retrospectively. The following formula shows how the necessity for the formation of commutation tables on two joint lives, of which one is select, may be avoided:

First, to calculate the full reserve on a continuous payment survivorship annuity by u and k columns on the insured life alone; let x be the insured life, y the annuitant, V_t the reserve at the end of t years, π the net premium, then

$$\pi a_{xy} = a_y - a_{xy} \quad \text{or} \quad a_y = (1+\pi)a_{xy};$$

$$V_t = a_{y+t} - a_{x+t:\,y+t} - \pi a_{x+t:\,y+t} = a_{y+t} - (1+\pi)a_{x+t:\,y+t};$$

$$V_{t+1} = a_{y+t+1} - (1+\pi)a_{x+t+1:\,y+t+1}.$$

Multiply each side of the last equation by $vp_{x+t}p_{y+t}$; then remembering that $a_x = vp_x a_{x+1}$, we have

$$vp_{x+t}p_{y+t}V_{t+1} = p_{x+t}(a_{y+t} - 1) - (1+\pi)(a_{x+t:\,y+t} - 1)$$

$$= p_{x+t}(a_{y+t} - 1) + 1 + \pi - a_{y+t} + V_t,$$

or

$$vp_{x+t}p_{y+t}V_{t+1} = V_t + \pi - (a_{y+t} - 1)(1 - p_{x+t});$$

$$\therefore V_{t+1} = (V_t + \pi)\frac{1}{p_{y+t}} \cdot \frac{1}{vp_{x+t}} - \frac{q_{x+t}}{p_{x+t}}\frac{a_{y+t} - 1}{vp_{y+t}};$$

$$\therefore V_{t+1} = (V_t + \pi)\frac{1}{p_{y+t}}u_{x+t} - a_{y+t+1} \cdot k_{x+t};$$

$$= I'_{t+1}u_{x+t} - a_{y+t+1} \cdot k_{x+t},$$

where

$$I'_{t+1} = I_{t+1} \times \frac{1}{p_{y+t}}.$$

This equation is adapted to select and ultimate notation and the tables of $u_{[x]+t}$ and $k_{[x]+t}$ given in Vol. X, pp. 143–44, of the *Transactions* may be used.

To find the mortality gains it should be remembered that according to the select and ultimate method the net premium of the first year is $\pi - G_x$* and thereafter is π.

It is therefore only necessary to proceed to use the above formula, omitting the unknown G_x, and by successive calculations get the fifth terminal value. If from this terminal value we subtract $G_x \times D_{[x]}/D_{x+5} \times l_y/l_{y+5}$, we obtain the fifth terminal reserve under the contract. From the equation thus formed the value of G_x may be obtained.

The following example shows the working of the formula for the

* G_x = present value at issue of assumed mortality gain.

special case $x = 40$, $y = 30$, the functions being select and ultimate for the insured, ultimate for the annuitant, American $3\frac{1}{2}$ per cent. Table.

Formulas used $V_{t+1} = I'_{t+1} \cdot u_{[x]+t} - a_{y+t+1} \cdot k_{[x]+t}$ where $I'_{t+1} = (V_t + \pi)\frac{1}{p_{y+t}}$, $x = 40$, $y = 30$, $\pi = 28.050$, $\pi \div p_{30} = 28.288$. American $3\frac{1}{2}$ per cent. Select and Ultimate.

t (1)	I'_{t+1} (2)	$u_{[x]+t}$ (3)	(2) × (3) (4)	$100a_{y+t+1}$ (5)	$k_{[x]+t}$ (6)	(5)×(6) (7)	(4)−(7) (8)	(8) $+\pi = I_{t+2}$
0	28.288	1.040122	29.423	1942.02	.004915	9.545	19.878	47.928
1	48.339	1.041741	50.357	1922.86	.006535	12.566	37.791	65.841
2	66.413	1.043014	69.270	1903.04	.007736	14.722	54.548	82.598
3	83.324	1.044377	87.022	1882.56	.009020	16.981	70.041	98.091
4	98.965	1.045708	103.488	1861.38	.010408	19.373	84.115	———

Hence we have $84.12 - G_x \frac{D_{[40]}}{D_{45}} \cdot \frac{l_{30}}{l_{35}} = 55.11$, the full reserve under the contract, whence $G_x = 22.51$ (which on the basis of an ordinary life policy at age at issue 40 corresponds to average sum assured of $1,926, which is almost equal to the claim of the second year).

The above formula is adapted to the case where another mortality table (for instance, Mr. McClintock's) is used for the life of the annuitant, if joint life commutation columns on the ultimate tables have already been calculated.

Having calculated the value of G_x the successive select and ultimate terminal reserves may be obtained by subtracting from the successive entries in col. (8) the product of G_x and

$$\frac{D_{[x]}}{D_{x+1}} \cdot \frac{l_y}{l_{y+1}}, \frac{D_{[x]}}{D_{x+2}} \cdot \frac{l_y}{l_{y+2}}, \frac{D_{[x]}}{D_{x+3}} \cdot \frac{l_y}{l_{y+3}} \text{ and } \frac{D_{[x]}}{D_{x+4}} \cdot \frac{l_y}{l_{y+4}}, \text{ respectively.}$$

DEFERRED SURVIVORSHIP ANNUITIES.

Let n years be the period of deferment, then, if the death of x take place in the tth year of the contract, the claim is an annuity on the life of $y + t$ deferred n years, hence the value of the single premium above given for the survivorship annuity is modified as follows:

$$\Sigma_{t=1}^{t=\infty} \frac{C_{x+t-1}}{D_x} \cdot \frac{l_{y+t}}{l_y} \cdot \frac{N_{y+t+n}}{D_{y+t}}.$$

Now

$$\frac{1}{D_{y+t}} = \frac{1}{D_{y+n+t}} \cdot \frac{D_{y+n+t}}{D_{y+t}} = \frac{1}{D_{y+n+t}} \cdot v^n \frac{l_{y+n+t}}{l_{y+t}}$$

$$= \frac{1}{D_{y+n+t}} \cdot \frac{l_{y+n+t}}{l_{y+n}} \cdot v^n \frac{l_{y+n}}{l_y} \cdot \frac{l_y}{l_{y+t}};$$

$$\therefore \text{value of single premium} = \frac{D_{y+n}}{D_y} \Sigma_{t=1}^{t=\infty} \frac{C_{x+t-1}}{D_x} \cdot \frac{l_{y+n+t}}{l_{y+n}} \cdot \frac{N_{y+n+t}}{D_{y+n+t}}.$$

In other words, the value of the single premium for the survivorship annuity deferred n years is equal to the discounted probability of the annuitant surviving n years multiplied by the single premium for the survivorship annuity to $y + n$ after the death of x.

It necessarily follows that the formulas above given for the calculation of the mortality profits in the case of a survivorship annuity will apply equally well to a deferred survivorship annuity if y is replaced by $y + n$ and the result multiplied by D_{y+n}/D_y.

The question of negative reserves has already been referred to under the head of survivorship annuities, but it becomes much more important in the case of deferred survivorship annuities on account of the greater fall in the value of the risk incurred from year to year through the greater age of the annuitant when the benefit is entered upon.

The following figures, based on Mr. McClintock's Tables at 3½ per cent., insured male, beneficiary female, show this clearly.

NET PREMIUM FOR CONTINUOUS PAYMENT, 20 YEAR DEFERRED SURVIVORSHIP ANNUITY OF $100.00, MCCLINTOCK'S TABLE, AT 3½ PER CENT., INSURED MALE, BENEFICIARY FEMALE.

Age of Insured.	Age of Beneficiary.				
	20	30	40	50	60
25	$6.01	$4.01	$2.35	$1.09	$.32
35	8.66	5.58	3.11	1.37	.39
45	14.34	9.30	5.05	2.12	.57
55	25.81	17.58	9.82	4.10	1.06

NET PREMIUM FOR FIRST YEAR'S RISK UNDER ABOVE CONTRACT.

Age of Insured.	Age of Beneficiary.				
	20	30	40	50	60
25	$6.73	$5.47	$3.91	$2.21	$.80
35	7.74	6.30	4.50	2.55	.91
45	10.28	8.36	5.97	3.38	1.21
55	16.60	13.50	9.65	5.46	1.96

It appears from the above table that, when the beneficiary is 20 years old, the insured must be between 25 and 35 for the level net premium to be sufficient to cover the first year's risk, if 30 years old, the insured must be between 35 and 45, if 40, between 45 and 55, if 50, the insured must be over 55, and so on.

It may be argued that, as the deferred survivorship annuity is now-a-days only a practical question when taken in conjunction with an instalment policy, it is quite legitimate to reduce the reserve on the instalment policy when the deferred survivorship annuity produces a negative reserve. The argument is based on the equality at the start of the net premiums for the payments and the benefits. Even from this point of view there is a limitation—viz: that the negative reserve should never be greater than the surrender charge; otherwise there is a loss to surplus through surrender which was not contemplated when the policy was issued. Furthermore, however, the change of beneficiary where permitted in the contract has the immediate result of raising the reserve to that under the instalments certain, so it seems to the writer that there are good arguments against taking advantage of the negative reserve under one of the constituent portions of the combination contract.

The question of the sufficiency of the reserve on continuous instalment policies as calculated under the rules approved by the New York Insurance Department was met approximately in the following way.

The recent issues under this form of contract in one of the large New York companies were found to be grouped as follows:

Age o Insured.	Age of Beneficiary.				
	0–25	26–35	36–45	46–55	55–65
up to 29	17	14	4	—	—
30 to 39	47	119	29	2	1
40 to 49	37	34	108	11	1
50 and up	20	9	12	16	4
Total	121	176	153	29	6

The total number of cases being 485, nearly one-half of the cases belong to the groups represented by average ages of the insured 35 with beneficiary 30 and insured 40 with beneficiary 35. The last two columns being relatively unimportant for the purpose of aver-

aging, except where the age of the insured is 50 and up where they balance against the young ages of the insured, were added to the third and the above table was modified as follows:

Average Age of the Insured.	Average Age of the Beneficiary.			
	20	30	40	Total.
25	17	14	4	35
35	47	119	32	198
45	37	34	120	191
55	20	9	32	61
Total	121	176	188	485

Net premiums for the continuous payment, 20 year deferred survivorship annuity, based on Mr. McClintock's Table at 3½ per cent. male for the insured, female for the beneficiary, were then calculated and the amount of ordinary life insurance purchased by the average premium for $50 per annum of annuity was found to be as follows:

Age of Insured.	Amount of Insurance Equivalent to $50 per Annum.
25	$183
35	146
45	133
55	175

the average weighted according to the number of cases in each group being $147. The corresponding amount based on American 3 per cent. is $115.

As the commuted value of 20 payments of $50 each is $735, assuming 3½ per cent., or $766, assuming 3 per cent., the present practice of reserving on the basis of $780 in the first case, or $800 in the second, thus allowing only $45 at 3½ per cent., or $34 at 3 per cent., for the average value of the deferred survivorship benefit, seems to the writer to indicate an understatement of the liability under this form of policy.

It may be argued that in the case of continuous payment deferred survivorship annuities, the majority of cases are those where the insured and beneficiary are both in middle life, so that it will be found that level net premiums lead to small positive or negative terminal reserves with the result that the mean reserve is practically constant from year to year, and bears a constantly diminishing ratio

to the mean reserve on ordinary life policies on the life of the insured, thus necessitating a smaller extra percentage of the total of the instalments certain than above indicated. The writer only claims, however, that an approximate method like that at present in use should be tested by comparison with exact values, basing the ages of insured and beneficiary on risks written by a representative company which writes policies in sufficient numbers to be used as a basis for other companies. It will be interesting to learn whether such a test has ever been made. The writer wishes to express his obligation to Mr. J. B. Maclean for his assistance in checking the formulas and calculations in this paper, and feels that though the points touched upon may have occurred to many of the members of the Society, their time will not be wasted if it leads to an interesting discussion.

On the Methods Used in the Construction of the l_x^{aa} Column, with a New Method of Calculating D_x^{aa} Values.

BY

SIDNEY H. PIPE.

The main object of this paper is to promote a discussion upon a question of theory which has arisen in determining the cost of the "waiver of premium" benefit. The theory of this subject has not come up for adequate discussion, and, while its practical side is well developed, there are conflicting opinions upon the correct method of calculating premiums for this benefit. Mr. Hunter has shown that the financial results of the two methods advocated are practically the same. In the following remarks the expression "single analyzed table" means a table which assumes that at the youngest age x, the l_x value of the basic mortality table is composed wholly of active lives, and that subsequent values of l_x are split up into active and disabled lives, the former representing active entrants at ages other than x.

The expression "separate analyzed table" means a series of tables constructed for each age at entry, and in which the number of entrants at each age is the l_x value of the basic mortality table, all of whom are assumed to be active, and in which the survivors are split up into active and disabled lives.

These two methods depend upon two different theories. The former assumes that p_x^{aa}, and consequently, a_x^{aa}, vary only with the age x, and are not affected by the age at entry. The determination of the cost of the waiver of premium benefit involves the difference between the values of an annuity payable for life and one payable during activity and it is necessary to determine a modified value of a_x which will conform to the assumption concerning a_x^{aa}.

This value, denoted by a_x^a, may be obtained as follows:

Of l_x^{aa} entrants at age x, ${}_np_x^a l_x^{aa}$ will survive, active or disabled, n years hence. Of these survivors l_{x+n}^{aa} will be active and $(l_{x+n}^{ii} - {}_np_x^i l_x^{ii})$ will be disabled.

$$\therefore {}_np_x^a l_x^{aa} = l_{x+n} - {}_np_x^i l_x^{ii},$$

and

$$ {}_np_x^a = {}_np_x \frac{l_x}{l_x^{aa}} - {}_np_x^i \frac{l_x^{ii}}{l_x^{aa}}; $$

$$ \therefore a_x^a = \frac{D_x}{D_x^{aa}} a_x - \frac{D_x^{ii}}{D_x^{aa}} a_x^i = a_x + \frac{D_x^{ii}}{D_x^{aa}} (a_x - a_x^i). $$

This is the formula obtained by Mr. Mead (*Transactions,* Vol. XI, p. 320). If x is the youngest age of the single analyzed table, then $\mathrm{a}_x{}^a = \mathrm{a}_x$.

If the entrant at age x survives as an active life for n years, the above formula, applied to age $(x+n)$, will give the value of an annuity payable till death, when separate analyzed tables are used.

The latter method assumes that the probabilities determined from the basic mortality table should not be altered after the l_x column has been split up into its component parts of active and disabled survivors. Since p_x is to remain unchanged, $p_x{}^{aa}$ will vary, by this method, with the age at entry.

I believe the latter method to be theoretically accurate. It is assumed in the use of the basic mortality table that ${}_np_x$ is the correct probability of a life aged x surviving n years, and computations of net premiums and reserves are made upon this assumption. A single analyzed table used for all ages at entry modifies this assumption and is therefore inaccurate.

An examination of the reserve formulas on pages 62 and 63 of Mr. Hunter's paper (*Transactions,* Vol. XII), to which no objection has been raised on theoretical grounds, shows that they are based upon the theory of separate analyzed tables.

The formula for the total reserves after n years is given as

$$ l_{x+n} A_{x+n} - l_{x+n}^{aa} (\pi_x + P_x^{ai}) \mathrm{a}_{x+n}^{aa}. $$

This formula assumes that l_{x+n} active and disabled lives survive out of l_x entrants at age x, and P_x^{ai} is the extra premium based upon this assumption. All the subsequent reserve formulæ in that paper are based upon this result.

In the discussion upon Mr. Hunter's paper, Mr. Mead, though disagreeing with separate analyzed tables as theoretically unsound, agrees with Mr. Hunter's reserve formulas which are based upon that theory.

The value A_{x+n} in the above formula is used to express the present

value of the sum assured payable at death to both active and disabled survivors, and is deduced from the basic mortality table. It therefore assumes that the general mortality amongst survivors n years after entry is the mortality of the basic table. This can only follow upon the assumption of separate analyzed tables.

CONSTRUCTION OF ANALYZED TABLES.

The construction of separate analyzed tables for each age at entry involves a great amount of labor if the l_x^{aa} column is calculated in each case. The following is a method by which the work is considerably shortened by calculating the D_x^{aa} column direct from the D_x value of the basic mortality table.

Let L_x be any number of entrants at age x subject to the mortality of the basic mortality table.

Then L_{x+n} will be the total number of survivors and will consist of L^{aa}_{x+n} active lives and L^{ii}_{x+n} disabled.

Assuming that those who become disabled in any year survive to the end of the year of disability, $q^{i}_{x+n} L^{ii}_{x+n}$ will be the number of "invalid" deaths in the $(n+1)$th year following entry; and $q_{x+n} L_{x+n} - q^{i}_{x+n} L^{ii}_{x+n}$ will be the number of "active" deaths in the same year.

The total "active" decrement in the $(n+1)$th year is

$$r_{x+n} L^{aa}_{x+n} + q_{x+n} L_{x+n} - q^{i}_{x+n} L^{ii}_{x+n}.$$

Since

$$L^{ii}_{x+n} = L_{x+n} - L^{aa}_{x+n},$$

this expression becomes

$$L^{aa}_{x+n}(r_{x+n} + q^{i}_{x+n}) - L_{x+n}(q^{i}_{x+n} - q_{x+n});$$

$$\therefore L^{aa}_{x+n+1} = L^{aa}_{x+n}(p^{i}_{x+n} - r_{x+n}) + L_{x+n}(p_{x+n} - p^{i}_{x+n}).$$

Putting L_{x+n} in the form ${}_{n}p_x L_x$,

$$D^{aa}_{x+n+1} = D^{aa}_{x+n} v(p^{i}_{x+n} - r_{x+n}) + \frac{L_x}{l_x} D_{x+n} v(p_{x+n} - p^{i}_{x+n}).$$

L_x is the radix of the analyzed table, and if this is made equal to l_x of the basic mortality table, the ratio $L_x \div l_x$ becomes 1.

The expressions $D_{x+n} v(p_{x+n} - p^{i}_{x+n})$ and $v(p^{i}_{x+n} - r_{x+n})$ are constant for the same attained age $(x+n)$, and, once computed, can be used for all ages at entry.

The legal reserve basis in Canada is now the $O^{M(5)}$, $3\frac{1}{2}$ per cent. table, so that the values in Table I are calculated upon that basis. The r_x and q_x^i values are those deduced by Mr. Hunter (*Transactions,* Vol. XII, p. 66), except that $q_{78}^{\ i}$ and $q_{79}^{\ i}$ have been altered to .123 and .129 respectively so as to run into the $O^{M(5)}$ table at age 80 with greater smoothness. Disability has been assumed to cease at age 60.

Table II shows the method of calculating the D_{30+t}^{aa} values for age at entry 30.

Table III gives the D_{x+t}^{aa} values for quinquennial ages at entry x and for values of t from 0 to $(80-x)$. The annuity value at age 80 is equal to the $O^{M(5)}$ value.

TABLE I.

Age.	$D_x \cdot v \cdot (p_x - p_x^i)$	$v(p_x^i - r_x)$	Age.	$D_x \cdot v \cdot (p_x - p_x^i)$	$v(p_x^i - r_x)$
20	9711.71	.76761835	50	1002.74	.87662222
1	8755.33	.77921062	1	955.216	.87550628
2	7904.81	.78983671	2	920.206	.87340386
3	7107.85	.80046280	3	884.154	.87127536
4	6402.65	.81012174	4	858.336	.86814686
5	5742.12	.81977971	5	820.522	.86594009
6	5123.97	.82943671	6	782.200	.86367632
7	4581.96	.83812657	7	743.237	.86134106
8	4074.69	.84681545	8	703.984	.85891690
9	3601.04	.85550434	9	664.433	.85638357
30	3189.90	.86322608	60	624.576	.85893719
1	2868.06	.86901352	1	584.604	.85700483
2	2596.67	.87383188	2	538.065	.85603864
3	2398.35	.87671497	3	486.119	.85603864
4	2238.62	.87862898	4	442.135	.85507246
5	2113.52	.87957294	5	393.755	.85507246
6	1994.05	.88051207	6	347.709	.85507246
7	1903.80	.88048019	7	303.847	.85507246
8	1793.41	.88140966	8	262.213	.85507246
9	1688.24	.88233430	9	223.155	.85507246
40	1587.69	.88325410	70	186.580	.85507246
1	1511.54	.88320289	1	155.869	.85410628
2	1437.66	.88314686	2	124.336	.85410628
3	1384.89	.88211980	3	95.713	.85410628
4	1315.07	.88205314	4	72.369	.85314009
5	1263.77	.88101352	5	51.465	.85217391
6	1213.45	.87996521	6	33.076	.85120773
7	1147.87	.87987343	7	18.810	.84927536
8	1099.15	.87880386	8	6.7447	.84734299
9	1050.67	.87772077	9	1.3518	.84154589

TABLE II.

CALCULATION OF $D_{x+t}^{aa}x = 30$.

Age. $x+t$	D_{x+t}^{aa}	$D_{x+t-1}^{aa}v(p_{x+t-1}-r_{x+t-1})$	$D_{x+t-1}v(p_{x+t-1}p_{x+t-1}^{\psi})$
30	33508.00		
1	32114.88	28924.98	3189.90
2	30776.32	27908.26	2868.06
3	29490.00	26893.33	2596.67
4	28252.67	25854.32	2398.35
5	27062.23	24823.61	2238.62

TABLE III.

VALUES OF D_{x+t}^{aa} AT $3\frac{1}{2}$ PER CENT.

DISABILITY CEASING AT AGE 60, MORTALITY BASIS $O^{M(5)}$

Age $x+t$	Age at Entry x.							
	20	25	30	35	40	45	50	55
20	50643							
1	48586							
2	46614							
3	44722							
4	42906							
5	41162	41240						
6	39486	39550						
7	37875	37928						
8	36326	36370						
9	34836	34874						
30	33404	33436	33508					
1	32025	32052	32115					
2	30698	30722	30776					
3	29422	29443	29490					
4	28193	28211	28253					
5	27010	27026	27062	27130				
6	25870	25885	25917	25976				
7	24773	24786	24814	24867				
8	23716	23727	23752	23798				
9	22697	22707	22729	22769				
40	21715	21723	21743	21779	21846			
1	20767	20775	20792	20824	20883			
2	19853	19860	19875	19903	19956			
3	18971	18977	18990	19015	19061			
4	18120	18125	18137	18158	18199			
5	17297	17302	17312	17332	17368	17440		
6	16503	16507	16516	16533	16565	16629		
7	15736	15739	15747	15762	15790	15846		
8	14993	14996	15003	15017	15041	15090		
9	14275	14278	14284	14296	14317	14361		

TABLE III (*continued*).

Age $x+t$	Age at Entry x.							
	20	25	30	35	40	45	50	55
50	13580	13583	13588	13598	13617	13655	13737	
1	12908	12910	12914	12923	12940	12973	13045	
2	12256	12258	12262	12270	12284	12313	12376	
3	11625	11626	11630	11637	11649	11675	11730	
4	11012	11014	11017	11023	11034	11056	11104	
5	10419	10420	10423	10428	10437	10457	10498	10593
6	9842.4	9843.5	9845.9	9850.4	9858.7	9875.4	9911.2	9993.4
7	9282.9	9283.8	9285.9	9289.7	9296.9	9311.3	9342.3	9413.3
8	8739.0	8739.8	8741.5	8744.9	8751.1	8763.5	8790.1	8851.3
9	8210.0	8210.7	8212.2	8215.1	8220.4	8231.1	8254.0	8306.5
60	7695.4	7695.9	7697.3	7699.7	7704.3	7713.4	7733.0	7778.0
1	7234.4	7234.9	7236.0	7238.1	7242.1	7250.0	7266.7	7305.4
2	6784.5	6785.0	6785.9	6787.7	6791.1	6797.8	6812.2	6845.3
3	6345.9	6346.3	6347.1	6348.6	6351.5	6357.2	6369.6	6397.9
4	5918.4	5918.8	5919.5	5920.8	5923.2	5928.2	5938.7	5963.0
5	5502.8	5503.1	5503.7	5504.8	5506.9	5511.1	5520.2	5540.9
6	5099.1	5099.3	5099.8	5100.8	5102.6	5106.2	5113.9	5131.7
7	4707.8	4708.0	4708.4	4709.2	4710.8	4713.9	4720.5	4735.7
8	4329.3	4329.5	4330.0	4330.6	4331.9	4334.5	4340.2	4353.2
9	3964.1	3964.3	3964.6	3965.2	3996.3	3968.6	3973.4	3984.5
70	3612.8	3612.9	3613.2	3613.7	3614.6	3616.6	3620.7	3630.2
1	3275.8	3275.9	3276.1	3276.5	3277.4	3279.0	3282.5	3290.7
2	2953.7	2953.8	2954.0	2954.4	2955.1	2956.5	2959.5	2966.4
3	2647.1	2647.2	2647.4	2647.7	2648.3	2649.5	2652.1	2658.0
4	2356.6	2356.7	2356.8	2357.1	2357.6	2358.7	2360.9	2365.9
5	2082.9	2083.0	2083.1	2083.3	2083.8	2084.6	2086.5	2090.8
6	1826.5	1826.5	1826.6	1826.8	1827.2	1827.9	1829.5	1833.2
7	1587.8	1587.8	1587.9	1588.1	1588.4	1589.0	1590.4	1593.5
8	1367.3	1367.3	1367.4	1367.5	1367.8	1368.3	1369.5	1372.1
9	1165.3	1165.3	1165.4	1165 5	1165.7	1166.2	1167.2	1169.4
80	982.0	982.0	982.1	982.2	982.4	982.8	983.6	985.5

On the Determination of the "Expected Mortality on Net Amount of Risk" and "Interest Required to Maintain Reserve."

[Note on Gain and Loss Exhibit.]

BY

MERVYN DAVIS.

These items are the most difficult to calculate of any required in the Gain and Loss Exhibit, and their determination involves a large amount of additional labor on the part of the actuarial department. It is hoped, then, that the following method, which will produce the results required in a very short time without any previous calculations, may prove of interest.

It was suggested at a meeting of the Society some time ago that the "Expected" be determined as the item required to balance the Exhibit. This is evidently unsatisfactory, for, as pointed out at the time, the "Interest required" is even more difficult to calculate; moreover, the calculation of the one involves that of the other, for, knowing the mean reserve, we can at once obtain the initial when given the tabular cost, and vice versa.

If, however, we consider both quantities unknown and that their difference is to be equated to the item required in the gain column to balance the Exhibit, we have,

Expected — Interest required $= A$ (the balance required).

Or, in symbols,

$$\tfrac{1}{2}(S_1K_1 + S_2K_2) - \frac{i}{2}(S_1I_1 + S_2I_2) = A, \qquad (1)$$

where $S_1 =$ amount of insurance in force at beginning of calendar year,

$S_1I_1 =$ initial reserve for the then current policy year,

$S_1M_1 =$ mean reserve for the then current policy year,

$S_1K_1 =$ tabular cost for the then current policy year,

and $S_2, I_2, \ldots$ are similar figures for the end of the calendar year.

Now,

$$I\left(1+\frac{i}{2}\right)=M+\frac{K}{2};$$

$$\therefore Ii=\frac{M+\frac{K}{2}}{1+\frac{i}{2}}i;$$

and

$$\frac{K}{2}-\frac{Ii}{2}=\frac{1}{2}\left[K-\frac{M+\frac{K}{2}}{1+\frac{i}{2}}i\right]=\frac{K+\frac{Ki}{2}-Mi-\frac{Ki}{2}}{2+i}=\frac{K-Mi}{2+i}.$$

Equation (1) may therefore be written

$$S_1\frac{K_1-M_1i}{2+i}+S_2\frac{K_2-M_2i}{2+i}=A,$$

which readily reduces to

$$\tfrac{1}{2}(S_1K_1+S_2K_2)=\left(1+\frac{i}{2}\right)A+\frac{i}{2}(S_1M_1+S_2M_2), \quad (2)$$

giving the "Expected," while the "Interest required" is then determined as "Expected"—A.

Adjustments.—The formula just obtained is mathematically correct on the assumptions that the business of each year is paid for during that year and that terminations occur on policy anniversaries. As these assumptions will not hold in practice, the following corrections will, theoretically, be required:

1. For S^D losses occurring during the year after their policy anniversaries,

$$+i\cdot S^D\cdot M_2.$$

2. For S^P insurance "unplaced" at the end of the previous year and subsequently paid for.

$$+iS^P\cdot M_1.$$

3. For S^L insurance (other than death losses) terminating $1/m$th

of a year $\begin{matrix}\text{after}\\ \text{before}\end{matrix}$ policy anniversaries

$$\pm \frac{i}{m} S^L \cdot M_{\frac{1}{2}}.$$

Practically speaking, however, it will be necessary to consider only the first of these adjustments, and it will be sufficiently approximate to calculate this on terminal instead of mean reserves; this adjustment may, therefore, in the case of a large business be determined as one half year's interest on the total reserves released by death.

The method, therefore, may be described briefly as follows: take *A*, the gain item required to balance the Exhibit, add to it one half year's interest, and also one year's interest on the mean of the mean reserves and one half year's interest on the reserves released by death (or more correctly, one year's interest on the reserves released on policies terminated by death after their policy anniversaries during the year).

The result obtained is the "Expected mortality on net amount at risk" and its excess over the quantity *A* is the "Interest required to maintain reserve."

Different Interest Bases.—In the case of a company using more than one interest reserve basis, an average rate of interest for use in the above formula may be determined, taking as weights the mean of the different mean reserves as given in the statements.

As an example of the method, the Gain and Loss Exhibit of a company for the year 1910, as given in the New York State Report, was taken, the work being as follows:

Rate of Interest.	Mean of Mean Reserves.			
4 per cent.	$21,832,193	× .04	=	873,288
3 per cent.	40,012,901	× .03	=	1,200,387
3½ per cent.	213,623	× .035	=	7,477
	62,058,717			2,081,152

average rate of interest = .033535

Gain item required to balance Exhibit

A = $344,566.60

A, increased by $\frac{1}{2}$ year's int.		
$344,566.60 \times 1.016768$		$ 350,344
1 year's int. on mean of mean reserves		
$62,058,717 \times .033535$		2,081,139
$\frac{1}{2}$ year's int. on reserves released by death		
$2,544,602 \times .016768$		42,668
Expected	=	$2.474,151
Subtract A	=	344,567
Interest required	=	$2,129,584

The actual figures given in the company's statement being

Expected	=	$2,484,208
Interest required	=	2,129,979

Mortality Experience of the Ætna Life Insurance Company Under Its Ten Year Renewable Term Policies.

BY

MAXIMILIAN H. PEILER.

In 1868 the Ætna introduced a ten year renewable term policy which is still in limited operation. It was adopted with the approval of Mr. H. W. St. John, by advice of the late William Scheffler, consulting actuary, a former army officer, who was instrumental in introducing several new measures which experience has proved to have been of great value. This plan, by which the company has been enabled to supply $86,000,000 of insurance at a low price and to distribute death benefits aggregating over $7,000,000, is now gradually approaching termination, as it is no longer possible to restore the waste by death and withdrawal, owing to the laws of certain states prohibiting methods of profit allotment contained in the policy.

CONTRACT.

The policy is of the ten year form, expiring at the end of ten years from date of issue, but granting the right of renewal, without medical examination, on the basis of an ascending premium scale at the expiration of successive decennial periods unless the insured has reached an age falling between 70 and 79. The profits are applicable to the purchase of the reduction of the premium unless withdrawn by surrender. Under this provision the policies entering the second, third, fourth and fifth decennial periods have been charged with the premium in the contract of the first period according to the age at date of issue. When the insured has reached an age falling between 70 and 79 at the expiration of any decennial period, the amount of allotted profits has not been quite sufficient to purchase a like reduction in premium, and the insured has been required to either pay an increased premium rate or to accept a slight reduction of insurance for the remainder of life.

SELECTION.

The utmost care was exercised in the selection of risks during the entire history of 41 years, and, with very few exceptions, only superior male lives residing in the salubrious sections of the United States and Canada were accepted. An additional safeguard against a high mortality consisted of a limitation of insurance on one life to a maximum of $10,000 and to a minimum requiring $15 of premium payment, annual, semi-annual or quarterly. As the company was able to renew the policies for several successive decennial periods without increasing the rate of premium, there was no selection against the company such as generally exists under ten year term policies in which there is a marked increase in the premium at the end of each period. The low price, the method of selection and the uniformity of risks constitute the peculiarity of this group, and lend additional interest to the low rate of mortality which was experienced throughout the period of observation.

PREMIUM RATES.

While the policy was issued as a ten year term contract, a scale of premium was charged which was expected to provide against whole life contingencies, provided the rate of interest earned was good, the rate of mortality low, and the rate of expense moderate. In order to bring out this point a table is given showing the two scales of premium charged by the company, and showing also whole life net rates on the American Table with 4½ per cent. interest and whole life net rates based on the experience of the ten year renewable term policies with 4½ per cent. interest.

Age.	R. T. Rates 1868 to 1894.	R. T. Rates 1894 to Present.	Whole Life Net Rates Am. Ex., 4½ Per Cent.	Whole Life Net Rates Present Experience, 4½ Per Cent.
20	$11.09	$12.20	$11.97	$ 9.46
30	14.67	16.10	15.34	13.00
40	21.02	22.81	21.30	19.45
50	33.17	35.28	32.49	30.83
60	55.64	58.00	54.14	51.94
70	98.03	100.62	97.00	89.97
75	131.83	134.59	132.64	123.40
79	168.34	171.30	174.67	145.92

RANGE OF INVESTIGATION.

The investigation covered the years of issue 1868 to 1908, inclusive, carried to the anniversaries of the policies in 1909. There were 38,054 policies insuring $86,613,546, the exposures being 252,337 years for $582,726,217 of insurance. There were terminated by death 2,883 policies insuring $7,056,120. A separate investigation was made of the policies which were changed to whole life insurance whenever the insured had reached an age falling between 70 and 79 at the expiration of any decennial period.

METHOD OF TREATMENT.

The age was taken at the nearest anniversary of birth.

The policies and amounts insured were traced by policy years and the duration was taken as the difference between the calendar year of exit and the calendar year of entry, except in the case of terminations by death where the death was taken in the policy year in which it occurred.

One of the most difficult questions to determine from published experience of life insurance companies is the extent to which there has been an improvement in mortality among insured lives. In order to trace such improvement the investigation was divided into two sections, the issues of the years 1868 to 1884, inclusive, and of the years 1885 to 1908, inclusive.

The exposed to risk were tabulated in the usual way but have not been given as they would not be of interest to actuaries at large. If any actuary desires to see these tables I should be glad to show them to him.

The ultimate object of the investigation was the construction of a mortality table on the basis of monetary experience for the purpose of allotment of profits. This table will be referred to in a later part of the paper, but, in the first place, comparisons with standard tables are given in order that the mortality experience can be readily seen. Comparison with the $O^{[M]}$ Table is given. This table was selected because it is the most modern table which traces the effect of selection.

COMPARISON OF RESULTS.

In Table I is given a comparison of the actual deaths and the expected deaths both by amounts insured and by policies at ages

15–29, 30–39, 40–49 and 50–60, and according to policy years. A brief synopsis of that table is as follows:

COMPARISON OF ACTUAL TO EXPECTED DEATHS BY O[M] TABLE.

Amounts.			
Policy Years.	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.
1	$ 481,350	$ 372,213	1.29
2–5	1,469,075	1,477,712	.99
6–10	1,436,207	1,706,263	.84
11 and subsequent	3,297,101	3,876,403	.85
	$6,683,733	$7,432,591	.90
Policies.			
Policy Years.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
1	180	160.7	1.12
2–5	609	640.0	.95
6–10	588	731.6	.80
11 and subsequent	1,351	1,641.0	.82
	2,728	3,173.3	.86

It has been pointed out in the *Transactions of the Actuarial Society* that the mortality in Britain is lower in the first policy year than in this country. The foregoing confirms this statement. It is probable that the British experience is higher in the later policy years than in the United States. (*Transactions of the Actuarial Society,* Vol. VII, pp. 127–128.) There are two deductions which may be drawn from the foregoing:

1. That the mortality by policies is slightly less than by amounts. A larger difference would have been expected had policies for a greater amount than $10,000 been issued.

2. That the O[M] Table is better adapted to the Ætna experience during the first five policy years than thereafter.

In order to obtain a comparison by ages at entry the following synopsis has been made, the figures referring to the 1st to the 10th policy years being taken according to ages at entry, and for the 11th and subsequent years according to attained ages.

The comparison indicates that the experience (on policies) of the Ætna on its term insurance does not reach the O[M] standard until the later ages at entry. It may be said, therefore, that the O[M] does not fit the experience of the Ætna either at early ages of entry or in the later policy years at the average ages of entry.

FIRST TO TENTH POLICY YEARS.

Amounts.			
Ages at Entry.	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.
15–29	$ 495,000	$ 556,091	.89
30–39	1,083,160	1,144,318	.95
40–49	946,950	1,001,432	.95
50–59	861,522	854,347	1.01
	$3,386,632	$3,556,188	.95
Policies.			
Ages at Entry.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
15–29	220	255.2	.86
30–39	419	493.8	.85
40–49	376	407.5	.92
50–59	362	375.8	.96
	1,377	1,532.3	.90

ELEVENTH AND SUBSEQUENT POLICY YEARS.

Amounts.			
Attained Ages.	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.
20–39	$ 84,400	$ 140,741	.60
40–49	527,130	726,133	.73
50–59	1,005,250	1,219,358	.82
60–69	1,293,021	1,354,447	.95
70–79	387,300	435,724	.89
	$3,297,101	$3,876,403	.85
Policies.			
Attained Ages.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
20–39	40	63.4	.63
40–49	215	318.6	.67
50–59	376	506.1	.74
60–69	550	566.7	.97
70–79	170	186.2	.91
	1,351	1,641.0	.82

In order to show the degree of the improvement in mortality the following synopsis is presented of the mortality divided into issues of 1868 to 1884 and those pertaining to the issues of 1885 to 1908.

RATIO OF ACTUAL TO EXPECTED MORTALITY BY $O^{[M]}$ TABLE.

Policy Years.	1868–1884.		1885–1908.	
	Amounts.	Policies.	Amounts.	Policies.
1st–5th	1.21	1.12	.99	.94
Subsequent	.94	.90	.78	.75
All policy years	.98	.94	.84	.81

The foregoing shows an unmistakable improvement in the mortality which may be due among other causes partly to improved sanitary conditions, partly to the increasing efficiency of the medical examiners, and partly to increasing knowledge regarding medical selection of lives.

Comparisons are also shown with the American Table and with the Modified Healthy English Table as used in the Specialized Investigation of the Actuarial Society, the former being an ultimate and the latter a select table. A synopsis of the comparison with the $O^{[M]}$ is also given in order to make the exhibit complete:

TEN YEAR RENEWABLE TERM MORTALITY EXPERIENCE; RATIO OF ACTUAL TO EXPECTED.

Years.	American.		Modified H. English.		$O^{[M]}$	
	Amounts.	Policies.	Amounts.	Policies.	Amounts.	Policies.
1st	.56	.48	1.16	1.00	1.29	1.12
2d	.74	.69	1.13	1.04	1.16	1.08
3d	.69	.67	.88	.85	.94	.92
4th	.81	.71	.92	.81	1.03	.90
5th	.71	.76	.76	.81	.84	.90
1st to 5th	.69	.64	.97	.90	1.05	.99
Subsequent	.88	.85	.95	.91	.86	.83
All years	.81	.78	.95	.91	.90	.86

The foregoing proves that the mortality experience under the ten year term policy was considerably below the American for all policy years and below the Modified Healthy English Table for the third and subsequent policy years. From a comparison of ratios based on amounts and on policies, it may also be inferred that a table which is suitable for obtaining the expected deaths under amounts insured may not be so applicable to an investigation based upon policies.

WHOLE LIFE POLICIES ISSUED IN EXCHANGE FOR TEN YEAR RENEWABLE TERM POLICIES

As already mentioned those policies which finished a decennial period when the insured was between 70 and 79 were changed to whole life policies and an increased premium or a lower amount of insurance granted. We, therefore, in this case had a mild form of selection against the company, and accordingly it would be interesting to determine the mortality experience as compared to a standard table such as the O[M]. The following is a synopsis of the experience:

WHOLE LIFE POLICIES ISSUED IN EXCHANGE FOR TEN YEAR RENEWABLE TERM POLICIES AT END OF TEN YEAR PERIOD TERMINATING AT AGES 70 TO 79.

Amounts.			
Attained Ages.	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.
70–74	$123,973	$133,236	.93
75–79	175,658	156,777	1.12
80 and over	72,756	82,207	.89
	$372,387	$372,220	1.00
Policies.			
Attained Ages.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
70–74	50	53.3	.94
75–79	72	69.1	1.04
80 and over	33	34.9	.95
	155	157.3	.99

The Ætna experience was remarkably close to the O[M].

METHOD OF GRADUATION.

The method of graduation selected was one of Mr. King's, i. e., the method of finding quinquennial values and inserting intermediate values by osculatory interpolation. The rates of mortality showed such an unusually light progressive tendency that it was necessary to abandon select graduation. This may be seen by a comparison of the aggregate mortality with the mortality excluding the first five policy years.

GRADUATED RATES OF MORTALITY, AGGREGATE EXPERIENCE.

Age.	All Years.	First Five Years Excluded.
25	.00464	.00413
30	.00495	.00510
35	.00544	.00552
40	.00653	.00683
45	.00888	.00911
50	.01204	.01221
55	.01643	.01633
60	.02558	.02580

The small difference between the aggregate and the ultimate mortality, leaving out the first five years, is noticeable.

In Table II appears a comparison of the graduated rates of mortality of the Ætna experience on ten year renewable term policies compared with the American, O^{M} and $O^{M(5)}$ tables. It is evident that the experience of the Ætna on its term policies has been much more favorable than the aggregate experience of the British companies on their whole life participating contracts.

I am indebted to Mr. E. E. Cammack for compiling these statistical data and supervising the construction of the tables, and to Mr. Arthur Hunter for suggesting synoptical tables and outlining these explanations.

TABLE I.

MORTALITY EXPERIENCE OF THE ÆTNA LIFE INSURANCE COMPANY UNDER ITS TEN YEAR RENEWABLE TERM POLICIES.

(Exposure to Anniversary Date of Policies in 1909.)

COMPARISON OF ACTUAL DEATHS AND EXPECTED DEATHS BY O[M] TABLE.

Age at Issue.	Amounts.			Policies.		
	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
			Policy Year 1.			
15–29	$ 76,700	$ 70,177	1.09	29	32.3	.90
30–39	135,500	126,512	1.07	55	55.3	.99
40–49	149,250	101,338	1.47	59	41.	1.44
50–60	119,900	74,186	1.62	37	32.1	1.15
15–60	$481,350	$372,213	1.29	180	160.7	1.12
			Policy Year 2.			
15–29	$ 74,300	$ 70,993	1.05	35	32.5	1.08
30–39	161,500	132,579	1.22	63	57.	1.11
40–49	90,700	103,275	.88	37	42.	.88
50–60	115,200	74,973	1.54	43	33.4	1.29
15–60	$441,700	$381,820	1.16	178	164.9	1.08
			Policy Year 3.			
15–29	$ 68,100	$ 68,094	1.00	32	31.	1.03
30–39	130,975	129,188	1.01	49	55.5	.88
40–49	87,000	105,130	.83	39	42.8	.91
50–60	73,000	77,845	.94	31	35.1	.88
15–60	$359,075	$380,257	.94	151	164.4	.92
			Policy Year 4.			
15–29	$ 65,100	$ 59,938	1.09	26	27.8	.94
30–39	125,000	120,892	1.03	46	52.4	.88
40–49	103,300	102,861	1.00	38	42.0	.90
50–60	78,250	78,865	.99	32	35.3	.91
15–60	$371,650	$362,556	1.03	142	157.5	.90
			Policy Year 5.			
15–29	$ 56,850	$ 55,579	1.02	24	25.9	.93
30–39	85,800	115,278	.74	39	49.6	.79
40–49	86,000	99,992	.86	38	41.	.93
50–60	68,000	82,230	.83	37	36.7	1.01
15–60	$296,650	$353,079	.84	138	153.2	.90

TABLE I (*continued*).

Age at Issue.	Amounts.			Policies.		
	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
Policy Years 6 to 10.						
15–29	$ 153,950	$ 231,310	.67	74	105.7	.70
30–39	444,385	519,869	.85	167	224.	.75
40–49	430,700	488,836	.88	165	198.7	.83
50–60	407,172	466,248	.87	182	203.2	.90
15–60	$1,436,207	$1,706,263	.84	588	731.6	.80
Attained Age.	Amounts.			Policies.		
	Actual Death Loss.	Expected Death Loss.	Ratio of Actual to Expected.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
Policy Years 11 and Subsequent.						
25–39	$ 84,400	$ 140,741	.60	40	63.4	.63
40–49	527,130	726,133	.73	215	318.6	.67
50–59	1,005,250	1,219,358	.82	376	506.1	.74
60–69	1,293,021	1,354,447	.95	550	566.7	.97
70–79	387,300	435,724	.89	170	186.2	.91
25–79	$3,297,101	$3,876,403	.85	1,351	1,641.	.82
All Policy Years.						
15–79	$6,683,733	$7,432,591	.90	2,728	3,173.3	.86

TABLE II.

MORTALITY EXPERIENCE OF THE ÆTNA LIFE INSURANCE COMPANY. TEN YEAR RENEWABLE TERM GRADUATED RATES OF MORTALITY COMPARED WITH THOSE OF STANDARD TABLES.

Age.	Ætna Life.	American.	O^M	$O^{M(5)}$	Ratio of Ætna Life Mortality to		
					American.	O^M	$O^{M(5)}$
20	.00535	.00781	.00404	.00652	.69	1.32	.82
21	.00530	.00786	.00416	.00659	.67	1.27	.80
22	.00512	.00791	.00431	.00665	.65	1.19	.77
23	.00487	.00796	.00446	.00672	.61	1.09	.72
24	.00472	.00801	.00463	.00680	.59	1.02	.69
25	.00464	.00807	.00481	.00689	.57	.96	.67
26	.00459	.00813	.00500	.00698	.56	.92	.66
27	.00458	.00820	.00523	.00709	.56	.88	.65
28	.00464	.00826	.00544	.00721	.56	.85	.64
29	.00478	.00835	.00569	.00732	.57	.84	.65
30	.00495	.00843	.00595	.00747	.59	.83	.66
31	.00513	.00851	.00620	.00762	.60	.83	.67
32	.00527	.00861	.00648	.00777	.61	.81	.68
33	.00535	.00872	.00677	.00796	.61	.79	.67
34	.00539	.00883	.00706	.00816	.61	.76	.66
35	.00544	.00895	.00738	.00837	.61	.74	.65
36	.00552	.00909	.00771	.00860	.61	.72	.64
37	.00567	.00923	.00804	.00886	.61	.71	.64
38	.00591	.00941	.00838	.00915	.63	.71	.65
39	.00620	.00959	.00877	.00945	.65	.71	.66
40	.00653	.00979	.00915	.00978	.67	.71	.67
41	.00692	.01001	.00956	.01015	.69	.72	.68
42	.00734	.01025	.01001	.01056	.72	.73	.70
43	.00781	.01052	.01048	.01099	.74	.74	.71
44	.00832	.01083	.01099	.01146	.77	.76	.73
45	.00888	.01116	.01153	.01200	.80	.77	.74
46	.00947	.01156	.01213	.01256	.82	.78	.75
47	.01009	.01200	.01277	.01320	.84	.79	.76
48	.01071	.01251	.01345	.01388	.86	.80	.77
49	.01136	.01311	.01422	.01463	.87	.80	.78
50	.01204	.01378	.01504	.01545	.87	.80	.78
51	.01280	.01454	.01595	.01634	.88	.80	.78
52	.01365	.01539	.01693	.01731	.89	.81	.79
53	.01453	.01633	.01799	.01839	.89	.81	.79
54	.01542	.01740	.01918	.01956	.89	.80	.79
55	.01643	.01857	.02045	.02083	.88	.80	.79
56	.01764	.01989	.02184	.02222	.89	.81	.79
57	.01917	.02134	.02338	.02375	.90	.82	.81
58	.02104	.02294	.02505	.02541	.92	.84	.83
59	.02320	.02472	.02689	.02722	.94	.86	.85

TABLE II (*continued*).

Age.	Ætna Life.	American.	O^M	$O^{M(5)}$	Ratio of Ætna Life Mortality to		
					American.	O^M	$O^{M(5)}$
60	.02558	.02669	.02887	.02921	.96	.89	.88
61	.02812	.02888	.03105	.03138	.97	.91	.90
62	.03076	.03129	.03344	.03373	.98	.92	.91
63	.03351	.03394	.03603	.03632	.99	.93	.92
64	.03642	.03687	.03886	.03912	.99	.94	.93
65	.03946	.04013	.04196	.04221	.98	.94	.93
66	.04263	.04371	.04532	.04554	.98	.94	.94
67	.04591	.04765	.04900	.04918	.96	.94	.93
68	.04879	.05200	.05299	.05317	.94	.92	.92
69	.05129	.05676	.05735	.05748	.90	.89	.89
70	.05417	.06199	.06207	.06219	.87	.87	.87
71	.05816	.06767	.06723	.06731	.86	.87	.86
72	.06403	.07373	.07281	.07290	.87	.88	.88
73	.07275	.08018	.07892	.07896	.91	.92	.92
74	.08382	.08703	.08548	.08553	.96	.98	.98
75	.09577	.09437	.09264	.09267	1.01	1.03	1.03
76	.10714	.10231	.10043	.10043	1.04	1.07	1.07
77	.11646	.11106	.10882		1.05	1.07	
78	.12319	.12083	.11795		1.02	1.04	
79	.12830	.13173	.12782		.97	1.00	
80	.13261	.14447	.13844		.92	.96	
81	.13692	.15861	.15000		.86	.91	
82	.14204	.17430	.16240		.81	.87	
83	.14691	.19156	.17573		.77	.84	
84	.15098	.21136	.19014		.71	.79	
85	.15586	.23555	.20569				
86	.16320	.26568	.22213				
87	.17460	.30302	.24001				
88	.19160	.34669	.25887				
89	.21311	.39586	.27881				

Select Rates of Mortality Amongst Impaired Lives and the Probabilities of Lives Becoming Impaired.

BY

PERCY C. H. PAPPS.

In Volume XXII of the Journal of the Institute of Actuaries, page 391, will be found an article by Dr. Sprague dealing with the "Construction and Use of a Series of Select Mortality Tables." So far as I know, but little has been written upon the subject covered in the title to this paper since the article by Dr. Sprague; and as it is both interesting and instructive to carry the analysis of select tables even further than has been done, I decided to put in the shape of a paper the results of some investigations I have made. Dr. Sprague has pointed out that it is possible to determine the premium which should be charged for the right to obtain insurance a certain number of years hence without medical examination. It may also be mentioned, in passing, that this premium depends upon the plan on which the insurance may be taken, although this point is not clearly stated by Dr. Sprague in his description of the method of obtaining the premium. It occurred to me that it might be interesting to show how it is possible to ascertain from select tables the numbers becoming non-select and the rates of mortality which prevail amongst such lives. It will be shown that the assumption that the effects of selection are exhausted in ten years, for example, results in all non-select lives dying of necessity within ten years after becoming non-select; unless it be assumed that a certain number regain their health and become once more select lives. If it were true that those who become non-select lives are really invalid lives, then the select tables would furnish us with the means of ascertaining the rates of invalidity, and mortality amongst invalids.

My remarks will be more easily understood if we have before us a portion of a select table. Since the $O^{[NM]}$ table shows a period of selection extending for five years only, this will save space and be quite sufficient for purposes of illustration. A portion of this table is shown in Table A. It may be mentioned that if any line be followed across the table the attained age increases by one year

with each column. In the older form of select tables the figures on any line related to the same attained age.

TABLE A.

Age at Entry.	Years Elapsed since Date of Insurance.						Age Attained.
	0	1	2	3	4	5 or More.	
$[x]$	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	$l_{[x]+4}$	l_{x+5}	$x+5$
20	100,000	99,580	99,003	98,333	97,616	96,879	25
21	99,264	98,844	98,267	97,596	96,877	96,137	26
22	98,530	98,109	97,530	96,857	96,135	95,392	27
23	97,794	97,369	96,790	96,115	95,389	94,641	28
24	97,055	96,630	96,048	95,370	94,639	93,886	29
25	96,316	95,887	95,302	94,619	93,884	93,124	30
26	95,567	95,135	94,547	93,862	93,122		31
27	94,818	94,382	93,791	93,100			32
28	94,059	93,618	93,023				33
29	93,300	92,854					34
30	92,529						35

In Table A $l_{[20]}$ is the radix and is composed of select lives only. If the line opposite age 20 at entry is followed across the table, the numbers living at ages 21 to 25 will be found. These numbers are composed of select and what we may call impaired lives. If the first column of figures be followed down we will find the number of select lives at each age. Now, the first point to bear in mind is that the number living at any age $([x]+t)$ is composed of $l_{[x+t]}$ select lives, and, let us say, $l^{i}_{[x]+t}$ impaired lives; and the number of select lives, i. e., $l_{[x+t]}$, can be found from the first column of the table. We have, therefore,

$$l^{i}_{[x]+t} = l_{[x]+t} - l_{[x+t]}. \qquad (1)$$

For example $l^{i}_{[25]+3} = 94{,}619 - 94{,}059 = 560$.

From Table A we have $l_{[20]} = 100{,}000$ and $l_{[20]+1} = 99{,}580$; and the difference, $d_{[20]}$, or 420, is the number of select lives dying between ages [20] and $([20]+1)$. But $l_{[20]+2} = 99{,}003$ and $d_{[20]+1}$, or 577, is the number of both select and impaired lives dying between ages $([20]+1)$ and $([20]+2)$. Now, $l_{[21]} = 99{,}264$ and this is the number of select lives at age 21 out of 100,000 who were select at age 20. The difference, i. e., 736, is the number of select lives who die or become impaired between ages 20 and 21. If $i_{[x]}$ represents the number of select lives aged x who become impaired

between ages x and $(x+1)$, then it will be seen that

$$i_{[x]} = l_{[x]+1} - l_{[x+1]} = l^i_{[x]+1}. \tag{2}$$

For example, $i_{[25]} = 95{,}887 - 95{,}567 = 320$.

It will also be seen that,

$$l_{[x+1]} = l_{[x]} - i_{[x]} - d_{[x]},$$

so that

$$i_{[x]} = l_{[x]} - l_{[x+1]} - d_{[x]},$$

which would follow directly from formula (2).

Now, if $d^i_{[x]+t}$ represents the number of deaths occurring between ages $([x]+t)$ and $([x]+t+1)$ amongst impaired lives, we have

$$l^i_{[x]+t+1} = l^i_{[x]+t} + i_{[x+t]} - d^i_{[x]+t},$$

or

$$d^i_{[x]+t} = l^i_{[x]+t} - l^i_{[x]+t+1} + i_{[x+t]}, \tag{3}$$

and substituting the value of $l^i_{[x]+t}$ found by formula (1) and of $i_{[x+t]}$ found by formula (2) we get

$$d^i_{[x]+t} = l_{[x]+t} - l_{[x+t]} - l_{[x]+t+1} + l_{[x+t]+1}. \tag{4}$$

The above formulæ are quite sufficient to enable us to work out tables giving all the information required. It must be clearly understood that the function $i_{[x]}$ represents the number becoming impaired between ages x and $(x+1)$; and since the impaired come from the lives remaining select, no matter how long they have been insured, the function $i_{[x]}$ depends only upon the attained age. On the other hand, the function $l^i_{[x]+t}$ is being constantly added to by lives becoming impaired and depleted by deaths occurring amongst impaired lives, so that the values of $l^i_{[x]+t}$ will vary for both x and t within the period of selection. The same thing holds true for $d^i_{[x]+t}$. Select tables showing the values of $l^i_{[x]+t}$ and $d^i_{[x]+t}$, such as Tables B and C, are hereafter shown; and a table showing the values of $i_{[x]}$, such as Table D, is easily prepared.

In order to test the above formulas I decided to make use of the $O^{[M]}$ data, as it gives the most complete select tables graduated by Makeham's law, so far as I am aware. The volume published jointly by the Institute and Faculty gives the values of $d_{[x]+t}$ for one or two places of decimals, whereas the values of $l_{[x]+t}$ are given generally to the nearest integer; and since the values of $i_{[x]}$, $l^i_{[x]+t}$

and $d^{i}_{[x]+t}$ are small compared with the values of $l_{[x]+t}$, it seemed desirable to make use of the values of $d_{[x]+t}$. It was necessary, therefore, to modify somewhat the above formulas.

When the period of selection lasts for s years, it will be seen that

$$l_{[x]+t} = l_{[x]+t+s} + \Sigma_{s=s-1}^{s=0} d_{[x]+t+s},$$

and that

$$l_{[x+t]} = l_{[x+t]+s} + \Sigma_{s=s-1}^{s=0} d_{[x+t]+s}.$$

Now, since $l_{[x]+t+s}$ and $l_{[x+t]+s}$ are each equal to l_{x+t+s}, it follows from formula (1) that

$$l^{i}_{[x]+t} = \Sigma_{s=s-1}^{s=0} d_{[x]+t+s} - \Sigma_{s=s-1}^{s=0} d_{[x+t]+s}. \tag{5}$$

The function $d_{[x]+t}$ includes lives dying between ages $(x+t)$ and $(x+t+1)$ out of $l_{[x+t]}$ select lives and $l^{i}_{[x]+t}$ impaired lives, so that

$$d^{i}_{[x]+t} = d_{[x]+t} - d_{[x+t]}. \tag{6}$$

Having obtained the values of $l^{i}_{[x]+t}$ and $d^{i}_{[x]+t}$, the values of $i_{[x+t]}$ may be found by rearranging formula (3) so that

$$i_{[x+t]} = l^{i}_{[x]+t+1} - l^{i}_{[x]+t} + d^{i}_{[x]+t}. \tag{7}$$

Now, if ${}_{t}r_{[x]}$ represents the probability of a life now select at age x being impaired at age $(x+t)$, then

$${}_{t}r_{[x]} = \frac{l^{i}_{[x]+t}}{l_{[x]}} \text{ or } \frac{l_{[x]+t} - l_{[x+t]}}{l_{[x]}}. \tag{8}$$

Again, if ${}_{t}|r_{[x]}$ represents the probability of a life now select at age x becoming impaired between ages $(x+t)$ and $(x+t+1)$, then

$${}_{t}|r_{[x]} = \frac{i_{[x+t]}}{l_{[x]}} \text{ or } \frac{l_{[x+t]+1} - l_{[x+t+1]}}{l_{[x]}}. \tag{9}$$

Data derived from Experience of British Offices O[M].

Table B shows the number of impaired lives in existence according to age at issue and duration. The values have been computed by means of formula (5).

TABLE B.

VALUES OF $l^{i}_{[x]+t}$ DERIVED FROM $O^{[M]}$

$[x]$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
21	391.85	623.03	799.66	947.40	1,072.94
25	399.66	641.87	829.96	989.19	1,125.74
30	418.45	683.24	894.05	1,075.11	1,232.66
35	451.59	752.66	998.89	1,213.67	1,403.13
40	505.29	861.92	1,161.52	1,426.18	1,662.27
45	586.82	1,025.12	1,401.42	1,736.93	2,037.78
50	704.14	1,256.28	1,737.40	2,167.07	2,551.41
55	860.80	1,561.17	2,173.52	2,716.43	3,194.31
60	1,049.31	1,919.68	2,673.72	3,327.22	3,884.14
65	1,234.20	2,256.80	3,118.40	3,832.40	4,404.40
$[x]$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$
21	1,178.03	1,263.46	1,329.96	1,377.98	1,408.13
25	1,241.67	1,337.74	1,414.68	1,473.08	1,513.64
30	1.368.67	1,484.17	1,579.92	1,656.66	1,715.30
35	1,569.40	1,713.45	1,836.48	1,939.08	2,022.65
40	1,871.85	2,056.10	2,216.33	2,353.58	2,469.37
45	2,306.16	2,543.05	2,750.46	2,929.47	3,082.21
50	2,891.93	3,190.67	3,449.02	3,669.23	3,852.61
55	3,609.61	3,964.28	4,260.12	4,499.82	4,685.04
60	4,347.50	4,720.00	5,004.30	5,204.40	5,325.00
65	4,839.00	5,142.30	5,321.60	5,386.00	5,345.70

The values of $d^{i}_{[x]+t}$ have been computed by means of formula (6) and are shown in Table C.

TABLE C.

VALUES OF $d^{i}_{[x]+t}$ DERIVED FROM $O^{[M]}$

$[x]$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
21	162.16	218.48	249.36	274.12	297.37
25	160.25	217.58	250.25	277.10	302.52
30	158.95	218.89	255.20	286.03	315.58
35	159.44	224.04	266.09	303.10	339.02
40	162.37	234.57	285.53	331.79	377.24
45	168.78	252.68	316.78	376.59	435.76
50	179.75	280.81	363.58	442.37	520.28
55	196.22	321.06	428.88	532.43	634.01
60	218.11	372.88	511.42	643.62	770.84
65	242.20	429.40	598.50	756.50	902.80
$[x]$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$
21	320.24	342.98	365.63	388.30	401.20
25	327.67	352.76	377.86	403.02	418.22
30	345.01	374.52	404.13	433.92	453.29
35	374.95	411.14	447.59	484.31	509.61
40	422.83	468.75	515.04	561.65	595.15
45	495.00	554.52	614.24	673.97	717.98
50	597.85	675.06	751.58	826.93	882.41
55	733.81	831.08	925.22	1,015.32	1,079.24
60	892.30	1,006.70	1,112.40	1,207.90	1,268.10
65	1,035.20	1,151.60	1,249.70	1,327.30	1,359.60

From Tables B and C the values of $i_{[x+t]}$ may be prepared by means of formula (7). It is interesting to notice that,

$$\begin{aligned} i_{[40]} &= l^i_{[35]+6} - l^i_{[35]+5} + d^i_{[35]+5} \\ &= 1569.40 - 1403.13 + 339.02 \\ &= 505.29; \end{aligned}$$

and

$$\begin{aligned} i_{[40]} &= l^i_{[30]+11} - l^i_{[30]+10} + d^i_{[30]+10} \\ &= 1767.30 - 1715.30 + 453.29 \\ &= 505.29; \end{aligned}$$

so that the values of $i_{[40]}$ are identical, no matter what portion of the table is made use of in computing the values.

Assuming age 21 to be the youngest age at entry, the value of $l_{[21]}$ has been ascertained correct to two places of decimals from the graduation constants, and the following table prepared. Table D shows the number of select lives in existence at each age, the number of such lives becoming impaired, and the number dying for each age. It also shows the values of $r_{[x]}$, or the probability of becoming impaired at each age.

It will be advisable to investigate not only the rates of impairment but the rates of mortality prevailing amongst impaired lives. Now, if $q^i_{[x]+t}$ represents the rate of mortality prevailing between ages $(x+t)$ and $(x+t+1)$ amongst impaired lives who entered at age x, then $q^i_{[x]+t}$ is equal to $d^i_{[x]+t}/l^i_{[x]+t}$. From Tables B and C it will be found that $q^i_{[21]+1} = .41383$, $q^i_{[21]+2} = .35067$, $q^i_{[21]+3} = .31183$, etc. This shows that the rate of mortality is very high in the first year following impairment and it also shows that the rate $q^i_{[21]+2}$, for example, is of little value, since the high mortality of those entering at 21 and becoming impaired between 22 and 23 entirely obscures the mortality between ages 23 and 24 of those becoming impaired between ages 21 and 22. It is necessary, therefore, to separate from $l^i_{[x]+t}$ the survivors at age $(x+t)$ of those who become impaired between ages x and $(x+1)$, $(x+1)$ and $(x+2)$, etc.

This separation was not made by Dr. Sprague and the graduation of the data at his disposal would not have been sufficiently smooth to enable him to do so had he wished.

TABLE D.

VALUES DERIVED FROM $O^{[M]}$

x	$l_{[x]}$	$i_{[x]}$	$d_{[x]}$	$r_{[x]}$
21	93,060.25	391.85	246.08	.00421
2	92,422.32	393.34	247.72	.00426
3	91,781.26	395.11	249.64	.00430
4	91,136.51	397.10	251.84	.00436
25	90,487.57	399.66	254.35	.00442
6	89,833.56	402.46	257.18	.00448
7	89,173.92	405.67	260.35	.00455
8	88,507.90	409.48	263.91	.00463
9	87,834.51	413.65	267.87	.00471
30	87,152.99	418.45	272.27	.00480
1	86,462.27	423.74	277.14	.00490
2	85,761.39	429.70	282.53	.00501
3	85,049.16	436.26	288.46	.00513
4	84,324.44	443.58	294.96	.00526
35	83,585.90	451.59	302.12	.00540
6	82,832.19	460.51	309.94	.00556
7	82,061.74	470.27	318.49	.00573
8	81,272.98	480.87	327.82	.00592
9	80,464.29	492.56	337.96	.00612
40	79,633.77	505.29	348.98	.00635
1	78,779.50	519.00	360.95	.00659
2	77,899.55	534.17	373.90	.00686
3	76,991.48	550.19	387.90	.00715
4	76,053.39	567.88	403.01	.00747
45	75,082.50	586.82	419.29	.00782
6	74,076.39	607.08	436.78	.00820
7	73,032.53	628.98	455.54	.00861
8	71,948.01	652.29	475.61	.00907
9	70,820.11	677.44	497.05	.00957
50	69,645.62	704.14	519.85	.01011
1	68,421.63	731.89	544.10	.01070
2	67,145.64	761.93	569.78	.01135
3	65,813.93	793.25	596.86	.01205
4	64,423.82	826.71	625.33	.01283
55	62,971.78	860.80	655.12	.01367
6	61,455.86	896.59	686.15	.01459
7	59,873.12	933.41	718.34	.01559
8	58.221,37	971.79	751.52	.01669
9	56,498.06	1,010.31	785.47	.01788
60	54,702.28	1,049.31	819.99	.01918
1	52,832.98	1,088.48	854.69	.02060
2	50,889.81	1,126.92	889.32	.02214
3	48,873.57	1,164.92	923.38	.02384
4	46,785.27	1,200.54	956.38	.02566
65	44,628.35	1,234.20	987.76	.02766
6	42,406.39	1,264.80	1,016.90	.02983
7	40,124.69	1,291.00	1,043.10	.03217
8	37,790.59	1,312.50	1,065.60	.03473
9	35,412.49	1,328.50	1,083.60	.03752
70	33,000.39	1,337.40	1,096.30	.04053
1	30,566.69	1,338.50	1,102.90	.04379
2	28,125.29	1,330.90	1,102.50	.04732
3	25,691.89	1,314.10	1,094.40	.05115
4	23,283.39	1,287.00	1,078.10	.05528
75	20,918.29			

We may now enlarge the notation so as to be able to distinguish between the total number of impaired lives arising out of those entering at a certain age and the survivors of those becoming impaired at a certain age.

Let $l_z^{i[y]}$ = number living at age z out of those becoming impaired between ages y and $(y+1)$,

and $d_z^{i[y]}$ = number dying between ages z and $(z+1)$ out of the $l_z^{i[y]}$ living at age z.

Now,

$$d^i_{[x]+t} = \Sigma_{r=t-1}^{r=0} d^{i[x+r]}_{x+t},$$

and

$$d^i_{[x+1]+t-1} = \Sigma_{r=t-1}^{r=1} d^{i[x+r]}_{x+t}.$$

Therefore $d^i_{[x]+t} - d^i_{[x+1]+t-1} = d^{i[x]}_{x+t}$, if we assume that the rate of mortality amongst lives becoming impaired depends only upon the age at which impairment occurs and not at all upon the age at original entry.

The formula

$$d_z^{i[y]} = d^i_{[y]+z-y} - d^i_{[y+1]+z-y-1} \qquad (10)$$

may be used to obtain the values of $d_z^{i[y]}$ directly from Table C. It may be noticed that if s be the period of selection, l_{x+s} represents the survivors at age $(x+s)$ of $l_{[x]}$ select lives, and it also represents the survivors of l_x mixed lives, i. e., $l_{[x]}$ select and l^i_x impaired lives. It is evident, therefore, that all the l^i_x impaired lives must die within s years and the values of $d_z^{i[y]}$ will be zero for all values of $z-y$ greater than s.

The values of $d_z^{i[y]}$ are shown in Table E and the values of $l_z^{i[y]}$ in Table F. The values of $l_z^{i[y]}$ may be derived by starting with the values of $i_{[x]}$ and continually subtracting the values of $d_z^{i[y]}$, and it will be found that the entire number becoming impaired die within the period of selection. It is evident, therefore, that the assumption that the death rate amongst impaired lives depends entirely upon the age at date of impairment and not upon the age at the date of entry, is justified by the results. The values of $l_z^{i[y]}$ may also be obtained by continued addition of the values of $d_z^{i[y]}$ and the accuracy of the work will be ascertained by the values finally running into the values of $i_{[x]}$.

TABLE E.

VALUES OF $d_z^{i[y]}$ DERIVED FROM $O^{[M]}$

y	$z=y+1$	$z=y+2$	$z=y+3$	$z=y+4$	$z=y+5$
21	162.16	56.85	31.24	24.75	22.78
25	160.25	57.73	32.67	26.24	24.15
30	158.95	60.01	35.64	29.18	26.90
35	159.44	64.25	40.47	33.88	31.25
40	162.37	71.20	48.03	41.07	37.95
45	168.78	82.11	59.36	51.83	47.83
50	179.75	98.26	75.73	67.30	61.90
55	196.22	120.88	98.30	88.20	80.70
60	218.11	150.00	127.20	114.50	103.70
65	242.20	182.80	158.80	142.40	127.10
y	$=y+6$	$z=y+7$	$z=y+8$	$z=y+9$	$z=y+10$
21	21.91	21.32	20.73	20.19	9.92
25	23.18	22.43	21.71	21.01	10.29
30	25.69	24.70	23.68	22.66	11.04
35	29.61	28.22	26.80	25.39	12.28
40	35.72	33.65	31.60	29.60	14.10
45	44.61	41.50	38.50	35.50	16.80
50	57.20	52.60	48.00	43.20	20.20
55	73.80	66.70	59.60	52.40	24.00
60	93.20	82.60	71.80	61.10	27.10
65	111.60	95.90	80.40	65.30	27.70

TABLE F.

VALUES OF $l_z^{i[y]}$ DERIVED FROM $O^{[M]}$

y	$z=y+1$	$z=y+2$	$z=y+3$	$z=y+4$	$z=y+5$
21	391.85	229.69	172.84	141.60	116.85
25	399.66	239.41	181.68	149.01	122.77
30	418.45	259.50	199.49	163.85	134.67
35	451.59	292.15	227.90	187.43	153.55
40	505.29	342.92	271.72	223.69	182.62
45	586.82	418.04	335.93	276.57	224.74
50	704.14	524.39	426.13	350.40	283.10
55	860.80	664.58	543.70	445.40	357.20
60	1,049.31	831.20	681.20	554.00	439.50
65	1,234.20	992.00	809.20	650.40	508.00
y	$z=y+6$	$z=y+7$	$z=y+8$	$z=y+9$	$z=y+10$
21	94.07	72.16	50.84	30.11	9.92
25	98.62	75.44	53.01	31.30	10.29
30	107.77	82.08	57.38	33.70	11.04
35	122.30	92.69	64.47	37.67	12.28
40	144.67	108.95	75.30	43.70	14.10
45	176.91	132.30	90.80	52.30	16.80
50	221.20	164.00	111.40	63.40	20.20
55	276.50	202.70	136.00	76.40	24.00
60	335.80	242.60	160.00	88.20	27.10
65	380.90	269.30	173.40	93.00	27.70

From Tables E and F the values of $q_z^{i[y]}$ may be found and these values are shown in Table G. This table shows the rates of mortality amongst lives becoming impaired according to years elapsed since date of impairment.

TABLE G.

VALUES OF $q_z^{i[y]}$ DERIVED FROM $O^{[M]}$

y	$z=y+1$	$z=y+2$	$z=y+3$	$z=y+4$	$z=y+5$
21	.41383	.24751	.18075	.17479	.19495
25	.40097	.24113	.17982	.17610	.19671
30	.37985	.23125	.17866	.17809	.19975
35	.35306	.21992	.17758	.18076	.20352
40	.32134	.20763	.17676	.18360	.20781
45	.28762	.19642	.17670	.18740	.21282
50	.25527	.18738	.17772	.19207	.21865
55	.22795	.18189	.18080	.19802	.22592
60	.20786	.18046	.18673	.20668	.23595
65	.19624	.18427	.19624	.21894	.25020
y	$z=y+6$	$z=y+7$	$z=y+8$	$z=y+9$	$z=y+10$
21	.23291	.29545	.40775	.67054	1.00000
25	.23504	.29732	.40955	.67125	1.00000
30	.23838	.30093	.41269	.67240	1.00000
35	.24211	.30446	.41570	.67401	1.00000
40	.24691	.30886	.41965	.67735	1.00000
45	.25216	.31368	.42401	.67878	1.00000
50	.25859	.32073	.43088	.68139	1.00000
55	.26691	.32906	.43823	.68586	1.00000
60	.27755	.34048	.44875	.69274	1.00000
65	.29299	.35611	.46367	.70215	1.00000

From general reasoning it would appear that a life might cease to be select and yet be very far from becoming an invalid, so that we would expect that the values of $i_{[x]}$, or the rate of becoming impaired, would be much greater than the rate of becoming disabled based upon totally disabled lives. A comparison of these rates in the following table is of interest.

TABLE H.

PROBABILITY OF BECOMING IMPAIRED OR DISABLED.

Age.	Rate of Impairment.	Rates of Disability.		Age.	Rate of Impairment.	Rates of Disability.	
x	$O^{[M]}$	Hunter.	Mead.	x	$O^{[M]}$	Hunter.	Mead.
25	.00442	.000528	.00025	55	.01367	.002752	.00255
30	.00480	.000561	.00037	60	.01918	.005402	.00830
35	.00540	.000642	.00048	65	.02766	.012388	.02802
40	.00635	.000832	.00069	70	.04053		.11133
45	.00782	.001151	.00090	75			.26920
50	.01011	.001696	.00124	80			.92776

Mr. Hunter's rates are given on page 46 of Volume XII, *T. A. S. A.*, and represent the ultimate rates of disability. Mr. Mead's rates are given on page 326 of Volume XI, *T. A. S. A.* The rates derived from the $O^{[M]}$ data are shown in Table D.

From the above table we would assume that the lives becoming impaired according to the $O^{[M]}$ data, would show a much lower rate of mortality than the lives becoming totally disabled according to the tables of Mr. Hunter and Mr. Mead. On pages 51 and 79 of Volume XII Messrs. Hunter and Mead have given us select rates of mortality amongst disabled lives. Mr. Hunter's rates show different rates for the first ten years following disability, and Mr. Mead's for eight years. As there is a reasonably close agreement between these two sets of rates, I have compared Mr. Hunter's rates with the rates derived from the $O^{[M]}$ data, in the following table.

TABLE I.

RATES OF MORTALITY AMONGST DISABLED LIVES.

Years.	Age 25.		Age 35.		Age 45.		Age 55.		Age 65.	
	$O^{[M]}$	Hunter	$O^{[M]}$	Hunter	$O^{[M]}$	Hunter	$O^{[M]}$	Hunter	$O^{[M]}$	Hunter
1	.401	.400	.353	.327	.288	.290	.228	.248	.196	.190
2	.241	.183	.220	.145	.196	.140	.182	.147	.184	.147
3	.180	.122	.178	.097	.177	.112	.181	.120	.196	.131
4	.176	.076	.181	.064	.187	.083	.198	.100	.219	.119
5	.197	.058	.204	.048	.213	.059	.226	.083	.250	.113
6	.235	.049	.242	.043	.252	.052	.267	.078	.293	.108
7	.297	.040	.304	.039	.314	.047	.329	.076	.356	.105
8	.410	.034	.416	.035	.424	.045	.438	.075	.464	.103
9	.671	.028	.674	.031	.679	.043	.686	.076	.702	.102
10	1.000	.024	1.000	.027	1.000	.044	1.000	.077	1.000	.103

The ages in the above table are those at which impairment or disability occurs.

There is a remarkably close agreement between the rates of impairment ($O^{[M]}$) and of disability (Hunter) in the year following the impairment or disability, but in the second and following years the mortality amongst impaired lives is in excess of that amongst disabled lives. This is so entirely at variance with what might be expected that it would be interesting to know the cause of the anomaly. I shall not attempt to solve this problem, but will suggest that it may be due partly to the fact that the period of selection really lasts much longer than the ten years assumed, and that con-

sequently impaired lives do not necessarily die within ten years. It is possible that we have not yet arrived at a correct understanding of the mortality amongst totally disabled lives. It may be that some lives who have become impaired may later on once more become select and so upset the formulas on which this paper is based. The great difference in the rates shown in Table I would make it appear that some simple explanation should be forthcoming.

If we could obtain the rate of mortality amongst select lives, the probability of a select life becoming impaired, and the select rates of mortality amongst impaired lives, it would be a very interesting problem to reverse the operations shown in this paper and build up a set of select mortality tables.

My object in presenting this paper was to draw attention to the nature of select tables of mortality and the assumptions on which they are based, and I trust that those who discuss this paper at the next meeting may have some explanations for the difference between the rates of impairment and disability shown in Table I.

Workmen's Compensation Benefits.

BY

W. ARTHUR WATT.

INTRODUCTION.

This branch of insurance dealing with compensation paid to workmen for injuries received in the course of their employment is one of the latest to come under the notice of actuaries in this country. But already a great deal of attention has been paid to it as a political and social factor, and, in some form or other, it is rapidly being incorporated in the State and Provincial Laws of the United States and Canada. In Europe each country has an elaborate system of workingmen's insurance, which is designed to meet the needs of that particular country. As a rule these systems cover the same ground, differing in the number of benefits available and the machinery by means of which they are made operative. In many countries laws have been in effect for a number of years, and a great body of statistics has been collected which is exceedingly valuable to one who wishes to make a study of the subject. The newness of the subject in this country and the lack of adequate data make it necessary to turn to the experiences of the European countries, and the present paper embodies a few deductions and extracts resulting from such an investigation.

It is improbable that there will be any material deviation in this country from principles known and tested in Europe. In several of the states the government has undertaken to insure the workmen, and it is likely that the number will be augmented in the not distant future. The European experiences will be of value as indicating the general tendencies of the business. It is not clear that similar tendencies will be found when the underwriting is done by regular insurance companies.

It would be quite impossible in the compass of a short paper to give any sort of historical sketch dealing with the systems found in Europe, but reference will occasionally be made to the development of the subject in some one or more countries, especially Austria. It has also been found impossible in an introductory essay to make a

thorough study of any special phase of the subject, the vastness of which offers so many points of interest. On the other hand, it was considered necessary to give a general idea of the ground to be covered, leaving for future discussion the more important points.

TYPES OR SYSTEMS OF COMPENSATION.

The insurance of workmen against injury may be carried by the employer himself, or he may pay an insurance company to carry it for him. As an alternative, the state may undertake the risk, and it is this alternative which gives to the question such importance from a political and social point of view. It is obvious that an employer who finds the compensation of his injured workmen making serious inroads upon his resources would gladly avail himself of the agency of an insurance corporation by paying the requisite premium. On the other hand, there is thus an assurance to the employee that when the day of disaster does overtake him he will not suffer through the inability or other failure of his employer to meet his needs. Accordingly, it is not desirable that the employer should undertake to carry his own risk, and, where he might be tempted to do so, the state *usually* offers inducement to insure by relieving him of his liability, or he is compelled to contribute to a fund kept by the state. In any case, the state exercises some supervision so as to provide that the law may be complied with. In some countries the employer is compelled to insure with a regular insurance company, or, if he prefers it, with a state-controlled institution. In other countries the state undertakes the risks, assessing the employers to cover the disbursements.

It is not the province of this paper to enter into any discussion as to the desirability or otherwise of state insurance, but it seems necessary to assume that the only feasible system which will satisfactorily comply with the requirements of the law is compulsory insurance either in a regular insurance company or in a state-controlled institution.

GERMANY.

Germany has the most complete system of state insurance of any country in the world. Its influence has extended to many other countries and to its experience many writers have turned for statistics. The system is complex, and, to the democratic mind, unnecessarily so. The administrative and supervisory head of the

workmen's insurance is the Imperial Insurance Office. Its province is to explain the various provisions of the law; to approve of the constitutions of accident associations, the schedules of risk ratings, the rules for the prevention of accidents, etc. It is the court of last resort in controversies respecting accident insurance. The important work, however, is performed by the Mutual Accident Insurance Associations, which are composed of employers. Each association is composed of employers engaged in the same or similar industries, and the insurance is conducted on the mutual plan, the benefits paid to workmen being assessed on the members of the association. Membership in the proper association is compulsory for all employers engaged in that industry. The associations are subject to the supervision of the Imperial Insurance Office. The most important feature of this plan of organization is that the employers manage their own insurance, private accident insurance companies being excluded from this business.

AUSTRIA.

In Austria as in Germany there is compulsory state insurance for accidents to workmen. For convenience of administration the country is divided into a number of districts with a separate institution for each district. Compensation is granted for practically all industrial accidents having direct connection with the employment. The administrative head is the Minister of the Interior, who is entrusted with the enforcement of the laws and with issuing the decrees authorized by the laws. Associated with him is the Insurance Council, a committee of experts whose function is to furnish technical advice on insurance matters. The actual work of accident insurance is performed by the organizations known as Insurance Institutions which are kept separate on geographical lines, with special treatment of the various industries on the basis of their trade risks. They are composed of the employers, workmen and administrative officials of the establishments covered by the law.

ITALY.

In Italy, as in Germany and Austria, there is compulsory insurance, but with this difference, the method of insurance is not prescribed, and there are many exceptions to the compulsion. Consequently, while there is the Cassa Nazionale d'Assicurazione per gl'Infortuni degli Operai sul Lavoro, employers may insure their

employees with authorized private insurance companies. In place of these there may be established employers' voluntary mutual accident associations organized by employers, or even private establishment funds organized by one or more employers in the same locality. The government may also form employers' compulsory mutual accident insurance associations. Private funds must have 500 members, voluntary mutuals 4,000 and compulsory mutuals 15,000 each at least. It is noteworthy that private insurance companies are declining in importance, the growth of the national institution being very rapid. The private funds and employers' mutual associations are also showing growth in relative importance.

GREAT BRITAIN.

In Great Britain there is no compulsory insurance of employees. Even if assurance is carried it does not relieve the employer from his liability. This is only obtained if the employer and workmen agree upon a substitute provision which is approved by the Registrar of Friendly Societies. The workman can elect whether he will demand redress under the Workmen's Compensation Act or under the Employers' Liability Act. Very few actions are now brought under the latter act.

FRANCE.

In France, as in Great Britain, insurance is not required by the law, but provision is made both for the liability of employers and the insurance of employees. All employers, whether insured or not, must contribute to the State Guarantee Fund which has been established to protect the insured in case of the insolvency of the employer or insurance company. The insurance companies are also subject to the supervision and control of the government and must maintain adequate reserve or guarantee funds. In 1899 the Caisse Nationale d'Assurance en Cas d'Accidents was reorganized and made an official fund to compete with the private accident insurance companies. So far the competition has not reached any great proportions nor does the business obtained appear to be profitable. The maximum premium rates which can be charged are stated, which may be reduced if the precautions taken diminish the risk.

SOURCES OF INCOME AND FINANCIAL ORGANIZATION.

The basis of income is the payroll. Here, unlike life insurance, the individual worker whose life is to be assured against accident occurring in the course of his employment is not considered. A factor of some importance is the size of the establishment, and another is the wage rate. Indirectly, the latter factor is of importance when the cost of the accidents has to be considered. When the age groups of the cost of accidents are examined, it is seen that the tendency is for the average cost to increase with age. This is to be expected, as the costs are based on the loss of earning power of the injured persons, and the mature and experienced workmen would have a higher wage rate than those in the younger groups. For each establishment there is a risk rating dependent on the class of industry and perhaps of locality. The tendency is for the employer to pay the cost of the insurance, though this is not universal.

In Austria the various establishments included in the insurance system are assessed various rates upon the amount of wages paid, the risk being quite different according to the nature of the industry or nature of operation. A general scheme of classification of establishments is used by all the insurance institutions, embracing fourteen classes, each subdivided into a series of percentage ratings varying somewhat according to the experience of the individual institutions. The rates are revised every five years. The employer is required to forward to the institution the entire amount of the insurance assessments, but is authorized to deduct 10 per cent. of the assessments from the wages of his insured employees. The sick funds provide for the first four weeks of all cases of disability, towards the cost of which the employers pay but one third, and this is responsible for the plan of having the workmen pay one tenth of the cost of the accident insurance. The official scheme of revision of the insurance system provides for an extension of the period for which the sick fund shall care for the injured persons from four to thirteen weeks, in which case the employers are to defray the entire expense of the accident insurance. Another provision is that employers shall be assessed on the basis of the rate of wages paid to the workmen in the various wage classes and not on the total pay roll. It is planned that the payments made by the employers from year to year shall cover the capitalized value of the pensions and other benefits arising each year, and also the expenditures of administration, of special reserves, etc.

In Germany the accident insurance is differentiated broadly between the following groups of industries:

(1) Persons engaged in manufacturing, mining and transportation;
(2) Persons engaged in the building trades;
(3) Seamen;
(4) Persons engaged in agriculture and forestry.

The employers are assessed to defray the cost of the insurance, the assessment being based on the amount of wages and salaries paid by each firm, and determined by means of the risk rates adopted by the associations for the establishments involved. An arbitrary rate of wages based on the average daily earnings of ordinary adult day laborers may be used instead of the actual amount of the wages and salaries earned. One reason for this is stated to be that, if the actual amounts earned are included, a somewhat heavier burden is imposed on establishments where there are a large number of highly paid workers in comparison with establishments in which there are a large number of low paid workers. Employers must forward to the association within six weeks after the close of each fiscal year a statement showing, (1) the insured persons employed in the establishment during the fiscal year and the salaries and wages earned by them; (2) a computation showing the amount of wages and salaries to be used in calculating the assessments; (3) the risk class in which the establishment is rated. Provision may be made in the constitution removing the necessity for the computation required in (2), also that the salary and wage list shall be made up quarterly or semi-annually for the purpose of calculating assessments. About the same time, the post office authorities send to the board of directors of the association a statement of the amounts paid out. Adding thereto the amount of the charge for the reserve fund and the probable amount of the costs of administration for the ensuing year, the total is that amount which is to be assessed on the members. The assessments are designed for the specific purposes of formation of reserve funds, payments of the benefits specified in the law, prevention of accidents, defraying the necessary expenses of administration, payment of premiums for the rescue of injured persons, and, with the consent of the Imperial Insurance Office, establishment of institutions for the treatment and care of injured persons.

In connection with the building trades the system of assessments to cover annual outgo is not applicable as in the case of the industrial accident insurance. Inasmuch as the amount of work done each year and the persons engaged in the building industries changed so frequently, it was found necessary to adopt a financial system based on premiums sufficient to cover the entire cost of all accidents arising, instead of the system of assessments for current expenses only. The basis for the calculation of the premiums is the capitalized value of the payments which the Insurance Institute will probably have to make for accidents on building operations which require more than six days to execute, to which must be added a sum sufficient to build up a reserve and an adequate loading for administration expenses. Interest on the reserve fund may be used for current expenses if not otherwise required.

STATISTICS OF ACCIDENTS AND MORTALITY.

From the statistics available it has been difficult to make a judicious selection to suit the limitation of an average paper. In several countries, particularly in Austria, Germany, France and England, there is a veritable mine of information available for those interested, especially from the social and political viewpoint. The actuary is not quite so fortunate, and it is rather disconcerting to find a considerable number of possible lines of attack upon the rough data, none of which is perfectly familiar and many of which leave one "stranded" because the data have not "panned out."

Some very interesting statistics referring to the Austrian system are included in convenient form in the Twenty-fourth Annual Report of the Commissioner of Labor of the United States of America, and it has seemed desirable to make use of these results of a highly efficient system of compensation, so far as they can be made to serve. But, in order to comprehend their significance at times, it is necessary to bear in mind the salient features of the Austrian law, since the experience may be of limited value in this country, to the extent, at least, that different benefits are promised. Table A shows the number of workers insured, the number of accidents reported and the number compensated for the years 1890 to 1907.

On account of the considerable difference between the average number of persons insured and the number of full-time workmen it is necessary to use the latter in discussions regarding accident rates, etc. The difference is chiefly due to the presence or absence

TABLE A.

Year.	Average Number of Persons Insured.	Number of Full-time Workers Insured.	Number of Accidents Reported.	Accidents Compensated.		
				Fatal.	Non-Fatal.	Total.
1890	1,231,818	823,166	16,041	548	6,193	6,741
1	1,369,763	857,284	21,316	565	8,219	8,784
2	1,380,881	893,882	26,298	574	8,848	9,422
3	1,466,270	940,943	32,917	649	10,252	10,901
4	1,598,404	989,987	40,259	670	11,882	12,552
5	1,877,194	1,216,731	54,562	835	15,560	16,395
6	1,974,644	1,286,433	64,655	929	17,615	18,544
7	2,077,473	1,331,957	69,283	929	18,732	19,661
8	2,200,112	1,395,710	75,146	977	19,999	20,976
9	2,334,561	1,438,584	79,260	1,044	21,622	22,666
1900	2,372,213	1,462,350	80,534	1,003	22,036	23,039
1	2,530,178	1,481,857	81,605	995	23,139	24,134
2	2,535,517	1,483,293	84,003	901	24,412	25,313
3	2,621,929	1,518,518	88,155	909	24,550	25,459
4	2,687,002	1,609,208	99,744	1,037	26,575	27,612
5	2,806,223	1,645,423	103,735	1,111	27,676	28,787
6	2,918,679	1,726,824	109,118	1,089	30,380	31,469
7	3,030,452	1,824,939	119,052	1,189	32,303	33,492

of seasonal industries, such as agriculture, the building trades, etc. There has been a continuous increase since 1890 in the number of accidents reported, and this is due in large measure to improvements in the system of reporting same. It will be noted that a

TABLE B.

Year.	Number of Accidents Reported per 10,000 Full-time Workers.	Number of Accidents Compensated per 10,000 Full-time Workers.					Number of Accidents not Compensated per 10,000 Full-time Workers.
		Fatal.	Non-fatal.			Total.	
			Temporary Disability.	Permanent Disability.	Total.		
1890	194.9	6.7	55.9	19.3	75.2	81.9	113.0
1	248.6	6.6	70.8	25.1	95.9	102.5	146.1
2	294.2	6.4	70.7	28.3	99.0	105.4	188.8
3	349.9	6.9	74.5	34.5	109.0	115.9	234.0
4	406.7	6.8	82.6	37.4	120.0	126.8	279.9
5	448.4	6.8	87.4	40.5	127.9	134.7	313.7
6	503.8	7.2	95.2	42.1	137.3	144.5	359.3
7	520.2	7.0	102.7	37.9	140.6	147.6	372.6
8	538.5	7.0	105.5	37.8	143.3	150.3	388.2
9	551.0	7.3	108.6	41.7	150.3	157.6	393.4
1900	550.7	6.8	109.9	40.8	150.7	157.5	393.2
1	550.7	6.7	113.4	42.8	156.2	162.9	387.8
2	566.4	6.1	124.7	39.9	164.6	170.7	395.7
3	580.5	6.0	126.7	35.0	161.7	167.7	412.8
4	619.8	6.4	124.9	40.3	165.2	171.6	448.2
5	630.4	6.8	Not shown separately for the years 1905–7.		168.2	175.0	455.4
6	631.9	6.3			175.9	182.2	449.7
7	652.3	6.5			177.0	183.5	468.8

very much smaller number of accidents are compensated than have been reported. This is chiefly due to the fact that the number compensated includes only those causing disability of more than four weeks' duration.

Analyzing the accidents in greater detail, as in Table B, we have the numbers on the basis of 10,000 full-time workers.

The number of accidents resulting in death per 10,000 full-time workers has remained practically constant during the years 1890 to 1907. The number of non-fatal accidents per 10,000 full-time workmen, causing disability for more than four weeks, has, with a single exception, increased every year throughout the period. This increase has been pointed out as due to increasing familiarity with the law on the part of the assured, who have, in consequence, not been slow in asserting their rights. Reference to the experience in other countries demonstrates that this is not the sole reason for the increase in the accident rate, and the conclusion is that the actual number of accidents has increased since 1890, due to modern industrial methods.

Differentiating between the non-fatal accidents, we find, on the basis of 10,000 full-time workers, 19.3 accidents in 1890 resulted in permanent disability. Up to 1896, when the number of accidents was 42.1 per 10,000 full-time workers, there was a steady increase, but from that time on till 1904 the number was fairly constant, averaging slightly under 40 per 10,000 full-time workers. In the case of accidents causing temporary disability of over four weeks there has been a decided numerical increase; from 55.9 per 10,000 full-time workers in 1890, the number had risen to 87.4 in 1895 and in 1904 the number was 124.9, an increase in the decade of 37.5.

Pursuing this line of argument still further, it is desirable to analyze the accidents resulting in permanent disability so as to ascertain the degree of disability caused thereby.

The following table shows the relative loss of earning power and the percentage which each bore to the total cases of permanent disability.

Turning now to industry groups, the accident insurance law provides that, at least once in five years, the charges shall be revised on the basis of experience gained by the operation of insurance institutions during that period. Special studies have accordingly been made of the accidents occurring to persons subject to the com-

pulsory insurance since the year 1897 and the results are here shown for the five-year periods 1897 to 1901 and 1902 to 1906.

TABLE C.

Year.	Two-sixths and Under.	Over Two-sixths to Three-sixths.	Over Three-sixths to Four-sixths.	Over Four-sixths to Five-sixths.	Six-sixths.	Total.
1890	49.6	23.1	9.0	12.0	6.3	100.0
1	59.1	17.3	6.5	12.2	4.9	100.0
2	62.1	14.8	8.5	9.9	4.7	100.0
3	63.1	18.1	7.1	8.1	3.6	100.0
4	70.4	14.2	6.5	6.0	2.9	100.0
5	66.1	14.1	7.7	8.0	4.1	100.0
6	67.0	15.1	6.4	7.7	3.8	100.0
7	58.2	14.3	7.1	12.9	7.5	100.0
8	62.5	14.7	6.6	10.7	5.5	100.0
9	62.9	14.4	6.1	10.8	5.8	100.0
1900	64.3	14.9	6.7	9.6	4.5	100.0
1	60.1	14.8	7.5	11.8	5.8	100.0
2	66.3	12.5	6.7	9.5	5.0	100.0
3	65.7	13.5	6.9	9.5	4.4	100.0
4	68.2	11.7	6.7	8.9	4.5	100.0
5	65.9	12.9	6.6	9.5	5.1	100.0
6	70.8	11.2	5.7	7.8	4.5	100.0
7	71.6	10.9	5.6	7.9	4.0	100.0

The industry groups are the following:

Group I. (a) Agricultural establishments using power.
I. (b) Flour mills.
II. Transportation and storage.
III. Smelting works, mining of "nonreserved" minerals, etc.
IV. Stones and earth.
V. Metal working.
VI. Machinery, tools, instruments, apparatus, etc.
VII. Chemical industries.
VIII. Heating and lighting materials, oils, heating and lighting establishments, etc.
IX. Textile industries.
X. Paper, leather, rubber, etc.
XI. Woodworking, carved materials, etc.
XII. Food and drinks.
XIII. Clothing and cleaning.
XIV. Building and construction.
XV. Printing, publishing, theaters, etc.

Groups I to XV. All insured establishments

TABLE D.

Industry Group.	Number of Full-time Workers Insured.		Total Number of Accidents.		Number of Accidents Compensated on Account of					
					Temporary Disability.		Permanent Disability.		Death.	
	1897–1901.	1902–6.	1897–1901.	1902–6.	1897–1901.	1902–6.	1897–1901.	1902–6.	1897–1901	1902–6.
I(a)	160,111	169,789	3,220	3,692	1,343	1,507	1,766	2,029	111	156
I(b)	107,818	112,271	1,730	1,982	905	962	721	891	104	129
II	249,190	248,671	5,646	6,410	3,094	3,349	2,243	2,747	309	314
III	174,420	157,138	4,863	5,585	3,495	4,055	1,227	1,369	141	161
IV	647,695	732,101	9,545	12,038	4,988	6,150	3,908	5,120	649	768
V	351,600	425,880	5,865	7,429	3,907	5,031	1,878	2,316	80	82
VI	471,154	514,364	12,346	15,487	8,189	10,306	3,982	4,989	175	192
VII	114,805	134,167	1,304	1,628	761	940	477	608	66	80
VIII	88,088	115,944	1,302	1,818	816	1,112	438	638	48	68
IX	1,308,686	1,467,875	6,701	8,190	3,785	4,517	2,768	3,494	148	179
X	286,413	325,791	3,170	4,188	1,905	2,401	1,145	1,657	120	130
XI	284,369	339,872	8,235	11,594	4,648	6,453	3,251	4,754	336	387
XII	716,766	763,997	7,596	9,786	4,556	5,723	2,680	3,682	360	381
XIII	134,247	182,464	470	676	258	375	206	290	6	11
XIV	935,107	1,134,409	22,682	27,949	12,433	14,941	9,035	11,577	1,214	1,431
XV	133,626	186,862	594	944	375	590	215	345	4	9
Total	6,164,095	7,011,595	95,269	119,396	55,458	68,412	35,940	46,506	3,871	4,478

TABLE E.

Group.	Accidents Resulting in													Total Accidents.
	Temporary Disability.		Permanent Disability.									Death.		
	Number.	Percentage of all Accidents.	Resulting in the Following Loss of Earning Power.							Total.	Percentage of all Accidents.	Number.	Percentage of all Accidents.	
			8 Percent and Under.	9 to 18 Percent.	19 to 32 Percent.	33 to 48 Percent.	49 to 65 Percent.	66 to 83 Percent.	Complete Loss.					
Under 16 years	2,463	67.8	130	355	200	151	86	152	22	1,096	30.2	74	2.0	3,633
16 to 20 years	8,806	67.4	457	1,362	758	551	291	443	76	3,938	30.2	318	2.4	13,062
21 to 30 years	14,741	62.1	1,017	2,712	1,592	1,237	683	778	179	8,198	34.6	784	3.3	23,723
31 to 40 years	12,415	56.8	871	2,846	1,648	1,235	766	885	241	8,492	38.8	953	4.4	21,860
41 to 50 years	8,901	52.0	676	2,399	1,405	1,164	734	779	218	7,375	43.0	852	5.0	17,128
51 to 60 years	5,213	47.4	359	1,466	1,085	890	550	639	207	5,196	47.2	591	5.4	11,000
Over 60 years	1,948	40.1	119	567	438	532	377	422	161	2,616	53.8	299	6.1	4,863
Group I(a)	1,307	40.6	55	389	254	224	177	576	127	1,802	56.0	111	3.4	3,220
" I(b)	879	50.8	53	213	138	114	91	115	23	747	43.2	104	6.0	1,730
" II	3,016	53.4	293	801	435	316	198	215	63	2,321	41.1	309	5.5	5,646
" III	3,466	71.3	129	397	239	216	117	117	41	1,256	25.8	141	2.9	4,863
" IV	4,885	51.2	203	1,066	835	751	492	496	168	4,011	42.0	649	6.8	9,545
" V	3,870	66.0	327	706	356	267	128	101	30	1,915	32.6	80	1.4	5,865
" VI	8,036	65.1	726	1,564	735	581	225	231	73	4,135	33.5	175	1.4	12,346
" VII	757	78.0	45	149	105	59	55	49	19	481	36.9	66	5.1	1,304
" VIII	806	61.9	49	158	80	71	35	46	9	448	34.4	48	3.7	1,302
" IX	3,728	55.6	241	978	610	359	235	329	73	2,825	42.2	148	2.2	6,701
" X	1,884	59.4	117	371	226	177	101	144	29	1,165	36.8	120	3.8	3,169
" XI	4,600	55.8	333	1,001	637	549	375	353	52	3,300	40.1	336	4.1	8,236
" XII	4,477	59.0	259	964	553	377	245	284	77	2,759	36.3	360	4.7	7,596
" XIII	249	53.0	28	85	30	19	14	34	5	215	45.7	6	1.3	470
" XIV	12,155	53.6	739	2,781	1,853	1,656	979	990	315	9,313	41.1	1,214	5.3	22,682
" XV	372	62.6	32	84	40	24	20	18	——	218	36.7	4	.7	594
Total	54,487	57.2	3,629	11,707	7,126	5,760	3,487	4,098	1,104	36,911	38.7	3,871	4.1	95,269
Males	50,530	57.6	3,358	10,704	6,464	5,282	3,157	3,623	980	33,568	38.2	3,712	4.2	87,810
Females	3,957	53.1	271	1,003	662	478	330	475	124	3,343	44.8	159	2.1	7,459

TABLE E (*continued*).

Group.	Distribution of Accidents Resulting in Permanent Disability.							
	Resulting in the Following Loss of Earning Power (Percentage of Total).							Total.
	8 Per-cent and Under.	9 to 18 Per-cent.	19 to 32 Per-cent.	33 to 48 Per-cent.	49 to 65 Per-cent.	66 to 83 Per-cent.	Com-plete Loss.	
Under 16 years	11.9	32.4	18.2	13.8	7.8	13.9	2.0	100.0
16 to 20 years	11.6	34.6	19.2	14.0	7.4	11.3	1.9	100.0
21 to 30 years	12.4	33.1	19.4	15.1	8.3	9.5	2.2	100.0
31 to 40 years	10.3	33.5	19.4	14.6	9.0	10.4	2.8	100.0
41 to 50 years	9.2	32.5	19.0	15.8	9.9	10.6	3.0	100.0
51 to 60 years	6.9	28.2	20.9	17.1	10.6	12.3	4.0	100.0
Over 60 years	4.6	21.7	16.7	20.3	14.4	16.1	6.2	100.0
Group I(a)	3.1	21.6	14.1	12.4	9.8	32.0	7.0	100.0
" I(b)	7.1	28.5	18.5	15.2	12.2	15.4	3.1	100.0
" II	12.6	34.5	18.8	13.6	8.5	9.3	2.7	100.0
" III	10.3	31.6	19.0	17.2	9.3	9.3	3.3	100.0
" IV	5.0	26.6	20.8	18.7	12.3	12.4	4.2	100.0
" V	17.1	36.9	18.6	13.9	6.7	5.3	1.5	100.0
" VI	17.8	37.6	17.8	14.0	5.4	5.6	1.8	100.0
" VII	9.4	31.0	21.8	12.3	11.4	10.2	3.9	100.0
" VIII	10.9	35.3	17.9	15.8	7.8	10.3	2.0	100.0
" IX	8.5	34.6	21.6	12.7	8.3	11.7	2.6	100.0
" X	10.0	31.8	19.4	15.2	8.7	12.4	2.5	100.0
" XI	10.1	30.3	19.3	16.6	11.4	10.7	1.6	100.0
" XII	9.4	34.9	20.0	13.7	8.9	10.3	2.8	100.0
" XIII	13.0	39.5	14.0	8.9	6.5	15.8	2.3	100.0
" XIV	7.9	29.9	19.9	17.8	10.5	10.6	3.4	100.0
" XV	14.7	38.5	18.4	11.0	9.2	8.2	—	100.0
Average	9.8	31.7	19.3	15.6	9.5	11.1	3.0	100.0

The accidents to employees of the state railroads are not included in Group II on account of the special organization of accident insurance for these workers.

Table E gives accidents compensated and rate of compensation for the different groups of industries, age groups and sex, for the five-year period 1897 to 1901. The division of accidents resulting in temporary and permanent disability differs somewhat from above, but the sum of the two agrees. No explanation as to the difference was given.

In this table is shown the influence of age in exposing the workers to the various degrees of risk. It also furnishes some indication of the influence of age upon the workers' ability to recover from accident. Referring to the cases of total disability, if the accidents causing loss of earning power of 49 per cent. and over are considered, it is seen that persons of a very young age suffer more

severely than those of greater age, and that after age 20 the proportion sustaining a loss of earning power of 49 per cent. or over increases with age. It is also shown that there is a decrease in the proportion of cases resulting in temporary disability as the age of the injured person increases. In cases of fatality, as in those resulting in permanent disability, the tendency is to increase with age. When the accidents resulting in permanent disability are distributed according to degree of disability, the lower degrees of disability occur most frequently among the younger age groups, and, as the workers grow older, permanent disablements tend to cause greater loss of earning power.

When comparison is made between male and female workers, it is found that the tendency is for accidents to the latter to result more frequently in permanent disability than in the case of the former. The opposite is found to hold in the case of accidents resulting in temporary disability. Females suffer a smaller proportion of fatal accidents due no doubt to the fact that they are not exposed to the highly dangerous occupations.

The data for the various industry groups show the same general tendencies as the totals.

Table F shows the summarized results of the injuries to workmen included in the insurance system in Austria for the years 1897–1901.

For each injury group the accidents resulting in permanent disability are shown according to the percentage of loss of earning power, and underneath the actual numbers are shown the percentages of the total number of such accidents.

BENEFITS.

The acts of nearly all the countries are framed with the view of obviating the necessity of instituting legal proceedings. If disputes arise, the acts specify the necessary procedure for settlement by special arbitration tribunals or by ordinary law courts. The laws in every case fix the compensation to be paid, and with but one or two unimportant exceptions this compensation is based upon either a wage rate or the actual wages received by the injured persons. It consists of allowances for temporary disability and annual pensions or lump sum payments for permanent disability or death, to which are added frequently the expenses of medical and surgical treatment and a funeral benefit.

TABLE F.

Result of Injury.	Accidents Resulting in Temporary Disability.	
	Number.	Percentage of all Accidents.
All injuries of the arm, right or left	3,486	53.1
Injury of right and left arm at same time	197	60.7
All injuries of the hand, right or left	4,158	68.7
All injuries of fingers, right and left hand	19,162	61.4
Injuries of fingers of both hands occurring at the same time	100	64.5
Loss of or injury to legs and feet, including toes	16,938	66.6
Loss of or injury to arm and leg in various combinations	371	64.0
All injuries of eyes	1,240	22.9
Injuries of the head	1,443	42.9
Injury of shoulder including those accompanied by injury of arm	690	45.6
Fractures of collar bone including those accompanied by injury of arm	369	49.7
Fractures of ribs	806	57.0
Injuries of trunk	2,518	57.8
Injuries of testicles	154	62.6
Ruptures	77	12.5
Injuries of several parts of the body	2,360	52.0
Internal injuries	190	16.4
Concussion of the brain	162	32.3
Miscellaneous (stroke, hemorrhage, blood poisoning, sun stroke, etc.)	67	12.7
Traumatic neurosis following injuries	19	9.3
Suffocation		
Drowning		

TABLE F (*continued*).

Accidents Resulting in Permanent Disability. Resulting in the Following Loss of Earning Power. 8 Percent and Under.	9 to 18 Percent.	19 to 32 Percent.	33 to 48 Percent.	49 to 65 Percent.	66 to 83 Percent.	Complete Loss.	Total.	Percentage of all Accidents.	Death. Number.	Death. Percentage of all Accidents.	Total Accidents.
152	809	593	445	297	637	104	3,037	46.3	40	0.6	6,563
5.0	26.6	19.5	14.7	9.8	21.0	3.4	100.0				
2	27	17	17	19	23	18	123	37.8	5	1.5	325
1.6	22.0	13.8	13.8	15.5	18.7	14.6	100.0				
163	540	314	192	170	395	92	1,866	30.8	31	0.5	6,055
8.8	28.9	16.8	10.3	9.1	21.2	4.9	100.0				
2,173	4,821	2,217	1,172	809	801	20	12,013	38.5	28	0.1	31,203
18.1	40.2	18.4	9.7	6.7	6.7	0.2	100.0				
6	15	7	6	11	7	3	55	35.5	—	——	155
10.9	27.3	12.7	10.9	20.0	12.7	5.5	100.0				
648	2,229	1,572	1,286	959	1,238	298	8,230	32.4	264	1.0	25,432
7.9	27.1	19.1	15.6	11.7	15.0	3.6	100.0				
8	39	32	32	32	36	19	198	34.1	11	1.9	580
4.0	19.7	16.2	16.1	16.2	18.2	9.6	100.0				
144	899	1,023	1,463	358	169	115	4,171	77.1	1	——	5,412
3.4	21.6	24.5	35.1	8.6	4.0	2.8	100.0				
87	401	250	178	119	94	60	1,189	35.3	733	21.8	3,365
7.3	33.7	21.0	15.0	10.0	7.9	5.1	100.0				
39	252	183	174	99	67	5	819	54.1	5	0.3	1,514
4.8	30.8	22.3	21.2	12.1	8.2	0.6	100.0				
13	151	86	62	30	26	4	372	50.2	1	0.1	742
3.5	40.6	23.1	16.6	8.1	7.0	1.1	100.0				
27	184	92	68	39	38	12	460	32.5	149	10.5	1,415
5.9	40.0	20.0	14.8	8.5	8.2	2.6	100.0				
85	473	271	222	197	174	80	1,502	34.5	335	7.7	4,355
5.7	31.5	18.0	14.8	13.1	11.6	5.3	100.0				
5	45	13	6	8	1	1	79	32.1	13	5.3	246
6.3	57.0	16.4	7.6	10.1	1.3	1.3	100.0				
6	410	47	43	10	1	3	520	84.7	17	2.8	614
1.2	78.8	9.0	8.3	1.9	0.2	0.6	100.0				
51	312	287	268	195	253	141	1,507	33.2	670	14.8	4,537
3.4	20.7	19.0	17.8	12.9	16.8	9.4	100.0				
8	42	37	37	38	38	32	232	20.1	733	63.5	1,155
3.4	18.0	16.0	16.0	16.4	16.4	13.8	100.0				
1	28	39	35	23	32	28	186	37.0	154	30.7	502
0.5	15.0	21.0	18.8	12.4	17.2	15.1	100.0				
4	17	26	33	40	28	36	184	34.9	276	52.4	527
2.2	9.3	14.1	17.9	21.8	15.2	19.5	100.0				
1	12	22	24	33	42	39	173	84.8	12	5.9	204
0.6	6.9	12.7	13.9	19.1	24.3	22.5	100.0				
									219	100.0	
									166	100.0	

GERMANY.

For thirteen weeks medical and surgical treatment are provided by the sick benefit funds, also benefit payments from the third day. To these funds the employers contribute one third and the employees the other two thirds. During the fifth to the thirteenth weeks, inclusive, the payments are increased by one third at the expense of the employer in whose establishment the accident occurred. After thirteen weeks, as also in case of death from injuries received, the entire expense of the compensation is borne by the employers' associations supported by the contributions of the employers.

The compensation allowable is as follows:

In case of total disability, temporary or permanent, 50 per cent. of daily wages of persons similarly employed, but not exceeding three marks, is paid by the sick benefit funds from the third day to the end of the fourth week, and during the rest of the thirteen weeks an additional $16\frac{2}{3}$ per cent. of said wages is paid, being a direct contribution from the employer. After thirteen weeks $66\frac{2}{3}$ per cent. of average annual earnings of the injured workman is paid by the employers' associations, and, in case of complete helplessness necessitating attendance, this may be increased to 100 per cent. of said earnings.

In case of partial disability the percentage is reduced.

In case of death a funeral benefit of one fifteenth of the annual earnings of deceased but not less than 50 marks is payable, and to dependent heirs pensions not exceeding 60 per cent. of annual earnings of deceased. If the widow remarries she draws a final sum equal to three annual payments.

If the annual earnings exceed 1,500 marks only one third of excess is considered in computing pensions.

AUSTRIA.

For the period of twenty weeks medical and surgical treatment is paid by the sick funds, also for four weeks compensation for disability. The cost of this is borne one third by the employers and two thirds by the employees.

After four weeks the compensation is paid by the Territorial Insurance Associations, also the death benefits. The employers contribute 90 per cent. and the employees 10 per cent.

Compensations are payable as follows:

In case of total disability, temporary or permanent, 60 per cent. of average daily wages of insured workmen in the locality is paid by the sick benefit funds during the first four weeks, and thereafter the territorial accident insurance institutions pay 60 per cent. of the average annual earnings of the injured person.

In case of partial disability, up to but not exceeding 50 per cent. of said wages or earnings is paid.

In case of death, funeral expenses not to exceed 25 florins are paid, and pensions to members of family of deceased not to exceed 50 per cent. of earnings. If widow remarries she is paid a lump sum equal to three annual payments.

If earnings exceed 1,200 florins said excess is ignored.

ITALY.

In Italy the entire cost of the compensation rests upon the employer, including cost of first medical and surgical treatment.

In case of total permanent disability, the amount of compensation paid is six times the annual earnings, but not less than 3,000 lire. In case of partial permanent disability the compensation is six times the loss of annual earning capacity, based on the assumption of a minimum earning capacity of 500 lire per annum.

If the injured workman is only temporarily disabled the compensation is 50 per cent. of wages or of reduction in wages for a period not exceeding three months.

In the event of death from the accident within two years, five times the annual wages of the deceased, not exceeding 10,000 lire, is payable to heirs, or, if there are no heirs, then the amount is turned into a special fund.

GREAT BRITAIN.

In Great Britain all injuries caused by accident arising out of and in the course of the employment, which cause death or disable a workman for at least one week from earning full wages at the work at which he was employed, are compensated, and the entire cost rests upon the employer.

If disabled the employee receives compensation during incapacity of not more than 50 per cent. of his average weekly earnings during the previous twelve months, the maximum being a pound a week. No payment is allowed for the first week if incapacity lasts less than two weeks.

If only partially disabled, the compensation does not exceed the difference between the average weekly earnings before injury and the average amount which he is earning or is able to earn after injury. In the case of minors full earnings not exceeding 10 shillings a week are allowed. The employer may, if incapacity continues beyond six months, be relieved of further liability on payment of a sum sufficient to purchase a life annuity through the Post Office Savings Bank of 75 per cent. of the annual value of the weekly payments.

In the event of death, the compensation payable to those entirely dependent on the earnings of the deceased is the amount of three years' earnings, but not less than £150 nor more than £300. A less amount to be agreed upon by the parties, or fixed by arbitration, is payable when there are only partial dependents. If there are no dependents, a funeral benefit not exceeding £10 is payable.

FRANCE.

In France, workmen or salaried employees are compensated for injuries resulting from accident, unless produced intentionally by the victim, during or on account of labor, if death ensues or disability lasts at least five days. The employer is liable for the entire cost of the compensation paid during disability including expenses of medical or surgical treatment. In the event of total permanent disability 66⅔ per cent. of annual wages is payable, but this may be increased or decreased by order of court if the accident is due to the inexcusable fault of employer or employee, the maximum amount payable, however, being the actual earnings.

If only partially disabled, one half the loss of earning capacity is paid to the injured workman. If demanded, one fourth of the purchase price of the annuity is payable in cash, the annuity being reduced accordingly.

In the event of temporary disablement, the allowance is 50 per cent. of the daily wages, beginning with the fifth day and including Sundays and holidays. If the disablement lasts more than ten days, payments are due from the first day. If the annual wages exceed 2,400 francs, only one fourth of the excess is considered in computing pensions. If beneficiary is of age, by mutual consent payments of pensions of not over 100 francs per annum may be replaced by a cash payment.

In the event of death, funeral expenses not exceeding 100 francs

are payable, and to dependent heirs pensions not exceeding 60 per cent. of annual wages of deceased. If the widow remarries, a final payment equal to three annual payments is made.

In all cases, provision is made for revision of compensations granted.

One of the most important points to be considered in dealing with the subject of compensation for injury is that concerning the method of treatment of cases in the first few weeks of illness. There is a great deal of diversity found to exist in Europe, and each country seems to have considered the subject on *a priori* grounds. The tendency is, however, as a result of several years' experience, to adopt more uniform treatment in all of the countries, and on account of its practical value as a guide to legislation in this country it seems well to examine rather carefully the prevalent systems.

In Germany and Austria it has been found advantageous to consider the relief granted during the earlier stages as sickness benefits, such being quite distinct from the regular system of compensation. By so doing there is speedy relief, and, moreover, an opportunity is afforded to consider carefully the applications for benefits. The greatest evil to be combatted is malingering, and there is thus ample scope for its detection before the compensation is awarded.

In Germany during the first thirteen weeks of disability the injured workman is given medical treatment and pecuniary relief from the sickness insurance associations. In Austria it is provided that the injured workman shall be cared for during the first four weeks only. An interesting point may be raised as to the desirability of such a waiting period and whether it should be short or fairly long. As a matter of fact the accidents causing disability for less than thirteen weeks form a large proportion of the total number, and under the Austrian system involve costs of administration which make an unduly large percentage of the total expenses of administration.

It is noteworthy that Austria proposes to follow Germany by extending the period of waiting to thirteen weeks, the four weeks' period having been found unsatisfactory. If this is done, the employees are to be relieved of their 10 per cent. contributions.

In Germany and Austria, with state-controlled institutions, difficulties which would confront one in this country are not met with in connection with the two-fold system of sickness and accident benefits. In the case of Germany, and the same might be postu-

lated of Austria, from the practical point of view, the transfer of the multitude of minor injuries caused by modern industrial methods from the accident associations to the sickness insurance organizations was almost imperative. Accident insurance requires the use of organizations with large numbers of members in order to properly distribute the cost of such an expensive type of insurance, but at the same time large organizations are at a distinct disadvantage in dealing with cases of disability of short duration. Now, as regards the period of waiting, at one time in Germany this was reduced to four weeks but subsequently was restored to thirteen weeks, as statistics of the duration of cases of injury showed that the average annual cost to the sick funds was but slightly higher if the waiting time was placed at thirteen weeks instead of four weeks. The relative share of accident relief borne

TABLE G.

Year.	Funeral Benefits.	Pensions to Survivors.			Pensions to Injured Persons.	Settlements to Widows on Remarriage and Commutation Payments.	Total.
		Widows.	Children.	Parents, etc.			
	$	$	$	$	$	$	$
1892	3,885	21,441	24,993	2,835	298,299	8,893	360,346
3	4,694	31,253	37,899	3,952	417,000	11,996	506,794
4	5,349	41,241	48,925	4,857	570,867	15,388	686,627
5	6,608	55,572	65,184	6,292	759,539	24,084	917,279
6	8,092	75,070	84,651	8,606	1,064,901	38,968	1,280,288
7	8,053	94,151	105,454	9,989	1,353,076	26,618	1,597,341
8	8,118	118,708	129,171	11,447	1,630,368	48,141	1,945,953
9	8,925	141,527	155,580	13,973	1,936,004	47,549	2,303,558
1900	8,995	169,107	180,714	15,201	2,274,191	79,424	2,727,632
1	8,618	192,174	199,363	16,120	2,588,118	78,106	3,082,499
2	8,785	218,811	222,059	18,267	2,921,335	103,047	3,492,304
3	8,803	243,608	238,809	19,863	3,198,535	93,070	3,802,688
4	9,850	271,802	259,100	21,244	3,491,592	106,791	4,160,379
5	10,146	298,511	275,917	23,726	3,840,656	120,129	4,569,085
6	10,552	332,241	300,281	24,029	4,172,246	108,771	4,948,120
7	11,324	368,350	328,274	25,304	4,624,822	102,527	5,460,601

by the sick funds and by the accident associations in Germany is shown by the duration of disability of the accident cases. From 1886 to 1895 the proportion of all accidents causing disability of thirteen weeks or less was 84 per cent. These cases required but 16½ per cent. of the total cost for accident insurance, which means that the insured workman who pays two thirds of the expense of

the sick funds provides approximately 11 per cent. of the cost of the accident insurance, the balance being provided by the employer.

The pecuniary benefits paid to persons insured in Austrian Insurance Institutions for the years 1892 to 1907 are shown in Table G.

The number of pensioners, the average amount of the pensions granted, and the relation which this amount bears to the earnings of the injured persons, are shown in Table H for the five years 1903 to 1907.

TABLE H.

Class of Pensions.	1903.			1904.		
	Number of Pensioners.	Average Annual Amount of Pension.	Average Annual Amount of Pension in Percentage of Earnings	Number of Pensioners.	Average Annual Amount of Pension.	Average Annual Amount of Pension in Percentage of Earnings
Widows.....	745	41.48	20.39	835	41.70	20.77
Children	1,412	26.22	12.85	1,701	25.10	12.83
Parents, etc...	89	26.19	17.81	93	30.51	20.08
Injured persons	7,746	41.41	22.36	8,407	38.64	20.82
Total.....	9,992	39.13	20.72	11,036	36.72	19.53

Clas of Pensions.	1905.			1906.			1907.		
	Number of Pensioners.	Average Annual Amount of Pension.	Average Annual Amount of Pension in Percentage of Earnings.	Number of Pensioners.	Average Annual Amount of Pension.	Average Annual Amount of Pension in Percentage of Earnings.	Number of Pensioners.	Average Annual Amount of Pension.	Average Annual Amount of Pension in Percentage of Earnings.
Widows	857	44.20	21.18	899	47.52	21.41	981	47.17	21.66
Children....	1,590	26.45	13.18	1,839	29.48	13.36	1,860	29.91	13.58
Parents, etc.	87	34.06	21.23	76	31.26	18.40	84	34.79	17.36
Injured persons...	9,449	41.87	22.80	10,010	39.74	21.75	10,722	39.32	21.06
Total	11,983	39.94	21.29	12,824	38.76	20.32	13,647	38.59	19.93

An interesting study is presented by Table I for the two five-year periods 1897 to 1901 and 1902 to 1906. The data consist of wages paid, insurance contributions and cost of accidents for the industry groups already considered.

The last column of Table I giving the net cost to the institutions of the accidents as a percentage of the wages of the injured persons is very important. Group I (a) refers to workers in agricultural establishments exposed to the risk of machinery. Here, rate of cost is higher than in any other group, being more than double that of

TABLE I.

Industry Group.	1897-1901.				1902-1906.			
	Wages of Insured Persons.	Insurance Contributions.	Net Cost of Accidents.		Wages of Insured Persons.	Insurance Contributions.	Net Cost of Accidents.	
			Amount.	Percent of Wages.			Amount.	Percent of Wages.
	$	$	$		$	$	$	
Group I(a)	10,099,412	496,431	645,720	6.39	11,317,250	562,603	719,939	6.36
" I(b)	12,901,909	274,025	358,492	2.78	14,403,845	325,454	420,238	2.92
" II	39,916,863	998,205	1,251,516	3.14	40,789,946	1,251,828	1,420,172	3.48
" III	36,323,906	872,038	892,488	2.46	35,681,473	964,517	980,614	2.75
" IV	88,116,292	1,526,581	1,896,154	2.15	107,270,092	1,989,439	2,374,681	2.21
" V	63,443,326	810,753	896,835	1.41	78,951,004	1,044,148	1,024,164	1.30
" VI	105,128,869	2,093,862	2,200,426	2.09	118,736,933	2,501,163	2,536,551	2.14
" VII	16,923,947	262,441	273,645	1.62	21,700,964	343,112	322,833	1.49
" VIII	16,164,241	207,853	256,490	1.59	21,738,904	330,151	358,448	1.65
" IX	151,627,683	1,051,327	1,007,273	.66	182,037,104	1,265,076	1,267,689	.70
" X	40,847,437	524,942	541,379	1.33	50,058,521	673,972	742,079	1.48
" XI	38,266,515	1,097,912	1,412,441	3.70	49,758,629	1,577,343	2,009,171	4.04
" XII	94,105,806	1,364,793	1,391,512	1.48	112,236,671	1,717,120	1,704,527	1.52
" XIII	19,688,544	108,064	93,720	.48	27,858,786	154,076	144,119	.52
" XIV	139,353,451	3,941,851	4,528,621	3.25	184,274,224	5,737,473	5,748,529	3.12
" XV	30,488,956	157,241	112,068	.37	44,153,596	201,256	177,219	.40
Total	903,397,157	15,788,319	17,758,780	1.97	1,100,967,942	20,638,731	21,950,973	1.99

Group I (b) referring to flour mills' employees. Besides Group I (a), three groups, II, transportation and storage, XI, woodworking, carved materials, etc., and XIV, building and construction, have an average cost exceeding three per cent. of the wages for the years 1897–1901 and also 1902–1906. In both cases the highest percentage is given in Group XI. Three other industries have an average cost of from two to three per cent. of the wages, groups III, IV and VI. In all six groups, there is a large trade risk.

PREMIUM RATES AND RESERVES.

In contrast to the scientific accuracy of life assurance premiums, the premiums for workmen's compensation benefits are merely such average rates as it is hoped will be sufficient. These consist of assessments which are required to cover either the capitalized value of the current year's pensions or the year's expenditure. In Austria a standard or basic rate applicable to the establishment having the highest risk rating is used, a percentage of this basic rate being used for other establishments. To obtain the basic rate a preliminary investigation of the accidents in the establishments included in the insurance scheme was made, and it is important to note that the rate was increased at the end of the five-year period. Subsequent increases have been made, but it is stated that these have not been as large as the situation required, owing to the opposition of the employers, and that they have not been sufficient to offset the growth of the deficit. The experience of the Austrian system gives emphasis to the argument that there is a decided tendency to charge too little at the outset and that steps taken to increase the rates are decidedly unpopular. This shows the necessity of charging adequate premiums at the outset, especially if reserves are to be accumulated. Further, as the premium is merely an assessment, no account is taken of the ages of the assured workmen, and, consequently, the younger men pay more than their share, the older men less than their share of the cost. Whether or when an attempt will be made to secure adequate data on which may be based graded premiums it is not easy to predict. Against the cost and trouble of such a step is urged the frequency with which men change from one establishment to another. The remedy is in some such system as that in force in Italy, whereby an employee has his own policy and may take it with him to his new position.

GOVERNMENT SUPERVISION AND PREVENTION OF ACCIDENTS.

One of the most striking of the conditions under which insurance of workmen against injuries suffered in the course of their employment may be conducted, exhibits it in strong contrast to life insurance. The latter from an obscure position as a social and economic factor has been developed into almost the most important in modern civilization, and, where there is the greatest freedom, there the moral caliber of the management is greatest. In opposition to this great principle, almost universally there seems to be a demand for the utmost degree of supervision over the insurance of workmen, as if the lack of restriction would entail a lack of good faith on the part of the insurers. The difficulty has arisen perhaps because of the lack of clear ideas upon the economic question involved, viz., who in the last analysis is to bear the cost of the insurance, and how is that cost to be assessed. Mr. Miles M. Dawson recently in an admirable paper delivered before the American Academy of Political Science argued that insurance to provide compensation to injured workmen must either be conducted by the state or else according to rates and rules laid down by the state after consultation with underwriters. He also affirmed that the cost of this insurance, if conducted by the state, must be levied as a tax upon all the people.

Against this we may say that it introduces a new idea as to the basis of the compensation and one which will not generally be acceptable.

By assuming, on the other hand, that the employer is bound to manifest the utmost good faith towards his employees and that he is responsible for injuries resulting from accident in the course of the employment, it is also assumed that the cost of these may be levied as part of the cost of production. What is the result? The employer finds it to his advantage to pay a regular premium to an insurance company, the company finds it to its advantage to urge that a maximum of care be taken to prevent accidents occurring, which the employer recognizes in the reduced premiums, if not compelled by law to maintain a standard of efficiency.

Life insurance, as much if not more than any other institution, is teaching and compelling people to take care of themselves so as to continue select lives, and may we not anticipate that workmen's compensation insurance will have a like wholesome effect upon employers and employees? Private insurance companies can be

conducted with a minimum of waste giving a maximum benefit. The premiums charged must be adequate to allow of reserves being accumulated, any other system entailing needless waste in future years. This is concealed in a system of taxation but exists nevertheless. The managements of life companies have recognized their duty to society and have willingly met the obligation. Has any one the right to say that the managements of workmen's compensation insurance companies will be less zealous and upright? The tendency towards more extensive government control exists, however, and, even in Great Britain, is in decided contrast to that governing life assurance.

CONCLUSION.

It is a disappointment to the writer that the present paper adds so little to our actuarial knowledge of the subject. There are few subjects which offer so many points of attack, and it is possible that scientific exactitude can only be attained after each point has been considered thoroughly.

Gill's Mortality Table.

BY

S. A. JOFFE.

Since the publication of my paper "Concerning the American Experience Table of Mortality" in the *Transactions* (XII, 253–260) I have been urged by several members of the Society to give greater details of Gill's Mortality Table, regarding which so little has been published.

Its author, Professor Charles Gill (1805–1855), was considered a mathematician of the first rank both in this country and in England. The "Historical Sketch" of his life, published in the "Journal of the Institute of Actuaries" (VI, 216–227), contains among other testimonials of his mathematical achievements one from the celebrated American mathematician, Professor Benjamin Peirce, and one from the eminent Woolwich Professor, T. S. Davies, F.R.S., Auditor of the Gresham Insurance Society. The first addressed Gill as follows: "I am able to say, with entire confidence, that you have not your superior, as a mathematician, in the United States, either in powers of analysis or in elegance of solution." The second wrote regarding Gill in the following terms: "Did any difficulty occur in my own researches that I deemed it necessary to refer to another mathematician, there are few men living to whom I should refer it with the same confidence of receiving efficient aid, in a full and original form, as Mr. Gill. In England he is considered to be the first of the mathematicians of America."

There is no question that a table constructed by a mathematician of so high a standing deserves serious consideration. Furthermore, the table in question has a considerable historical value, owing to the fact that the premium rates computed by Gill from his table were in use by The Mutual for fifteen years (1853–1868), and for certain periods by several smaller companies, as for instance The Globe Mutual, The United States Life, etc.

Notwithstanding its importance, however, it does not appear that any writer who referred to this table ever saw it or had first-hand information regarding it.* We find, for instance, a refer-

* Excepting of course Homans.

ence to Gill's Table in the well known paper "American Tables of Mortality" of Professor McCay, published in the "Spectator" for July, 1870, pp. 12–16, and reprinted in the "Journal of the Institute of Actuaries" for October, 1870 (XVI, 20–33), where he says: "Mr. Gill constructed an average table from the Actuaries', the Swedish, and other good tables." Another authority, Cornelius Walford, in his "Insurance Cyclopædia," under the article "American Tables of Mortality," writes: "The way he (Gill) accomplished his task was this: He took the Equitable Experience, the Swedish, and such other good tables as he was familiar with, and obtained an average of their results." In view of this scant and misleading information, and especially as this historical table has never before appeared in print, it would seem advisable to reproduce it in the Society's *Transactions* in full, as it appears in the manuscript volume "Assurance Tables," calculated by Gill (see page 86).

From the sequence of the columns in Gill's Table one would conclude that the values of $1/(1-p_x)$ were computed as reciprocals of $(1-p_x)$ in all instances; but this does not hold for the ages which are multiples of 5 between 10 and 70 inclusive; for those 13 ages the values of $1/(1-p_x)$ were directly read off from the curve, and then their reciprocals were taken as the values of $(1-p_x)$ as follows:

Age.	Read off from Curve. $1/q_x$	Reciprocals of Preceding. q_x
10	190.0	.005263
15	162.5	.006154
20	140.8	.007102
25	122.2	.008183
30	106.1	.009425
35	91.8	.010893
40	79.1	.012642
45	67.2	.014881
50	54.0	.018518
55	42.0	.023809
60	31.3	.031949
65	22.1	.045249
70	15.4	.064935

The values of q_x for the intermediate 48 ages were, as Gill informs us in the introduction* to his volume, interpolated by the method of differences. In attempting to verify Gill's figures I

* Cf. T. A. S. A., XII, 256.

experimented with various sets of ages, and of all the tests the most satisfactory results were obtained by using the following sets:

(1) For ages 10 to 14, the set of ages: 10, 15, 20, 25, 30 and 35;
(2) For ages 15 to 19, the set of ages: 15, 20, 25, 30, 35 and 40;
(3) For ages 20 to 44, the set of ages: 20, 25, 30, 35, 40 and 45;
(4) For ages 45 to 69, the set of ages: 45, 50, 55, 60, 65 and 70.

The method of interpolation was in all instances the same uniform method by successive addition of the leading differences of $10^6 q_x$ for unit intervals, namely: if we denote the leading differences for the 5-unit intervals by $\Delta, \Delta^2, \Delta^3, \cdots$, and the leading differences for 1-unit intervals by $\delta, \delta^2, \delta^3, \cdots$, then the δ's are expressed in terms of the Δ's by the well known formulas:

$$\delta = .2\Delta - .08\Delta^2 + .048\Delta^3 - .0336\Delta^4 + .025536\Delta^5$$
$$\delta^2 = .04\Delta^2 - .032\Delta^3 + .0256\Delta^4 - .02112\Delta^5$$
$$\delta^3 = .008\Delta^3 - .0096\Delta^4 + .0096\Delta^5$$
$$\delta^4 = .0016\Delta^4 - .00256\Delta^5$$
$$\delta^5 = .00032\Delta^5$$

In the following tables I have given the principal results for the ages 10 to 70, embodying the leading differences for the several intervals and the values of $10^6 q_x$, as well as the discrepancies (to one decimal) between Gill's figures and mine.

LEADING DIFFERENCES FOR 5-UNIT INTERVAL.

Ages.	Δ	Δ^2	Δ^3	Δ^4	Δ^5
10–14	891	57	76	—48	85
15–19	948	133	28	37	—47
20–44	1,081	161	65	—10	164
45–69	3,637	1,654	1,195	1,116	—2,201

LEADING DIFFERENCES FOR 1-UNIT INTERVAL.

Ages.	δ	δ^2	δ^3	δ^4	δ^5
10–14	181.07136	—3.176	1.8848	—.2944	.0272
15–19	177.860608	6.36384	—.5824	.17952	—.01504
20–44	210.963904	.64032	2.1904	—.43584	.05248
45–69	558.737664	102.97472	—22.2832	7.42016	—.70432

VALUES OF $10^6 q_x$ FOR AGES 10 TO 14.

(1) Age.	(2) Joffe's Value.	(3) Gill's Value.	(4) (2) — (3) to 1 decimal.
10	5,263.	5,263	0.
11	5,444.07136	5,444	0.
12	5,621.96672	5,622	0.
13	5,798.57088	5,798	.5
14	5,975.47424	5,975	.4

VALUES OF $10^6 q_x$ FOR AGES 15 TO 19.

(1) Age.	(2) Joffe's Value.	(3) Gill's Value.	(4) (2) — (3) to 1 decimal.
15	6,154.	6,154	0.
16	6,331.860608	6,334	— 2.1
17	6,516.085056	6,516	0.
18	6,706.090944	6,706	0.
19	6,901.475392	6,902	— .5

VALUES OF $10^6 q_x$ FOR AGES 20 TO 44.

(1) Age.	(2) Joffe's Value.	(3) Gill's Value.	(4) (2)-(3) to 1 decimal.
20	7,102.	7,102	0.
21	7,312.963904	7,311	1.9
22	7,524.568128	7,525	— .4
23	7,739.003072	7,739	0.
24	7,958.023296	7,958	0.
25	8,183.	8,183	0.
26	8,414.973504	8,413	1.9
27	8,654.705728	8,652	2.7
28	8,902.732672	8,902	.7
29	9,159.416896	9,158	1.4
30	9,425.	9,425	0.
31	9,699.655104	9,699	.6
32	9,983.539328	9,983	.5
33	10,276.846272	10,276	.8
34	10,579.858496	10,580	— .1
35	10,893.	10,893	0.
36	11,216.888704	11,217	— .1
37	11,552.388928	11,552	.3
38	11,900.663872	11,900	.6
39	12,263.228096	12,263	.2
40	12,642.	12,642	0.
41	13,039.354304	13,040	— .6
42	13,458.174528	13,459	— .8
43	13,901.905472	13,904	—2.0
44	14,374.605696	14,376	—1.3

VALUES OF $10^6 q_x$ FOR AGES 45 TO 69.

(1) Age.	(2) Joffe's Value.	(3) Gill's Value.	(4) (2)-(3) to 1 decimal.
45	14,881.	14,881	0.
46	15,439.737664	15,440	− .2
47	16,101.450048	16,101	.4
48	16,843.853952	16,845	−1.1
49	17,652.086336	17,652	0.
50	18,518.	18,518	0.
51	19,439.459264	19,439	.4
52	20,419.635648	20,419	.6
53	21,466.303552	21,465	1.3
54	22,591.135936	22,592	− .8
55	23,809.	23,809	0.
56	25,137.252864	25,136	1.2
57	26,595.037248	26,596	− .9
58	28,202.577152	28,204	−1.4
59	29,980.473536	29,980	.4
60	31,949.	31,949	0.
61	34,127.398464	34,125	2.3
62	36,533.174848	36,533	.1
63	39,181.394752	39,179	2.3
64	42,083.979136	42,082	1.9
65	45,249.	45,249	0.
66	48,679.976064	48,678	1.9
67	52,375.168448	52,374	1.1
68	56,326.876352	56,331	−4.1
69	60,520.732736	60,518	2.7

As I pointed out in my preceding paper,* the values of q_x in the last 29 ages of Gill's Table were taken from the 17 Offices' Table, with the exception of 3 ages. These three exceptions are the following:

Age.	17 Offices' Table.	Gill's Table.
76	.1031794	.103159
88	.2652741	.265291
95	.5842697	.590917

The first and second discrepancies may possibly be explained as simple errors in copying; but it is difficult, if not impossible, to dispose of the third instance as a copyist's mistake. This discrepancy (at age 95) is especially surprising when one takes into consideration the fact that 95 is a multiple of 5, and therefore the value of q_{95}, if not copied from the 17 Offices' Table, should have been ob-

* T. A. S. A., XII, 257.

tained as a reciprocal of the original value $1/q_{95} = 1.7$, which is 0.588235. Nor was it obtained by dividing $d_{95} = 49$ by $l_{95} = 83$, which would give 0.590361. And I regret that I cannot advance any satisfactory hypothesis of the origin or computation of Gill's q_{95}.

The lack of success in this respect and the failure to reproduce exactly Gill's figures of q_x in several instances, have caused me considerable disappointment; but as a compensation there remains for me the consciousness of having been instrumental in the recent finding and bringing to light of the volume "Assurance Tables" which had been lost sight of and forgotten for half a century. I do not doubt that a careful analysis of Gill's Table will well repay the student, as I still believe that the method employed by Homans in constructing the American Experience Table from his Table No. 1 was merely a modification of the method by which Gill constructed his table from the Carlisle Table.

GILL'S TABLE.

x	l_x	$l_x - l_{x+1}$	$1-p_x$	Σl_x	λl_x	$\lambda(l_x - l_{x+1})$	$\frac{1}{1-p_x}$	$\epsilon_x (= \mathring{e}_x)$
10	100,000	526	.005263	4,800,694	5.0000000	2.7209857	190.0	47.51
11	99,474	542	.005444	4,700,694	4.9977096	.7339993	183.7	46.76
12	98,932	556	.005622	4,601,220	.9953368	.7450748	177.9	46.01
13	98,376	570	.005798	4,502,288	.9928892	.7558749	172.5	45.27
14	97,806	584	.005975	4,403,912	.9903655	.7664128	167.4	44.53
15	97,222	598	.006154	4,306,106	.9877646	.7767012	162.5	43.79
16	96,624	612	.006334	4,208,884	.9850850	.7867514	157.9	43.06
17	96,012	626	.006516	4,112,260	.9823255	.7965743	153.5	42.33
18	95,386	640	.006706	4,016,248	.9794846	.8061800	149.1	41.60
19	94,746	654	.006902	3,920,862	.9765609	.8155777	144.9	40.88
20	94,092	668	.007102	3,826,116	.9735527	.8247765	140.8	40.16
21	93,424	683	.007311	3,732,024	.9704585	.8344207	136.8	39.45
22	92,741	698	.007525	3,638,600	.9672718	.8438554	132.9	38.74
23	92,043	712	.007739	3,545,859	.9639908	.8524800	129.2	38.03
24	91,331	727	.007958	3,453,816	.9606182	.8615344	125.6	37.32
25	90,604	741	.008183	3,362,485	.9571474	.8698182	122.2	36.61
26	89,863	756	.008413	3,271,881	.9535809	.8785218	118.9	35.91
27	89,107	771	.008652	3,182,018	.9499118	.8870544	115.6	35.21
28	88,336	786	.008902	3,092,911	.9461377	.8954225	112.3	34.51
29	87,550	802	.009158	3,004,575	.9422562	.9041744	109.2	33.82
30	86,748	818	.009425	2,917,025	.9382595	.9127533	106.1	33.13
31	85,930	833	.009699	2,830,277	.9341448	.9206450	103.1	32.44
32	85,097	850	.009983	2,744,347	.9299142	.9294189	100.2	31.75
33	84,247	866	.010276	2,659,250	.9255544	.9375179	97.3	31.07
34	83,381	882	.010580	2,575,003	.9210671	.9454686	94.5	30.38
35	82,499	899	.010893	2,491,622	.9164487	.9537597	91.8	29.70
36	81,600	915	.011217	2,409,123	.9116902	.9614211	89.1	29.03
37	80,685	932	.011552	2,327,523	.9067928	.9694159	86.6	28.35
38	79,753	949	.011900	2,246,838	.9017470	.9772662	84.0	27.67
39	78,804	966	.012263	2,167,085	.8965483	.9849771	81.5	27.00
40	77,838	984	.012642	2,088,281	.8911917	.9929951	79.1	26.33
41	76,854	1002	.013040	2,010,443	.8856665	3.0008677	76.7	25.66
42	75,852	1021	.013459	1,933,589	.8799670	.0090257	74.3	24.99
43	74,831	1040	.013904	1,857,737	.8740815	.0170333	71.9	24.33
44	73,791	1061	.014376	1,782,906	.8680034	.0257154	69.5	23.66
45	72,730	1082	.014881	1,709,115	.8617136	.0342273	67.2	23.00
46	71,648	1106	.015440	1,636,385	.8552041	.0437551	64.8	22.34
47	70,542	1136	.016101	1,564,737	.8484478	.0553783	62.1	21.68
48	69,406	1169	.016845	1,494,195	.8413970	.0678145	59.4	21.03
49	68,237	1204	.017652	1,424,789	.8340199	.0806265	56.6	20.38
50	67,033	1241	.018518	1,356,552	.8262887	.0937718	54.0	19.74
51	65,792	1279	.019439	1,289,519	.8181731	.1068705	51.4	19.10
52	64,513	1317	.020419	1,223,727	.8096472	.1195858	49.0	18.47
53	63,196	1357	.021465	1,159,214	.8006896	.1325798	46.6	17.84
54	61,839	1397	.022592	1,096,018	.7912625	.1451964	44.3	17.22

GILL'S TABLE (*continued*).

x	l_x	$l_x - l_{x+1}$	$1 - p_x$	Σl_x	λl_x	$\lambda(l_x - l_{x+1})$	$\frac{1}{1-p_x}$	$e_x(=\mathring{e}_x)$
55	60,442	1439	.023809	1,034,179	4.7813388	3.1580608	42.0	16.61
56	59,003	1483	.025136	973,737	.7708741	.1711412	39.8	16.00
57	57,520	1530	.026596	914,734	.7598189	.1846914	37.6	15.40
58	55,990	1579	.028204	857,214	.7481105	.1983821	35.5	14.81
59	54,411	1631	.029980	801,224	.7356867	.2124540	33.4	14.23
60	52,780	1686	.031949	746,813	.7224694	.2268576	31.3	13.65
61	51,094	1744	.034125	694,033	.7083699	.2415465	29.3	13.08
62	49,350	1803	.036533	642,939	.6932872	.2559957	27.4	12.53
63	47,547	1863	.039179	593,589	.6771231	.2702129	25.5	11.98
64	45,684	1923	.042082	546,042	.6597641	.2839793	23.8	11.45
65	43,761	1980	.045249	500,358	.6410872	.2966652	22.1	10.93
66	41,781	2034	.048678	456,597	.6209788	.3083509	20.5	10.43
67	39,747	2082	.052374	414,816	.5993044	.3184807	19.1	9.94
68	37,665	2122	.056331	375,069	.5759380	.3267454	17.7	9.46
69	35,543	2151	.060518	337,404	.5507541	.3326404	16.5	8.99
70	33,392	2168	.064935	301,861	.5236424	.3360593	15.4	8.54
71	31,224	2191	.070158	268,469	.4944885	.3406424	14.3	8.10
72	29,033	2201	.075805	237,245	.4628919	.3426200	13.2	7.67
73	26,832	2197	.081883	208,212	.4286530	.3418301	12.2	7.26
74	24,635	2179	.088468	181,380	.3915526	.3382572	11.3	6.86
75	22,456	2146	.095560	156,745	.3513324	.3316297	10.5	6.48
76	20,310	2095	.103159	134,289	.3077099	.3211840	9.7	6.11
77	18,215	2030	.111469	113,979	.2604292	.3074960	9.0	5.76
78	16,185	1949	.120444	95,764	.2091127	.2898118	8.3	5.42
79	14,236	1852	.130065	79,579	.1533880	.2676410	7.7	5.09
80	12,384	1739	.140406	65,343	.0928609	.2402996	7.1	4.78
81	10,645	1612	.151436	52,959	.0271457	.2073650	6.6	4.48
82	9,033	1474	.163194	42,314	3.9558320	.1684975	6.1	4.18
83	7,559	1330	.175912	33,281	.8784643	.1238516	5.7	3.90
84	6,229	1182	.189679	25,722	.7944183	.0726175	5.3	3.63
85	5,047	1035	.205095	19,493	.7030333	.0149403	4.9	3.36
86	4,012	892	.222480	14,446	.6033609	2.9503649	4.5	3.10
87	3,120	756	.242234	10,434	.4941546	.8785218	4.1	2.84
88	2,364	627	.265291	7,314	.3736475	.7972675	3.8	2.59
89	1,737	508	.292382	4,950	.2397998	.7058637	3.4	2.35
90	1,229	398	.323730	3,213	.0895519	.5998831	3.1	2.11
91	831	300	.360987	1,984	2.9196010	.4771213	2.8	1.89
92	531	215	.405263	1,153	.7250945	.3324385	2.5	1.67
93	316	144	.457227	622	.4996871	.1583625	2.2	1.47
94	172	89	.516304	306	.2355284	1.9493900	1.9	1.28
95	83	49	.590917	134	1.9190781	.6901961	1.7	1.11
96	34	22	.648649	51	.5314789	.3424227	1.5	1.00
97	12	8	.692308	17	.0791812	0.9030900	1.4	.92
98	4	3	.750000	5	0.6020600	.4771213	1.3	.75
99	1	1	1.000000	1	0.0000000	0.0000000	1.0	.50

Legal Notes.

BY

WENDELL M. STRONG.

Dividend Estimates:—(Germania Life Insurance Co. *vs.* Bouldin, Supreme Court of Mississippi, 56 So. Rep. 609.) This case bears a considerable resemblance in the decision reached and some of the facts to that of Timlin *vs.* Equitable Life (see *Transactions,* Vol. XI, p. 339). The policy in question was signed on the face and incorporated the conditions following by this clause,

"This policy is issued and the same is accepted by the assured upon the express conditions and agreements printed on the back hereof, which are referred to and made a part hereof."

The policy was forwarded to the agent and by him forwarded by mail to the insured. While in his hands, however, he pasted on the back of it immediately over the printed conditions and agreements a slip reading as follows:

"To Live and Win.
An Investment Insurance.

Age 30.	Amount of policy, $3,000.00
Annual premium, $109.80, payable for fifteen years.	
Dividend period, fifteen years.	
If death occurs during dividend period, your heirs will receive	$3,000.00
At the end of the dividend period you can select one of the following options:—	
I. Surrender policy for cash... { Reserve, $1,284 / Surplus, 777 }	$2,061.00
Reserve guaranteed and amount stated in policy.	
Surplus guaranteed, but amount estimated based on past experience.	
Total premiums paid in fifteen years	$1,647.00
Fifteen years' life insurance free and profit	414.00
II. Paid-up policy for $4,770, having cost only	1,647.00
III. Paid-up policy for $3,000 and cash	777.00

You will receive dividends during life on paid-up policy under second and third options.

Policies nonforfeitable and incontestable after three years, and can be surrendered either for paid-up policy or their cash value."

The policy was a fifteen year tontine policy without the right to continue beyond the fifteen years; consequently options II and III did not apply to the policy at all.

Near the end of the fifteen year period the insured inquired of the company the options and was informed that the amount payable was the guaranteed reserve of $1284 and the accumulated dividend of $192.82, whereupon he brought action. The court held the slip to be a part of the contract, thus giving the insured the right to continue the policy under option II or III, and also held that options II and III as shown in the slip were contracts for the specific amounts there named, while holding that option I was not a contract for a specific amount of surplus because it was stated that the amount was estimated.

Much of the opinion of the court has to do with the authority of the agent and comparatively little with what was really the essential point, that is, the question whether a slip, pasted to a policy in the way this was and not incorporated by reference, could under any circumstances be considered a part of the contract. This slip certainly could not be considered as incorporated by reference for the clause quoted above, incorporating the conditions by reference, stated explicitly, "printed on the back hereof," and the slip was certainly not "printed on the back hereof."

Perhaps the reasons for excluding such a slip were not quite as strong as those for excluding the dividend estimate in the Timlin case, because in that the policy stated specifically that:

"The contract between the parties hereto is completely set forth in this policy and the application therefor taken together and none of its terms can be modified except . . . by an agreement in writing signed" by one of the authorized officers.

Again, here the slip was pasted to the policy, a permanent method of attachment, there attached only by a pin. These differences, however, are hardly more than superficial, and if the rule that a paper pasted to a contract and not incorporated by reference in any way were followed in regard to contracts in general it would open wide the door to fraud, while, if such a rule is not applicable to contracts in general, it is hard to see upon what ground it is applicable to insurance contracts.

Another question involved was whether the slip, even if it were a part of the contract, did guarantee a specific sum under option II

or option III. The court held that it did. A very similar question was discussed in the case of Langdon *vs.* Northwestern Mutual Life Insurance Company (*Transactions,* Vol. XI, p. 503). The illustration in the Langdon case quoted three options, under the first of which it was stated that the reserve was guaranteed and the amount of the surplus was estimated on past experience. Under the second and third options the figures only were stated without any reference to the fact that they included both guarantee and surplus. The New York Court of Appeals, however, reached precisely the opposite result regarding the effect of such wording from that reached in the present case, stating:

"While it (the estimate) doubtless would have been more complete and perfect if, in stating the third option and referring to the cash or tontine plan therein provided for, it had again been stated that this sum was estimated and based on experience, still the identity of this sum with the surplus item of the same amount stated in the first option was obvious, and as it had been stated in such first option that the sum was estimated, there could be no misunderstanding of this fact when such item was again given in the third option."

A third important question was involved in the case. The action was in equity and an alternative form of relief prayed for was that, if the court held that the minds of the parties never met with respect to the terms of a policy of insurance, it should hold that there had been no contract and give the insured a decree for the premiums paid together with 6 per cent. interest thereon. The decree of the lower court was in accordance with this prayer. It seems evident that such a decision is unjust to the company, since the company could not by a similar plea have escaped its liability in case the insured had died within the fifteen years the policy had been in force: thus the contract had been executed by the company to the extent of having furnished fifteen years' insurance thereunder. The Supreme Court reversed the lower court, saying in the opinion:

"We do not know of any principle upon which this decree can be sustained. 'It is a faithful saying, worthy of all acceptation,' that when a party comes into a court of equity he is required to do equity, and it is clear, under the facts of this case, that if the complainant had died at any time during the life of this policy—that is, within the fifteen years—the company would have been required

to have paid the beneficiary the full amount of the policy called for, to wit, $3,000. In other words, the company would not have been in position to have claimed that the policy was not in full force and effect."

Another point in the case is of interest. The insured claimed that he did not read the policy until near the end of the fifteen year period. The court showed no tenderness towards any plea founded on his not having read the policy, speaking as follows regarding this:

"It is his business to know what the contents of the written contract of insurance are, and there can be no difference in this respect between an insurance policy and any other contract. In the absence of any fraud in the making of an insurance policy, the insured must be held to a knowledge of the conditions of his policy, as he would be in the case of any other contract or agreement. It certainly would be inequitable and inconsistent with every safe, sound rule of conduct to permit one party to a contract to accept without examination a written contract, and lay it aside for a long period of time, and then, when the period of termination is about to expire, to, for the first time, inform himself of its terms."

Payment of Premium at Time of Making Application:— (Northwestern Mutual Life Ins. Co. *vs.* Neafus, Court of Appeals of Kentucky, 140 S. W. Rep. 1026.) The applicant paid the first premium at time of making application and was given a receipt by the agent reading,

"An application for a one thousand dollar policy having been made by T. J. Neafus to the Northwestern Mutual Life Insurance Company, there has been collected the sum of $27.71 to be considered the first annual premium on said policy; provided the application is approved by the company at its home office; and, in that event, the insurance as applied for will be in force from the date of the medical examination; if the application is not so approved, the sum collected will be returned."

The examiner not considering the applicant insurable at the time of the examination did not forward the report of the examination to the company, but held it to make further examination later; meanwhile the applicant died. Whether this holding was with the consent of the applicant or not is disputed; it was, however, shown that the application would have been rejected had it been forwarded to the company with a report of the examination as made.

It was claimed that, in-as-much as the application had not been rejected, the insurance was in force. The court in a well considered opinion held against this contention. Any other result would have interposed a serious obstacle to the taking of the premium before the application is passed upon by the company, as no company can afford to insure, even for a few days, any one who applies to it, puts the premium in the hands of the agent, and submits to an examination.

The following are extracts from the opinion:

"The receipt is not complicated or difficult to understand. It is a brief, plainly written paper that any person of ordinary intelligence could readily and easily comprehend the meaning of. . . . It is plain that to construe this receipt into an obligation on the part of the company that it had insured Neafus from the date of the medical examination until he was rejected would make a radical and material alteration in its terms. . . .

We recognize, and have often applied in the construction of insurance contracts, the rule that they must be liberally construed in favor of the insured, but we have never gone to the extent of putting into the contract words that would make a radical change in its meaning, or that would make for the parties a contract that they did not make for themselves. There is nothing unfair or unreasonable or oppressive in the conditions of this receipt. It is simply a proposition made by the company to the applicant. It in substance tells him that if he wants insurance he can pay the first premium, submit to a medical examination, and then, if the company regards him as a desirable risk it will issue to him a policy that will relate back to and be in effect from the date of the medical examination. When the applicant accepts this proposal, he agrees to its terms and conditions, and cannot be excused from knowing when his insurance will begin. If we should say that under the receipt the applicant was insured from the date of his medical examination, although the company might reject the application, we would put the company in the attitude of insuring a person, whether he was a desirable risk or not. It would place upon it a burden of carrying without its consent, a risk that it did not and would not assume.

Assignment Without Insurable Interest:—(A. H. Grigsby, Petitioner, *vs.* R. L. Russell and Lillie Burchard, Administrators of John Burchard, Deceased, U. S. Supreme Court, 222 U. S. 149.) It has been considered by many that the Supreme Court of the United States holds an assignment without insurable interest valid only to the amount actually paid for the policy (and in later premiums thereon), and does not regard a policy of insur-

ance as property which can be transferred like any other property. The cases of Warnock *vs.* Davis and Cammack *vs.* Lewis are often cited in support of this. As a matter of fact, in both these cases the contract in its inception was a wagering contract, so that the question of the transference of a policy whose inception did not bear the taint of a wager was not squarely before the court.

In the present case the assignment was made after the policy had been in force several years, and the question was whether the assignee was entitled to the entire proceeds of the policy which he had purchased or only to reimbursement for the money he had expended on it. The court took the unequivocal stand that a policy could be sold like other property. Extracts from the opinion are:

"Of course, the ground suggested for denying the validity of an assignment to a person having no interest in the life insured is the public policy that refuses to allow insurance to be taken out by such persons in the first place. A contract of insurance upon a life in which the insured has no interest is a pure wager that gives the insured a sinister counter interest in having the life come to an end. . . .

"But when the question arises upon an assignment, it is assumed that the objection to the insurance as a wager is out of the case. . . . But this being so, not only does the objection to wagers disappear, but also the principle of public policy referred to, at least, in its most convincing form. The danger that might arise from a general license to all to insure whom they like does not exist. . . . On the other hand, life insurance has become in our days one of the best recognized forms of investment and self-compelled saving. So far as reasonable safety permits, it is desirable to give to life policies the ordinary characteristics of property. This is recognized by the bankruptcy law, which provides that unless the cash surrender value of a policy like the one before us is secured to the trustee within thirty days after it has been stated, the policy shall pass to the trustee as assets. Of course the trustee may have no interest in the bankrupt's life. To deny the right to sell except to persons having such an interest is to diminish appreciably the value of the contract in the owner's hands. The collateral difficulty that arose from regarding life insurance as a contract of indemnity only long has disappeared. And cases in which a person having an interest lends himself to one without any, as a cloak to what is, in its inception, a wager, have no similarity to those where an honest contract is sold in good faith."

Rights of Stockholders:—(Blanchard *vs.* Prudential Life Ins. Co., New Jersey Court of Errors and Appeals. 83 Atl. Rep. 220.) This case, as decided by the lower court, was discussed in Vol. XII,

p. 88, of the *Transactions* where a synopsis of the facts is given. There were two separate questions involved.

The first one was that of the right of the company to give a special benefit, not a part of the contract, to holders of non-participating policies. The lower court had upheld such right and the Court of Errors and Appeals affirmed this part of the decision. An extract from the opinion is:

"The concessions made appear to have been a necessary outlay from the earnings of the company which must be considered as entering into the cost of conducting the business and though, as a result, the profits are diminished and policyholders receive the benefit of such concessions, it is not tantamount to a participation in the profits. Net profit is the gain which remains after all the costs and expenses of the business have been paid. Whether the company should have made or make these concessions involves a purely business proposition. Upon what business basis the company has made these concessions has already been amply shown. While these transactions may be the subject of legislative control they are clearly not amenable to judicial supervision. Judicial interference can only be invoked when it appears that some clear statutory policy or some legal rule has been violated. No claim has been made that the directors in making these concessions acted fraudulently or in bad faith.

"It is not in violation of the contractual rights of policyholders holding dividend-paying policies, because their contracts are subject to the fair business methods adopted by the company to maintain its business, to expand it, and to use all other legitimate business means adopted by it to increase its stability and prosperity."

The second question was whether stockholders could enforce a distribution of a part of the surplus set apart to the stockholders but held and added to the contingency surplus. The lower court held that this should be divided among the stockholders, the Court of Errors and Appeals reversed this. An extract from the opinion is:

"The complainants hold about twenty per cent. of the stock and insist that their share of $2,500,000 should now be distributed to them in the form of dividends. Since the capital stock of the company is only two millions this would amount to a dividend of over 125 per cent. The defendant company claims that it is necessary to retain this fund in order that the solvency of the company may be amply secured. This at once gives rise to a question which appertains to the wisdom of business management and foresight, and does not call for the interference of the court. The complainants are practically seeking to have the business management of a life insurance company put into the hands of the court. Upon what

basic legal principle the court is asked to substitute its judicial sense for the business sense and discretion which must rest with the directors of the company, has not been made clear. The complainants do not claim that the directors, in their action, were actuated by a fraudulent or improper purpose. . . . The directors having exercised their discretion in the matter, their judgment is not open to a successful attack, unless it appears that it was a result of fraud or bad faith on their part."

Authority to Extend Time for Payment of Premiums:— (New York Life Ins. Co. *vs.* O'Dom, Supreme Court of Mississippi, 56 So. Rep. 379.) One of the dangers to which all the insurance companies are subject is that the strictly limited authority of their agents or employes at branch offices may be extended by the courts in the question of binding the company beyond their real or apparent powers. In the present case the general law of agency has been impartially applied, and the opinion contains a valuable review of the principles covering the authority of agents to modify a contract.

The essential facts were; that it was claimed the cashier of the agency had promised on the due date of the premium to accept, two days later, a dated back check, the check to be for a quarterly premium, whereas the policy called for annual premiums; that insured had information from the policy itself and also from premium notices that only certain named officers had power to make or modify the contract of insurance or to extend the time for paying the premium or to waive forfeiture. The following are extracts from the opinion:

"The contention of appellee is that the stipulation in the policy, 'No agent is authorized to waive forfeitures, or to make, modify, or discharge contracts, or extend the time for paying the premium,' is void, and Wilson, the cashier of the company, had such powers and duties as authorized him to waive the forfeiture; in other words, that Wilson was the alter ego of the company. . . .

"The fallacy lurking beneath the position of appellee is as dangerous and deceptive as it is open and glaring. It consists in assuming that the cashier of the Jackson branch was a general agent of the company; that, because he had power to collect premiums, he also had the power to extend the time of payment; and that he could waive the forfeiture and modify or change the contract of insurance. . . .

"If, therefore, the insured had notice that only the general agents or officers of the company had power to grant the extension of time, how can he be heard to say that he was misled or prejudiced by dealing with Wilson, whose duties are merely clerical, and

limited, administrative, and not executive? It was his duty to ascertain the authority of the person with whom he was negotiating, or else take his chances on a ratification by the company. Insurance companies must have some efficient means of enforcing punctuality in the payment of premiums. Their contracts may provide for the forfeiture of a policy upon the default of the prompt payment of the premium. If they are not allowed to enforce this forfeiture, they are deprived of a means which they have reserved by the contract of compelling the parties insured to meet their engagements. The provision for the release of the company from liability on the policy of the insured, to pay the premium when due, is of the very essence and substance of the contract of life insurance. To hold the company to its promise to pay the insurance, notwithstanding the default of the insured in the making punctual payment of the premiums is to destroy the very contract of life insurance itself."

Power to Determine Dividends:—(Mutual Benefit Life Ins. Co. *vs.* Emig's Administrators, Court of Appeals of Kentucky, 141 S. W. Rep. 38.) While the right of the directors to determine the amount of dividends payable is not often called in question, any decision, such as the present one in Kentucky, definitely upholding the right, is important as helping to obviate any future doubt. The question arose in this case because the extended insurance purchased by the amount available therefor on the lapse of the policy expired only a few days before the death of the insured, and a slight addition to the amount available for such purchase would have carried the term beyond the date of the insured's death. The court held that the policy was entitled only to such dividends as were distributed, quoting with approval from an opinion of the U. S. Supreme Court as follows:

"By that contract he was entitled to participate in the distribution of some part of the surplus, according to principles and methods that might be adopted from time to time by the defendant for such distribution, which principles and methods were ratified and accepted by and for every person who should have or claim any interest under the policy. It has been held that under such a policy, how much of the surplus shall be distributed to the policyholder and how much shall be held for the security of the defendant and its members is to be decided by the officers and management of the defendant, in the exercise of their discretion to distribute, having in mind the present and future business, and, in the absence of any allegations of wrongdoing or mistake by them, their determination must be treated as proper, and their apportionment of the surplus is to be regarded prima facie as equitable."

Extension of Time for Paying Premium:—(Stewart *vs.* Home Life Ins. Co., Appellate Division of Supreme Court of New York, 131 N. Y. Supp. 504.) One question before the court was, whether, when an extension for a definite length of time had been granted by the company, the policyholder was entitled to the period of grace from the expiration of the extension period. In this case the insured had obtained, by the payment of cash considerations, two extensions, the first for one month from the end of the period of grace, the second for a month from the end of the first extension. The insured died shortly after the expiration of the second period of extension, and, if he was entitled to grace from the end of such extension, the policy was still in force. Extensions are made on the understanding that the premium must be paid within the period of the extension or else the policy lapses, and the extension agreements, as usually worded, cover this specifically, as did the one in the present case. The court upheld the contract for the extension as made, holding that it did not keep the insurance in force beyond the end of the period of extension.

Construction of Question in Application:—(Enright *vs.* National Council, Knights and Ladies of Security, Supreme Court of Illinois, 97 N. E. Rep. 681.) One of the difficulties insurance companies have to meet is that of making the questions of the application, particularly the medical part, so clear and simple that a different meaning from the one intended cannot be attributed. At the same time it is necessary to have such questions short. In this case the chief interest attaches to the attempt to make use of a perhaps somewhat unfortunate mode of expression to give to a question in a medical examination a different meaning from the evident one. The question was:

"Have either of your parents, or any of your uncles, aunts, brothers, or sisters, or other blood relatives, been afflicted with consumption, scrofula, cancer, insanity, epilepsy, gout, rheumatism, or any other hereditary disease?"

It was answered "No," and the application made the statements therein warranties. The fact was that a brother and a cousin of the insured had died of consumption. The following extract from the opinion shows the claim of the plaintiff in regard to this question and the common sense view taken by the Supreme Court:

"It is contended that the judgment [in favor of the plaintiff] was right, because the ordinary and plain meaning of the words used to specify the diseases was limited or qualified by the words 'or any other hereditary disease,' and that it was necessary, not only to prove that a brother and cousin had died of consumption, but that the consumption was of some kind that was hereditary, and was not acquired by the brother and cousin. The Appellate Court adopted the view that, in order to avoid liability on the certificate, it was necessary to show that the consumption of which the brother and cousin died was hereditary in them. . . . We regard the plain meaning of the question to be whether any of the specified relatives or blood relatives of the insured had been inflicted with either of the particular diseases mentioned, or with any other disease not mentioned that was hereditary. The question did not imply that the diseases mentioned were hereditary, and it would be irrational to say that the fraternal society was seeking information from the applicant as to whether gout, rheumatism, or consumption were hereditary. What the society wanted to know, and required him to answer, was whether any of the relatives had had any of those particular diseases."

Construction of Question in Application:—(Blenke *vs.* Citizen's Life Ins. Co., Court of Appeals of Kentucky, 140 S. W. Rep. 561.) An appeal from the verdict in favor of the defendant company was made because the jury had been directed to find a verdict for the defendant, if the answers to "said questions" were "substantially untrue" . . . "even though the jury shall believe that said untrue answers, or any of them, if any such there was, were not made with a knowledge of their falsity, or with the intention to mislead or deceive the defendant company." Certain of the questions referred to, and which were answered falsely, were such that the applicant could answer "Yes" or "No" positively and not simply on belief. For instance, as to whether he had ever made application without receiving policy as applied for, or without receiving the policy within thirty days, or whether he had ever been rejected by any company or association. A question whether the applicant was now in perfect health "so far as you know or believe" was also asked.

It was claimed that the instruction to the jury as quoted above was erroneous and that the jury should have been allowed to take into consideration the good faith of the applicant, in support of which it was argued that the words "so far as you know or believe" applied to all the questions. The Court of Appeals decided against

this claim and held that the instruction as given was correct. Extracts from the opinion are:

"It is contended, in effect, that the question of good faith and honest intention of the insured in answering these questions should have been submitted to the jury in connection with their materiality. In other words, it is argued that so much of the instruction as directed the jury if they believed the answers were untrue, although not made with knowledge of their falsity or with the intention to deceive or mislead, should have been omitted. . . . On the other hand, although these answers were untrue, yet, if the jury had been instructed that they might return a verdict for the plaintiff if they believed the answers were made in good faith without knowledge of their falsity and without any intention to mislead or deceive the company, there was evidence sufficient to warrant the jury in finding a verdict against the company.

"Taking up first the argument of counsel that the question, 34, 'Are you in perfect health, so far as you know or believe?' should be read in connection with and as a part of every other question and answer made by the applicant, and the effect of each other question and answer should be so qualified and limited as to make the question read similar to question 34, or, to illustrate, that question 2, reading, 'Have you ever applied to any agent for insurance without receiving a policy within thirty days?' should be construed to read, 'Have you, so far as you know or believe, ever applied to any agent for insurance without receiving a policy within thirty days?' To thus construe all the questions and answers would plainly make a material change in their meaning and effect. The questions as written in the application are simple, direct, and positive, and the answers made by the applicant were intended to be equally simple, direct, and positive. Many of the questions do not leave any room for doubt or speculation as to what is intended, nor do they leave the sufficiency of the answers to the good faith or honest intention or belief of the applicant."

Bankruptcy—Ownership of Proceeds of Policy:—(Partridge *vs.* Andrews, U. S. Circuit Court of Appeals, Third Circuit, N. J., 27 Am. B. R. 388.) The insured held policies payable to his estate. He died between the time the petition in bankruptcy was filed against him and the time he was adjudicated a bankrupt. The statute in regard to bankruptcy has the following specific provision in regard to insurance:

"Provided, that when any bankrupt shall have any insurance policy which has a cash surrender value payable to himself, his estate, or personal representatives, he may, within thirty days after the cash surrender value has been ascertained and stated to the

trustee by the company issuing the same, pay or secure to the trustee the sum so ascertained and stated, and continue to hold, own and carry such policy free from the claims of the creditors participating in the distribution of his estate under the bankruptcy proceedings, otherwise the policy shall pass to the trustee as assets," etc.

The question under this special provision regarding an insurance policy was whether the trustee could get only the surrender value or whether he would take the proceeds of the policy. It was held that the trustee was entitled to the proceeds of the policy. The present case overrules an earlier case in the District Court.

Extracts from the opinion which show the position taken by the court are:

"The contention of the appellee is, that by the payment of the surrender value, as it existed at the time of the filing of the petition, the personal representative of the deceased bankrupt is entitled to the matured policy and its proceeds. Not only the language of the proviso, but the plain implication therefrom, forbids such an interpretation. If, as stated above, all surrender value has been extinguished by the death of the bankrupt before adjudication, there can be no surrender value to be ascertained at or after that time. Moreover, the insuring company could not be called upon to pay a surrender value that had ceased to exist, or to recognize a right that had been extinguished. . . . The policies in question, as property having a real value at the time of filing the petition, passed as of the date of the adjudication in their then condition, as matured contracts, to the trustee, and as there was no surrender value attaching to them at that date, the situation was such as to preclude any assertion of the privilege conferred upon the insured bankrupt by the proviso of the fifth subdivision of section 70a of the Bankruptcy Act. The intent of the bankruptcy law would be frustrated by an opposite conclusion. If the bankrupt had died before the petition was filed, the money called for by the policies (if the same had not been assigned) would have gone to his personal representative and been applicable to his debts. There is no reason why the same result should not now be attained through the bankruptcy proceedings. Neither the widow nor any of the bankrupt's family are beneficiaries under either of the policies here in question, and the creditors should not be deprived of an important asset to which they are justly entitled."

Loan on Policy in Missouri:—(Christensen *vs.* New York Life Ins. Co., St. Louis Court of Appeals, 141 S. W. Rep. 6.) This was a case involving the Missouri non-forfeiture law. At the time

the policy was issued the statute in force allowed only notes or evidence of indebtedness to the company on account of past premium payments on the policy to be deducted from the three-fourths of the net value of the policy to be applied to purchase extended insurance. An amendment to this statute was made authorizing the deduction from three-fourths of the net value of "any evidence of indebtedness to the company" on "policies of insurance on life hereafter issued." After this amendment a cash loan was made by the company on the policy. It was claimed that, because of the amendment, the amount of the loan could be deducted from the three-fourths of the reserve which would otherwise go to purchase extended insurance. The court held regarding this that the amendment did not affect loans on policies issued prior to its passage.

The decision of the lower court against the company was, however, reversed on another ground. When the note became due and was not paid or extended, the company sent the policy to the insured, endorsed for extended insurance, the extended insurance being for the smaller amount and shorter time resulting from the loan. With the policy the company sent a letter explaining the transaction and stating that the indebtedness had been paid in the foreclosure of the loan. The insured lived nearly a year after this time and made no objection to this settlement of his indebtedness. The statute in force at the time the policy was issued specified that the non-forfeiture statute should not be applicable, "if the policy shall be surrendered to the company for a consideration, adequate in the judgment of the legal holder thereof." In this case the consideration was the cancellation of the indebtedness to the company which might have been enforced against the insured. The fact that the insured received the explanation of the transaction and acquiesced in such transaction by his long continued silence evidenced that he accepted the consideration as adequate, and the court so held.

Service Upon Insurance Commissioner:—(Chicago Life Ins. Co. *vs.* Robertson, Court of Appeals of Kentucky, 143 S. W. Rep. 740.) Service of summons was on the Insurance Commissioner; he failed to notify the company and judgment was entered by default. Action was brought to set aside the judgment by default and the question at issue was whether failure of the Commissioner to notify the company entitled the company to a new trial. The

section of the statute authorizing the service on the Insurance Commissioner is:

"Before authority is granted to any foreign insurance company to do business in this state, it must file with the Commissioner a resolution adopted by its board of directors, consenting that service of process upon the Commissioner of Insurance of this state, in any action brought or pending in this state, shall be a valid service upon said company; and if process is served upon the Commissioner it shall be his duty to at once send it by mail addressed to the company at its principal office."

The court held that to make the service complete it was necessary for the Commissioner to mail the summons to the principal office of the company, saying in part:

"The law is not complied with by service merely upon the Insurance Commissioner. Ordinarily, it is presumed that the Commissioner will do his duty, but when it is made to appear that he has failed in its discharge, and that the company has received no notice of the pendency of the suit, it would be a denial of substantial justice to hold that it was not entitled to a new trial under such circumstances."

Death by the Hand of the Law:—(Northwestern Mutual Life Ins. Co. *vs.* McCue et al., U. S. Supreme Court, 223 U. S. Rep. 234.) The insured was executed for murder. The policy contained no provision regarding death by the hand of the law. The question was whether a policy of life insurance insures against death by a legal execution for crime. It was argued that the contract was a Wisconsin contract and that the law in Wisconsin made a company liable under such circumstances. The court held that the company was not liable, quoting from a previous decision in the same court: "It can not be that one of the risks covered by a contract of insurance is the crime of the insured. There is an implied obligation on his part to do nothing to wrongfully accelerate the maturity of the policy. Public policy forbids the insertion in a contract of a condition which would tend to induce crime, and as it forbids the introduction of such a stipulation it also forbids the enforcement of a contract under circumstances which can not lawfully be stipulated for." In the course of the opinion the question whether the contract was a Virginia contract, the policy having been applied for, delivered and paid for in Virginia, or a Wisconsin contract, was discussed; regarding this the significant statement was made: "If the public policy of Virginia were the

same as it is contended that of Wisconsin is, whether this court should have to yield it we are not called upon to decide. Being of opinion that McCue's policy was a Virginia contract it may be unnecessary to review the [Wisconsin] cases relied on by the respondents."

NOTICE OF FORFEITURE:—(Jones *vs.* New York Life Insurance Co., Supreme Court of Oklahoma, 122 Pacific Rep. 702.) - As the controversy in this case arose while Oklahoma was a territory the result should be governed by the Federal law. The question involved was whether under the circumstances of the case a notice such as that required by the law of New York was a condition precedent to forfeiture for non-payment of premium. The policy was issued in 1894 and one of the provisions of the agreement contained in the application was, "That the contract contained in such policy and in this application shall be construed according to the law of the State of New York, the place of such contract being agreed to be the home office of such company in the City of New York." The court held that under this clause the law of New York governed and furthermore that the notice required by the law of New York for business written within the state was a condition precedent to forfeiture.* The present case was distinguished from certain others by the court, because such other cases contained in the policy a distinct provision excepting the giving of notice from the general agreement, and the policy in this case contained no such exception.

That such decision with regard to notice of forfeiture is far-reaching may be seen when we consider that the question of notice is a question of fact, and, if there is contradictory evidence, would be decided by a jury. Also it would apparently make a contract reading as this one does, subject to both the requirements of the New York law and the requirements of the law of the state in which it was issued.

The decision would seem to be in conflict with the spirit, at least, of The Mutual Life Insurance Company *vs.* Cohen, decided in the Supreme Court of the United States in 1900 (179 U. S. 262). In this it was said, "These considerations lead to the conclusion that the statute of New York, directed as it is to companies doing business within the state, was intended to be, and is, in fact, applicable only to business transacted within that state."

* The law was changed in 1897 to read "... address in this state."

Right to Change the Beneficiary, Effect of:—(In re Edward L. Loveland, Bankrupt, U. S. District Court, Mass., 192 Federal Rep., 1005.) The policy was an endowment policy payable to insured if living at maturity, or to his wife, if living, in case of his prior death. A policy in favor of the wife is exempt by the Massachusetts statute. The court held that the policy in this case passed to the trustee in bankruptcy.

It must be remembered, in contrasting the decision in this case with certain other decisions as to the effect of the change of beneficiary clause where the beneficiary is wife, that the law of the state determines what property is exempt. It is, however, clear that, had no right to change the beneficiary existed, the interest of the wife, or any other beneficiary, would not have passed to the trustee in bankruptcy, and the question of the effect of the state law regarding exemption would not have entered.

Effect of Payment in Another State:—(Mary A. Gleason, Appellant, *vs.* Northwestern Mutual Life Insurance Co., Respondent, 203 N. Y. 507.) The policy was assigned to Mary A. Gleason, but, after the death of the insured, the administrator of the insured made claim and recovered judgment in Vermont. The question before the Court of Appeals was whether the judgment in Vermont was a bar to recovery by the assignee in New York. The assignee not being a party to the action in Vermont, the Court of Appeals held that such judgment in Vermont was not a bar, thus reversing the Appellate Division. (The decision of the Appellate Division was reported in the *Transactions,* Vol. XI, p. 517.)

Statements in Application:—(Becker *vs.* Colonial Life Ins. Co., Supreme Court of New York, 133 N. Y., Supp. 481.) The questions and answers of the medical part of the application were not referred to in the policy and no copy was attached. The question was whether they could be used in defense: the Court held that they could not. In view of the wording of the New York statute under which the question was decided, the decision of a higher court will be awaited with much interest. This case cannot be considered to follow Wheelock *vs.* The Home Life because the controlling statute is quite different.

Abstract of the Discussion of Papers Read at the Previous Meeting.

LIBERALITY OF MODERN POLICIES—HENRY MOIR.

VOL. XII, PAGE 175.

WRITTEN DISCUSSION.

MR. RHODES:

With much that appears in Mr. Moir's paper I am in very hearty accord. The paragraph in which he says "It seems to me that when any new feature is under consideration it would be well to ask ourselves the question 'should not we give the same concessions to our present policyholders;' and if we answer this question in the negative as regards existing policyholders we should be very careful indeed about incorporating it in the new contracts," should be kept prominently before the eyes of every life insurance executive.

I agree with the view that competition among the established companies is being reduced rapidly to a basis of cost. In addition to the temptation to adopt a dividend formula which favors unduly the recent issues, to which Mr. Moir refers, there is also the temptation to withhold from older policies the privileges and benefits that are granted to policies now being issued. If this latter temptation be not withstood the dividend fund will be swelled at the expense of the old policyholders, who would seem by all principles of fair dealing to be entitled to as liberal treatment as the new policyholders receive. The experience obtained under the old policies has made possible the granting of concessions to new policyholders, and I cannot understand why they should be deprived of the fruits of their experience.

I also agree with Mr. Moir that extended insurance should be granted upon a nonparticipating basis. The granting of participating extended insurance is attended by some very peculiar features. I have before me the record of a lapsed policy issued by a company which grants participating extended insurance. Assuming that the present dividend scale of the company which issued the policy will be continued, and comparing the mortality surplus which will be credited to the policy in its extended insurance form with that which would have been credited if the policy had been continued in its original form, I find that for each dollar of mortality surplus which would have been paid if there had been no

lapse the mortality element in the dividends on the extended insurance will range from $1.70 in 1912 to $3.05 in the last year of the extended insurance. If in lieu of participating extended insurance the policyholder had taken the value of the lapsed policy in paid-up insurance, for each dollar of mortality surplus which would have been paid on the paid-up policy the mortality element in the dividends on the extended insurance will range from $3.48 in 1912 to $6.40 in the last year of the extended insurance. It thus appears that in its extended insurance form the policy will receive a much larger share of the mortality surplus than it would have received if continued in its original form or as paid-up insurance. This result is completely at variance with all known mortality experience on extended insurance, and would seem (to adopt the language of a noted present-day writer) to knock the principles of the Contribution Plan into a cocked hat.

I cannot follow Mr. Moir in his statement that surrender values now guaranteed are too large. The remark is too general. A certain scale of surrender values may be entirely unobjectionable when used by one company, and very dangerous when used by another. The business of an old company with ample resources and a satisfied membership, which has always conducted its business on a conservative basis, cannot be judged in the same way as the business of a company whose history and condition are entirely different. Again, reference might be made to the experience of a company which adopted nearly forty years ago the principle of automatic extended insurance for lapsed policies. The plan was liberal at the beginning, but is much more so now. The present liberality would not have been justifiable at the start, but the experience gained under the original plan, as modified from time to time, has fully justified the company's course. It does not, however, follow that every other company should adopt this particular plan. There has not been enough independence in the life insurance business. Altogether too much regard has been paid to meeting competition. It is neither necessary nor expedient for any company to do everything every other company does.

Premising that my statements shall be understood as relating only to old and established companies, I desire to say that I do not agree with the view that the values now guaranteed are too large except where all surrender charges have been abolished. Assuming that Mr. Moir's statement relates to the surrender values now commonly guaranteed, I will say that I advocate such liberal surrender values for the reason that Mr. Moir gives for fearing them. It is for the same reason that I believe in small premium loadings. To my mind, liberal policy contracts and low premium rates, combined of course with the age and strength of a company, offer the greatest possible assurance for the future. It is true that managements may change either in personnel or purpose, but I know of no greater guarantees of continued safe and economical management

than a considerable amount of outstanding business bearing liberal policy conditions and low premium rates.

A great American actuary and lawyer several years ago stated that the truth was too clear to be disputed, that reserves are, mathematically, and in morals, the property of the person from whose premiums they have come. If he were writing now he would undoubtedly refer to the law also. We can well afford to adopt the statement. If there is to be an adjustment of surrender values, based upon the health of individual policyholders, we shall have an era of speculation the probable results of which I dislike to contemplate. So far as I know, all writers have agreed that surrender values should be based upon the average reserve.

I cannot agree that in the concrete case cited by Mr. Moir the policyholder acted wisely. The case is that of a man who was giving up a lucrative business to take a professional appointment at a very moderate salary. His future income would not permit him to carry all the insurance which he held, but he was not in immediate need of funds. Under these conditions it is clear to me that his proper course was to retain the insurance which was best adapted to his future needs.

At least one company, and one which was probably subject to a greater demand for policy loans than perhaps any other company, went through the panic of 1907 and consummated without any delay all policy loans which it was asked to make, without selling a single security. As I look upon it, the danger does not lie in the agreement to make policy loans on demand, but in making the agreement and then failing to make any provision to meet it. Doubtless there were policyholders in 1907 who secured policy loans for the purpose of buying securities at the prevailing low prices. There were other cases, of which a single instance will suffice. A large business was in jeopardy. The owner applied to the insurance company in his home town for a policy loan and was told that he would have to wait ninety days. He applied to another company and received his loan immediately. He saved his business and paid off the loan within ninety days. If he could not have secured an immediate loan his business would have been sacrificed or he would have been driven into the hands of a money shark and his insurance would have been sacrificed.

I believe thoroughly in the omission from policy contracts of the provision that the policy is not valid unless delivered during the continuance in good health of the applicant. It surely ought not to be within the power of a company to declare a policy void because at the time of delivery there might be some ailment concerning which the applicant had made no concealment or misrepresentation in his application, or which might have been incurred since his examination, and of which he might not be at all aware.

There are life insurance companies which are strong, and there are those which are weak. It is the business of the weak to become

strong and not to emulate the strong while they are still weak. They can become strong only by husbanding their resources. A stripling may injure himself permanently by attempting to perform a feat which a trained athlete may accomplish with ease.

The companies which have an especial reputation for liberal treatment of policyholders have pursued practically from the beginning a steady and consistent course. Their present methods have been evolved from their own experience, and there has been no experimentation. The practice of such companies should not be judged by the standards applied to companies very differently situated. If I were to attempt to define the term liberality I would say that in life insurance management it meant the recognition, in the light of knowledge and experience, of strict equity as between old and new members, as between continuing and retiring members, and as between the varying classes of policies. In other words, liberality and equity are synonymous. Equity cannot be determined without full knowledge of conditions, past and present, and these vary so much in different companies that what might be equitable in one company would be grossly inequitable in another.

MR. DOW:

The life insurance contract differs fundamentally from every other form of contract in two particulars; first, when once made under any restrictions or conditions you please, it cannot be broken by the chief contracting party; and, secondly, by its very nature, it could not possibly be made and successfully carried out as between two individuals. While the insured has the recognized right to terminate the contract at his pleasure at any time his financial condition changes or his need for protection ceases, the company must carry out its part of the contract to the letter, no matter how the original conditions may alter, or how desirable it may be to stop carrying the risk it has assumed.

While each individual contract contains a definite agreement to fulfil its promise of payment at death or maturity, the contingencies of the case depend not on known facts in regard to the future of the insured, but rather on known facts in regard to a number of such individuals of which, at the time of making the contract, the individual must be accepted as equal in every particular to the average man of the group in which he is originally placed.

It seems to follow, therefore, that while each contract is on an individual life, the assumption must continue that as far as the company is concerned he always must remain as a perfect illustration of this average individual. The sinking fund or reserve necessary to pay for each death as it occurs and the final payment at maturity for all who survive so long, while it is an aggregate amount for the group, must also be theoretically divided as belonging to each average individual of that group.

Regardless of whatever opinions we may have to the contrary, the lessons learned from our text books, and the popular prejudice of legislators and the insured are in favor of individual ownership, and it is our duty either to recognize that fact and admit its truth, or else demonstrate successfully that such is not the case.

Popular prejudice again has been strengthened year by year by the assurances on the part of everyone engaged in the insurance business that mutuality is the key note of the insurance problem; that the very fact that the contract may extend far into the future and cannot be broken, involves such uncertainties that premiums more than adequate for present known conditions must be charged; and since all are equal partners in the uncertainty of the future, it is imperative that such statements of gain and loss should be made from time to time as would inspire confidence that mutuality was a reality, and that no more was being exacted for benefits given than ought to be charged under the known conditions.

Mr. Moir has pointed out the number of restrictions which were imposed upon early contracts, and these were undoubtedly made with an honest intent to keep within the bounds of absolute safety. As time went on, it was seen that no one, either of those who remained in the company, or of those who died, or failed for any reason to continue, was injured by omitting certain of the restrictions and by granting certain privileges which were no more than fair when the individual directly concerned was considered as an average policyholder in every particular.

Liberality does not consist in doing what is absolutely right and is found by experience to be within the bounds of equity and safety. Liberality is of three kinds. We may think of it as freedom; freedom to choose any one of a number of things offered as best suits the fancy or needs of persons exercising the privilege. We may think of it as breadth; breadth of opportunity where an abundance of good things is offered together or in proper sequence; many of these offers may involve additional expense, and if so, and it can be clearly shown that they involve additional risk, no one for a moment objects to pay the price, if it can be shown to be worth the while. We may think of liberality also as generosity. This may be praiseworthy or blamable according as the generosity is warranted by the price charged, and if it is at the expense of all benefited, and not of some for the benefit of others.

My first definition covers such cases as a choice to use such dividends as are declared, either to reduce the premium, add to the policy in additional insurance, add to the policy as additional reserve thereby shortening the time of maturity, or to be held as a fund on deposit to be used when needed, or as a level annuity to reduce future premiums. Such a choice is freedom to use funds for any one of an equal number of benefits that may be shown to be mathematically equivalent. Such a choice as applies to the use of dividends is also applicable to cash values, which may be taken as

cash in part or as a whole, or applied as a mathematical equivalent to purchase paid-up or extended insurance, pure endowments or annuities. This freedom of choice belongs to modern policies merely on the assumption that cash is cash, and as such is good to purchase its mathematical equivalent in such commodities as the company has to offer.

It may take a long time yet to convince the theorist that such freedom of choice is safe, and it is far from established that there may not be selection against the company in the exercise of such choice; but I have yet to learn of a single company whose failure or even temporary embarrassment was the result of liberal surrender values, or freedom in its offers to policy-holders to make the selection they wished.

While "it is generally admitted that the mortality on extended policies is higher than that applicable to paid-up policies or those maintained by premium payment," and while this may be absolutely true, if either one of these methods of settlement in case of non-payment of premiums is made automatic, the results are on the same lives, and we could not expect a far different experience. Even if it were different, the degree of difference would, on an ultimate table of mortality, probably be within such limits as could be easily adjusted by dividend returns. I differ from Mr. Moir in the opinion that such policies of extension in a mutual company should be non-participating. It seems to me that the assumption that the reserve belongs to the individual contract (an assumption which the statute prescribes and the public believes in), makes it evident that every policy which holds any reserve however small and contributes an adequate cost of insurance for the risk assumed on an average individual should share in excess interest and such mortality gain as can be determined on the entire group of such insurance. Anything less than that is unfair. The mere fact that the share would be a decreasing one is not an argument for non-participation, and a small share is certainly better than none at all, for allowing none violates the principle of mutuality.

It is unjust to segregate the healthy risks since each was accepted as an average man at the time of entry, and he should remain the equal of every other of the same age, and any adverse contingency should be accepted by members as a whole. Mr. Moir frankly states that he does not favor the annual dividend system, but he describes with great fairness the arguments for it, since we are forced by statutory enactment to maintain a full net level premium reserve and to spread evenly over the duration of insurance the expenses applicable to a particular year. I have been trained to believe thoroughly in the annual dividend principle, and to use such principle consistently as a contingent immediate liability even where the dividend was deferred for a few years; and while the deferred dividend, if honestly carried out, might have encouraged policy-holders in persistency and a habit of looking into the future, I

question whether the advantage is not more than offset by the disappointment which has too frequently come when the hopes for the future were far from being realized.

My second definition of "breadth of opportunity" covers such privileges as grace in payment of premium, reinstatement, right to travel and reside in foreign countries, disability advantages, loan values, adjustment of misstatement in application, and options in regard to settlement. All these and more too are new features which competition has introduced and one by one have been generally adopted or made legally necessary. Such liberality where it can be extended to every policy-holder regardless of his length of membership may be perfectly fair to all, even if they incur, as some do, an element of expense; for if the total expense incurred is not too great and if all can be included as they are liable to need the privilege, no principle of fairness is violated, and the only danger is that competition may lead to too great a length.

The third view point of liberality as I have defined it is that meaning which I call generosity. It is not a virtue to be generous with the property of others. The fundamental principle which must underlie all efforts on the part of the company to have a liberal contract to the point of generosity is that no one should have privileges which he does not ultimately pay for. No one should enter the company under favoritism or leave the company with a burden of debt which he or his class has incurred and others will have to pay. The payment of larger dividends to new policyholders than can be apportioned to all on the same equitable basis (allowances being made for kind of contract, gross premiums paid, and reserve basis maintained) should be condemned in the strongest manner. Where deferred dividend policy-holders or old policyholders of the annual dividend class are made to suffer that larger annual distributions may be made to new members for advertising purposes, there is the same species of injustice which tempted companies to incur extravagant expenses met from funds which should have been held in trust for the deferred dividends. I like particularly well what Mr. Moir has to say on this subject.

In this same class of doubtful liberality comes the subject of competition in surrender values. It is the duty of company officials to preserve justice as between individual members and also collectively. Mr. Moir says he has always been an advocate of liberal surrender values, and he further thinks that those who feared selection against the company in case of withdrawal were not warranted in their belief; but he states that he is strongly of the opinion that the pendulum has swung too far. Perhaps this is true, but I am not prepared to assert it yet.

The total business of any one year of issue on the books of a company has cost, it is true, an expenditure of a larger amount than the expense loadings of the business itself; but if that business was bought and paid for by an outside party on condition that he would

receive such portion of the loading as is not returned in dividends or required for annual expenses during the continuance of their persistence, I am convinced that the business would be an exceedingly profitable one. If this would be profitable for an individual, it is certainly profitable for the policy-holders. Business that has passed its first year has a tangible asset in the present value of future loadings in accordance with the usual persistency of the policies issued, and the actual mortality cost for the first three years is small.

All the policy-holders, from the instant they enter into partnership with others in the company, enjoy all the privileges of membership. The capital expended in securing their admission is not to be paid back at once. They share with others the interest earned on the mean invested assets, and so far as there is an undivided surplus, the benefit of interest earned thereon accrues to all.

The surplus which is not to be divided and constitutes the contingent reserve, may be 5 per cent. or larger, and everyone who withdraws even his full reserve in surrender leaves some charge behind in his share of the contingent reserve. The item of initial expense seems to me to be the only logical reason to justify a surrender charge in any case, and after that charge has been met by the entire group, the full reserve can surely be paid. Let me illustrate by taking an extreme case. Assume that one-half of all the business in force terminates by surrender. Suppose these surrenders to be evenly distributed by ages and kinds except that all who withdraw are healthy lives; and suppose also that as many deaths occur from those who remain as would have occurred if no surrenders had taken place. Suppose further that the contributed costs of insurance are reduced one-half by this wholesale termination, and compare statements that would be made under this supposition.

I.		II.	
$200,000,000	Insurance in force.	$100,000,000	Insurance in force.
50,000,000	Reserve.	25,000,000	Reserve.
2,500,000	5% contingent reserve.	2,500,000	Cont. reserve now 10%.
2,000,000	Death claims.	2,000,000	Death claims.
1,000,000	Net death loss.	1,000,000	Net death loss.
1,333,333	Costs of insurance.	666,667	Costs of insurance.
333,333	Mortality gain.	333,333	Mortality loss.

In the second case we have a mortality loss as great as the gain in the first instance, but observe that the contingent reserve is now 10 per cent. and that would free one-half and still keep the same margin of safety. If the dividend scale of the company remained unaltered, after making good the mortality loss, there would be $916,617 of released contingent reserve which would take the place of the mortality gain of $333,333. It would be difficult to imagine a more extreme case than this, and it would be extremely unlikely

that retiring members would not release more surplus contingent reserve than they would deplete the mortality savings.

I will not add anything to what Mr. Moir says on the matter of selection, for I agree with it all.

The great danger which is before us is not in the liberality of our present contracts as they are written, nor in our lack of ability to carry out all the benefits to the policy-holders with safety and fairness, but it lies rather in the chance of too great liberality in the items of expense or in the possibility of undesirable conditions enforced by further legislation. We must take the public so completely into our confidence as to make them feel that they are getting all that is theirs by right, and all that can be done in conformity to the insurance principles that are mathematically sound and so thoroughly established.

ORAL DISCUSSION.

MR. DAWSON: Mr. President, it seems to me that Mr. Moir's paper of last October was exceedingly timely, and it is particularly fortunate that it should have been written by one whose attitude towards liberality was so well known and so well known to have been favorable.

There certainly are possibilities where everything is set forth in the policy and little or no option is left to the company, that too much may be undertaken. That possibility is perhaps increased by the direction of competition in recent years to such features, instead of the more or less unreliable estimates of possible results in consequence of deferring payment of dividends. And it seems to me that very special attention may with great propriety, and doubtless with best results, be given to this subject.

It is desirable, it seems to me, that we take into account what has been the evolution of such policies in our country.

Unfortunately, in the early history of life insurance in the United States, our companies as a whole, on account of the action of several of them, indeed, of many of them, were in danger of achieving for themselves the reputation which was afterwards obtained by a great assessment society, of being "the great repudiators." There was at that time a marked disposition on the part of our life insurance companies,—a thing which is now almost impossible for us to conceive,—to be litigants, and to be particularly and peculiarly unfair as to anything which was not nominated in the bond. The law books up to about 1880 contain an enormous number of decisions concerning life insurance in matters where the companies were technically right in most cases, but were morally wrong. In addition, we were very slow, largely because we were attempting in those days to realize big estimates of annual dividends, to adopt even reasonable provisions in regard to the surrender values. Unfortunately, likewise, the first move in this country towards liberal surrender values took the form of compulsion by the state, a thing

always hateful, and even when exceedingly desirable, which I think most of you will now admit the Armstrong legislation was, it is very much and very naturally resented because it is compulsion, too. In consequence, the adoption of liberal surrender values was greatly delayed in this country, and, during that time, a great many of our companies were exceedingly illiberal when called upon to do anything that was not "nominated in the bond"; and it was out of this that the elaborate system of stating definitely the surrender values in life insurance policies grew up. It is a system not to be found, I think, in any other country, unless it be Canada, which has taken it from us.

For instance, the policy of a company which is generally believed to be the most liberal to its policyholders of all the companies in the world, the Australian Mutual Provident Society, does not contain the amount of a single surrender value. It contains no tables—nothing whatever to indicate what the company will do for the policyholder. And yet, the policyholder who asks anything of the Australian Mutual Provident Society, is as certain to receive terms as liberal as it is possible for that great mutual company to give, as he would be if everything were "nominated in the bond," as I have put it.

Now, that is the result of a different condition. It is not necessary and it was not necessary in competition with our American companies for that company to take the course of inserting these values in its policies.

I do not speak of this by way of reproach. We have travelled long and far; perhaps, as has been pointed out, too far, in some cases, in the direction of liberality; and it is not impossible that the evolution which we have undergone is better for the policyholders and for the companies themselves than it would have been had the other tradition been established among us.

I think it is well to give attention, however, to this. There is still enough fallacy concerning what is the basis upon which the values of a policy should be computed so that in some cases perhaps too large surrender values are being offered in the early years of life insurance policies. There is no question that those values should not be based upon anything artificial, and that due and very careful attention should be given to the proposition that all the initial costs should be repaid before a surrender value is allowed, and that the surrender value should not be greater than is possible after those costs have been repaid.

In regard to extended insurance, we have also taken a course of our own, although in recent years there has apparently been a great increase in the popularity of a different method of extending the insurance. It seems to me that many of the questions which have arisen in connection with that subject and which still arise to plague us, could never have arisen, had we taken the very much more natural way of continuing the policy in force when the premium failed to be paid when due, viz.: to charge that premium

against the policy, treat it as a policy of precisely the same character as before, credit it with the same dividends, and thereby not merely keep it in force so long as possible, but keep it in force in such a manner that the policyholder could go forward with it, without prejudice. It is perhaps too late to make that change now altogether, although I hate to think so.

I am not disclosing any confidences when I say to you that the chairman of the special sub-committee of the Armstrong Committee which was considering the amendment of the laws had selected (both things being presented in a bill prepared by myself) for the proposed laws of New York, this method of making premium loans automatically and that this, together with some other things which were actually preferred by the committee and by its counsel, was abandoned for the reason that no evidence had been taken by the Committee on which to base this recommendation. In other words, that hard and fast rule, doubtless a wise one, but which, had it been known earlier, would have resulted in evidence being put in to warrant it, changed that view. Had such a feature been adopted, this proposition of making extended insurance participating would not of course have been ventured upon, because our general form of extended insurance as an automatic feature would have passed out of the policy.

It seems to me that might have been a very good thing, but, unfortunately, the other course was taken. A marked demand for that particular form of extension given by the automatic loan has since then shown itself in the construction of policies, notwithstanding.

There is one criticism in Mr. Moir's paper which has been made by Mr. Rhodes, which I think ought not to be permitted to pass. I think it to be entirely unsound, and one to which your attention should be very particularly directed. It is, that it is safe and proper for one company, being an old company, well established, to offer things which a new company must not offer.

The equities in regard to life insurance are not based upon the company. They are based upon the premiums which the policyholder has paid and the contract which he has purchased. It is neither proper in my judgment, nor is it even feasible, for the smaller companies of the country to wait until they have become old and prosperous before they offer policies as desirable in all respects as those of the older companies. The only consequence, if they fail to do so, is that their policies must be sold either under actual misrepresentation or by concealment of the fact that they are not as good as the policies offered by the other companies.

The surrender values, and the other liberal features of life insurance policies are not paid for, or should not be, by other policyholders, but by the policyholder who purchased the contract; and if there is a sufficient number of policyholders who have joined together, to enable the company to have a safe average, and, therefore, to have a safe business, it seems to me that it is perfectly clear

that everything that reasonably and properly can be done, can be done by the new company as well.

This is not merely theory. It has been borne out by the history of life insurance in all countries. It is by no means exclusively the largest companies that have granted the most liberal surrender values and have safely done so. Thus, the Australian Mutual Provident Society, to which I have already referred, commenced this liberal feature of automatic policy loans and very liberal features in regard to cash and other surrender values when it was one of the smallest insurance companies in the world; and it was because, and chiefly because, in my judgment, its long experience with such a course proved to be safe and prudent, that we take a different view to-day from what we did twenty years ago, when that company's policies first came to my attention and I had the privilege of bringing it to yours. In our own country the most liberal surrender values were for many years offered by a small company, a very small company in those days, up in Vermont, on the recommendation of Elizur Wright; and, had Mr. Wright never become Commissioner of Massachusetts and never imposed upon any companies by force a system of surrender values, but had confined himself to recommending it to other companies as well as to the company which I have mentioned and which became one of his clients, it is not improbable that we would have been much further along that road at a much earlier date than proved to be the case.

It has not been my experience as a consultant that liberal surrender values have proved a source of annoyance to my companies. Small and young companies which have been liberal in this regard, if properly and prudently conducted, have got along as well as other companies. The very first client I had as a consultant was given that recommendation, and never had any unpleasant experience in connection with the policies which it so issued—policies which received very high praise from my revered friend, the predecessor of Mr. Rhodes as actuary of his company.

It should be added, perhaps, that nothing can be done which creates so great a probability that a company will be prudently managed, and so as to please its policyholders and retain their confidence, as that it gives hostages for such conduct in the form of so liberal surrender conditions that the policyholder at all times remains such *con amore* and not because of any feeling of compulsion.

We can scarcely do better, it seems to me, than to do this—Adopt for our slogan in connection with American life insurance policies: "They shall be as liberal, and even as generous, if you please, as is safe, prudent and equitable for all concerned."

MR. MOIR:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

As the time is limited I shall make my reply as brief as possible. The paper was prepared because of a general trend which had im-

pressed itself on my attention after the passage of the Armstrong Laws in 1906, when companies seemed immediately to strain and strive after new features of popular interest, in my judgment going entirely too far in one or two directions. I selected many incidental features in order to emphasize, and call attention to, the possibility of our being too bold as the result of pressure by agents and outsiders for liberal contracts.

The question of surrender values is that which has been most dealt with in the discussion. Where would one or two of the companies have been, which were subjected to so much undeserved criticism in 1905–6, if they had been then guaranteeing full reserves on cash surrender and also paying annual dividends? There would have been a stampede from some of the best companies. The Armstrong Investigation is now past; the excitement and feeling is over for the present; but other inflammatory incidents are likely, indeed they are certain to come, although the time, the cause, and the companies affected, may not be foreseen. At some time, some company, or a whole group of companies, will again be subjected to similar strain, with sensationalism which may cause an immense economic loss. The company may be very excellent now, yet, by imprudent or unfortunate investments the contingency reserve may disappear. If there were also a public clamor, there would be a run on such a company for surrender values and under such circumstances the full reserve guaranteed by so many companies is a real danger.

Mr. Dow practically admits the possibility that one-half of the entire business of a company might be discontinued under circumstances of this nature. This admission is enough to enable a broad gauge and well trained insurance man to work out the ultimate condition of the company, and perhaps I could properly leave the fallacies in Mr. Dow's figures to the observation of students and readers. When a company reaches this stage of public criticism, it usually has a noticeable padding of assets with many doubtful items, and rarely if ever has a 5 per cent. contingency reserve of free surplus. The fixed charges are not cut down automatically by the reduction of business; the death claims are not ended after one year, but the heavy rate may continue for a considerable time; a reduction in the dividend rate, if of serious magnitude, would encourage further surrenders; the production of new business would be more expensive because more difficult; and generally speaking the condition of that company would be a most unenviable one; the poor remnant of policyholders would feel continually unsafe, while those who gave up their policies have lost that which we are continually striving to strengthen—namely, their belief in the soundness of life insurance.

Mr. Rhodes quoted a remark with approval that the reserves in mathematics and in morals belong to individuals, setting this down as being almost a fundamental proposition; and Mr. Dow even says that the lessons learned from our text books favor individual

ownership. I am sure that both are mistaken as to the attitude of the best authorities on this subject. The weight of actuarial authority is undoubtedly against any such individual ownership, although the popular prejudice of legislators does lie in the other direction. The fact that Mr. Dow calls it "popular prejudice" is significant! Personally I have the old-fashioned notion that the reserve value, the individual reserve value, is only an incident, grossly incorrect in individual cases and only actuarially sound *on the average*. But there is no time to enter on any extended discussion of this subject.

Like Mr. Dawson, I am of the belief that the doctrine that a strong company may properly do those things which a weaker company may not do is a very dangerous one, and one which we, as a body of scientists, should scrutinize with suspicious care. Mr. Rhodes' argument regarding the delivery of the policy whether the health of the applicant may be impaired after the date of application seems to ignore the vital points in this question. In the first place, any applicant in good faith can secure from any company a policy which will go into force immediately on acceptance, by the simple expedient of paying the first premium when he makes application. The plan to which I object allows men to get an expert medical opinion on their health as well as some temporary insurance protection from the company without paying anything for such benefits—a condition wrong fundamentally.

The only other point to which I shall refer relates to Mr. Dow's discussion where he says "that cash is cash, and as such is good to purchase its mathematical equivalent, etc." With this as a general proposition I entirely agree; our difference arises in the decision as to what the mathematical equivalent may be. For example, until quite recently it was considered that if paid-up insurance was purchased by cash on the basis of the American Experience Table with 3 per cent. interest, the insured got a mathematical equivalent although the company might be earning nearly 5 per cent. interest and have a mortality experience of 80 per cent. of the American Table. But we have quite left behind us any such idea of mathematical equivalents. Theoretically it may be all right to make extended insurance participate in the surplus; but in such case the expense of caring for the business, as well as the extra mortality to which such policies are subject, should be met by the extended insurance class. If these extras are thus properly assessed my claim is that dividends will be relatively small if in many instances they do not disappear entirely. This feature, combined with the dissatisfaction and disappointment which will exist in the most favorable circumstances on account of the decrease in dividends from year to year, leads me to the conclusion that the wisest course is to leave such extended insurance on the non-participating basis to which we are all accustomed.

A PENSION FUND METHOD—C. C. FERGUSON.

VOL. XII, PAGE 192.

WRITTEN DISCUSSION.

MR. DAWSON:

The method advanced by Mr. Ferguson is, in short, this: Assuming a contribution, by the employe, the employer, or both, of a certain percentage of the salary, to be applied to purchase (*a*) life insurance to age 65 for a given amount, (*b*) disability insurance to the same age for a given amount, and (*c*) a pension for whatever amount the remainders of the annual contributions will purchase, apply the contributions each year to the purchase of these several benefits. He gives also a card account with such an employe, showing how this would work out, upon certain assumptions concerning salary, life and disability premiums, and annuity rates.

Undoubtedly this method is specially suitable for the case of a small force of employes, altogether insufficient to supply a basis for safe averages. What Mr. Ferguson does in applying it to his own company, is presumably to transfer these employes, as regards their life and disability insurance, into the body of the company's policyholders, and, as regards pensions, into the body of its annuitants, thus assuring a basis for safe averages. Such a method may even be necessary when the number of employes is small; indeed something of that nature, i. e., calling for the purchase elsewhere of the insurance and the annuities usually is deemed requisite in such case.

There is also a great deal of flexibility in the method, as presented; as, for instance, that the insurance could be increased or diminished, leaving less or more to be applied to the purchase of pensions.

It could be employed, of course, in connection with an enterprise employing a large number; there is nothing in the fundamental nature of the proposition that would render this impracticable.

There is the very obvious advantage that, when an account is kept in this fashion, the employe can at any moment be permitted to inspect it, and will see at a glance how the contributions have been paid out for his benefit.

Mr. Ferguson is also right in remarking that "the special difficulties in this connection have usually arisen on account of the failure of the originators of the schemes to provide a definite actuarial basis." The literature respecting actuarial valuation of pension funds has been produced chiefly in response to demands for valuation and readjustment of benefits and contributions in funds

which were established without consultation with competent actuaries, and the difficulties are largely, but by no means solely, due to that.

Mr. Ferguson's method, however, will hardly answer the requirements when a corporation employing many thousands desires to provide a scale of definite retirement pensions for superannuated employes. Such an employer, if desirous of doing this by creating a fund, will perhaps put before the actuary the problem of advising what percentage of the payroll must be set aside each year to provide a pension equal to a certain percentage of the average wages for the last ten years of service, or even the wages for the last year of service. In such event it is expected of the actuary that he name such a percentage—or, it may be, percentages, varying with age upon entering the service. This problem, also, will often be further complicated by fixing the pension at a different percentage per year of service for different terms of service. It will not be acceptable, of course, that the actuary merely respond: "If you will make the contribution such a percentage of the wages, I can apply it to the purchase of deferred annuities and whatever annuity it buys, depending upon the way the employe's actual wages run, will be paid as a pension." The employer, though talking of contributions and a fund, virtually always has in view a simple, definite scale of retirement allowance and wants that worked to, instead of giving the actuary the easy task of buying with a given contribution whatever annuity it will pay for. Though the employer talks of a pension fund, the idea really before both employer and employe is service pensions; and computations which do not supply pensions having a definite relation to the wages and term of service will not answer in most cases.

If it be said that, with the rates for life and disability insurance and for deferred annuities supplied, the percentages of contributions can be computed under Mr. Ferguson's method very readily so as to produce any desired amount of pension, it is at once seen that this is not true, because there must also be assumptions as to rates of increase in salary, as to rates of withdrawal, etc., before the problem is soluble. All Mr. Ferguson will really have supplied is guarantees as to mortality and interest rates and as to disbursements for insurance and annuity expenses. These are three factors which, when the numbers are large, are not the most difficult to approximate; when the numbers are small, they will, of course, be found by purchasing these benefits from insurance companies. But one difficulty is that some employes may be uninsurable. To be sure, if a very large number are taken at one time, a life insurance company might accept them all without medical selection; but that is precisely the case when it is not necessary to purchase insurance and few companies would so accept a small group.

Mr. Ferguson remarks, naively: "As only a portion of the reserves on the pensions would be allowed on resignation, the with-

drawals would leave a profit in the fund which might be applied in liberalizing the disability or sickness benefits. Also the funds could sometimes be invested at a higher rate than 4 per cent. and the profit from this source, as ascertained by a periodical valuation, could be applied in adding bonuses to the pensions."

This seems to imply that the annual remainders are not actually applied to purchase deferred annuities and that merely an account is kept as if they were. If so, and if the fund accumulates at interest only, it would not produce so large a pension and, if individual shares were forfeited at death, whether it would or not, would depend upon whether in this case $O^{M(5)}$ mortality rates were realized in practice.

Mr. Ferguson must here be thinking of a peculiar, private arrangement of his own office with its employes, whereby these annuitants are granted surrender values—which is not usual—and also participate in interest earning over 4 per cent. per annum. These are much more favorable terms, of course, than can usually be secured. Thus, if I mistake not, Mr. Ferguson's company values at 3 per cent. Certainly rates at $3\frac{1}{2}$ per cent., without participation, are as good as could usually be secured.

As conceived by employers, a pension fund plan involves much more than merely incidental gains from withdrawals. The thing inquired about usually is the required rate of contribution as a percentage of the *entire* payroll though all employes who fail to serve a certain number of years, as ten or even twenty, are not to receive any benefit. Mr. Ferguson's comment, therefore, that in such cases "the desired result"—i. e., the rate of contribution required to furnish a given rate of pension—" is only achieved in a very rough way" is merely a statement of what must be, if the problem is to be solved at all; his method merely evades solving it.

MR. FERGUSON:

(AUTHOR'S REVIEW OF DISCUSSION.)

I desire to express my thanks to Mr. Dawson for the lengthy and, on the whole, favorable criticism he has given of my paper. I gather, however, that he thinks my method unsatisfactory in that it does not apply to pension funds conducted on the usual lines where the amount of pension is not actuarially related to the individual contribution. My suggestion, however, was that employers might be induced to accept a plan with a better scientific basis, and I think that the results from a system such as I have proposed could, without difficulty, be translated into the terms with which employers have been familiar.

Certainly the usual plan of determining a pension by allowing for each year of service a certain percentage of, say, average salary, is not very simple and not very definite when viewed in advance. An illustration of what the average pension would be on the basis

of an assumed average salary would be quite as illuminative. If, then, a certain employe received say twice the average salary, his pension would be just twice the pension shown in the standard illustration. The only factor to disturb the proportion would be a variation in the rate of salary increases, which would not be so very important in obtaining a general view of the outcome. Further, the illustration could be made on several different bases showing various assumed types of increasing salaries. In any event the amount of the pension cannot be exactly foretold, since under all pension methods it depends upon the future salary which will always vary according to the individual's ability and success.

The various improvements which have been made in pension schemes by bringing in the factors of length of service and average salary are simply attempts to roughly relate individual pensions with individual contributions, and the question still stands: Why not make the relation exact? This was the problem to which I addressed myself and I do not think that it has been evaded.

My purpose was to describe a method which could be used in a large concern without necessarily purchasing the insurances and annuities from an insurance company, though it might be found advantageous to do so. If the group were small, the insurances could, no doubt, be obtained from a company if the members were in average good health; otherwise, the insurance feature would require to be dispensed with—a result which would necessarily follow as well under any other scheme.

Where a firm was large enough to administer its own fund without recourse to an insurance company, there would naturally be a profit from withdrawals (even where surrender values are allowed on resignation, which is frequently the case) and from interest, if the rate earned proved to be in excess of the rate assumed. There might, on the other hand, be a loss from mortality. The balance of this profit and loss could be applied periodically to augment the annuities which would be annually set up according to the original tables. Of course there might not be any profit from interest over an assumed rate of 4 per cent. This would depend largely upon locality, and I may be personally excused for suggesting a profit over 4 per cent., placed as I am in a locality where insurance companies have been earning over 7 per cent. for many years. To avoid any misunderstanding, I may say that in the plan adopted by my own office, the contracts issued are non-participating and the rates include the usual loading. On this account a larger proportion of the cost is borne by the company than would likely have been the case had participating contracts been issued.

MASSACHUSETTS SAVINGS BANK INSURANCE.—ROBERTSON G. HUNTER.

VOL. XII, PAGE 196.

WRITTEN DISCUSSION.

MR. H. E. RYAN.

It is perhaps natural that the views expressed by Mr. Hunter on this subject should be shared very largely by one who was closely associated with the Massachusetts scheme during his incumbency as State Actuary, and who, therefore, has observed it from much the same standpoint. There are one or two points that have occurred to me, however, which possibly may add something to the general fund of information on the subject.

The savings bank plan was conceived as a substitute for that form of insurance commonly known as industrial insurance. To fulfill properly its mission therefore it would seem that its primary object should be to reach the masses—those people who now patronize the regular industrial companies. The banks are able to furnish their insurance at a low cost to the policyholder. This is due of course to state aid, private contributions to the propaganda work, and to selection.

There is no question that any insurance institution—whether it be a company, a bank, or other organization—can save in cost of production and maintenance by making a careful selection of business. In savings bank insurance this is done in several ways. Applications are not much sought in industries where the wage is low or where the grade of employment is poor. The immediate effect of this form of selection is to increase the average premium to be handled. More remotely there is a saving on account of better mortality. Again, the monthly, not the weekly, premium is the unit of collection. By soliciting active workers a good preliminary selection is effected which keeps down the declination rate and hence lessens the cost of examining poor risks. As no small children are accepted for insurance by the banks, another means is found of dealing with relatively large units of insurance and premiums. The greater part of the infantile insurance written by the companies is at premiums of from three to five cents per week, whilst a large percentage of the insurance issued by the banks is on the twenty year endowment plan and at quarterly premium rates.

The difficulty of handling monthly premium collections without agents is no small one. Were it not for an ingenious substitution of the policyholder's employer for the house-to-house collector the State probably would have to supply the necessary machinery and bear the consequent expense. The banks could not employ collectors

and furnish insurance materially cheaper than the industrial companies can do. The method of collecting premiums through factory payrolls makes for efficiency and low ratio of expense. Another means to the same end is the conjunction of savings bank insurance with the work of industrial relief associations.

Many of the larger manufacturing corporations have amongst their employees some form of mutual aid association, and the tendency seems to be in the direction of encouraging such organizations. The usual form of association is voluntary as to membership, and provision is made for payment of disability and death benefits. Here savings bank insurance should find a field for useful work. The "natural death" benefit promised by these associations should be safeguarded by adequate premium charges and reserve accumulations. Such matters are hardly functions of voluntary associations unless placed under state supervision. A number of such associations have arranged with savings-insurance banks for policies on their members, and it is not unlikely that the corporate powers of the banks may be enlarged so that the disability indemnities may be similarly underwritten by them.

Mr. Hunter mentions that the banks do not attempt to insure children. Under such a system children are not easy to reach. This brings out a phase of industrial insurance that cannot be disregarded. The present system of weekly premium insurance owes a very large part of its success to the woman in the home. She counts on spending a certain weekly sum from the family earnings for insurance, both of the children and of the adult members. She will often forego many small luxuries and even some of the necessaries that the insurance may be kept in force. She is the one responsible dependent of the family, and self preservation inculcates a certain habit of thrift which, forced out of its state of inertia by the insurance agent, results in protection for the family to such extent as she can afford the outlay. It may be argued that the same instinct which urges her to maintain the small insurance can be developed in the direction of saving in sums large enough to maintain a savings bank account, in which event she would come into contact with savings bank insurance. It must be remembered, however, that industrial insurance deals with a non-saving class of people. Many of those who are in far better circumstances fail to save voluntarily. It has been well said that "human nature is weak and good resolutions to save frequently come to naught." If this is true generally, how much more is it true of industrial workers whose lot in life is the least prosperous—to whom so many of the necessaries appear as luxuries?

One can hardly fail to observe the significant reference in Mr. Hunter's paper to state insurance. At the time when savings bank insurance was first promulgated, the United States was little concerned with that question. Within the past three or four years we have seen a great public sentiment develop with regard to the com-

pensation of workmen injured in the pursuit of their daily occupations. In the various states and by the federal government there have been legislative inquiries looking to the better protection of industrial workers. The old common law defenses of contributory negligence, assumption of risks incident to the trade, and the fellow servant rule are being swept away. In the states of Washington and Ohio there have been established quasi-state insurance systems, and in many of the most important industrial states there have been passed or proposed laws which indicate how determined an interest the state is taking in the question of compensation. Many to-day believe that it is the function of the state to supply insurance or compensation to its workmen. In view of these recent changes in public sentiment, Mr. Hunter's references to state insurance are exceedingly pertinent. In the field of life insurance the State of Wisconsin takes a somewhat longer step than Massachusetts has done. Here the State itself will be the insurer through a system conducted by the Insurance Commissioner. There is no intermediary, like the savings banks, for issuing policies and keeping the accounts. The agency medium is provided by law, which designates certain county and town officers and state banks to receive applications and premiums for transmission to the Insurance Department.

One interesting and significant phase of savings bank insurance which may be mentioned is the seeming hopelessness of interesting young women and girls employed in industry in matters pertaining to thrift generally and especially insurance or annuities. It is said of Massachusetts workingwomen that fully fifty per cent. either continue in industry after marriage or else do not marry at all. This means that fifty per cent. of these women have their own future to provide for, and yet it has been the remarkable and discouraging experience of savings bank insurance that they will not listen seriously to attempts made to stimulate thrift until, because of advanced age or physical disability, insurance passes beyond their reach. Annuities have not become popular because of the apparently high cost and the long period of deferment. Most of the young women seem to look upon age 60 or 65 as a time too remote to worry about.

In an effort to popularize old age annuities, a special form of policy, combining term insurance to age 65 with a life annuity commencing at that age, was put forward. From the outset, the policy has proved the opposite of popular. Perhaps one reason for this was the fact that no cash surrender values are granted. But the main objection appears to be its composite (and hence intricate) character, combined with the long period of deferment. People who consider such matters at all seem to feel that their money can be invested to far greater advantage than by purchasing deferred annuities. Annuities without insurance features are seldom purchased from the banks and then not by industrial workers.

MR. J. M. CRAIG:

Mr. Hunter says in his paper,

> "The interest manifested in this movement not only by insurance men, but also by economists and sociologists has led me to give a brief sketch of its history before analyzing the problem that was presented to those who were entrusted with carrying the law into effect."

It is pertinent to give a few extracts from the chief advocate of the scheme as found in *Collier's* for September 15, 1906.

> "Industrial insurance is simply life insurance in small amounts of the kind commonly taken by the wage-earners. In the United States the policies average now about $140. They serve mainly to provide funds to meet the wage-earner's heavy expenses of the last illness and a decent burial.
>
> . . . . . .
>
> The peculiar features of industrial as distinguished from ordinary life insurance are:
>
> (*a*) That the premiums are fixed for all ages at five cents or multiples thereof, the variations for different ages being in the amount of insurance so purchased; whereas in ordinary life insurance the variation is in the amount of premium.
>
> (*b*) That the premium is payable weekly; whereas in ordinary life insurance the premium is payable annually, semi-annually, or quarterly.
>
> (*c*) That the premium is collected from house to house; whereas in ordinary life insurance the payments of premiums are commonly remitted by mail, or are made at the office of the company or of its agents."

We quote from the same person as given by Mr. Hunter,

> "The sacrifice incident to the present industrial insurance system can be avoided only by providing an institution for insurance which will recognize that its function is not to induce working people to take insurance regardless of whether they really want it or not and can afford to carry it, but rather to supply insurance on proper terms to those who do want it and can pay for it; an institution that will recognize that the best method of increasing the demand for life insurance is not eloquent, persistent persuasion, but, as in the case of other necessities of life, to furnish a good article at a low price."

It is quite evident from the above that the class of people for which this savings bank life insurance was intended was the same class that was reached by industrial companies.

The People's Savings Bank and the Whitman Savings Bank combined reported in force, for the year ending October 31, 1910, 3,318 policies covering $1,367,363, or an average of $412 per policy.

Whatever else this may be, it certainly is not industrial insurance, and it requires a wide stretch of the imagination to place a group of policies where the average is $412 in the same class with another group of policies with an average of $140. Moreover, the Whitman Savings Bank for the year ending October 31, 1910, cancelled 814 policies, which represented 77½ per cent. of the number written, and the People's Bank, which did not begin business until November 2, 1908, or over four months later than the Whitman Bank began, cancelled 522, which was the equivalent of 60 per cent. of the number written. The combined gain for the year was 797 policies for $374,602.

As against this record four districts in the city of Boston of one industrial company made a gain during the year 1910 of 7,549 industrial policies for $1,069,096. Including five hundred dollar intermediate policies, the number would be increased to 8,268 and the amount to $1,428,596. The ordinary business, including intermediate, of the same company in the same territory, showed a gain of $1,750,690.

It appears from the above that this company, in the city of Boston alone, showed a gain of more than ten times the number of industrial policies (including intermediate) and nearly four times more in amount insured than the total gain throughout the State of the two banks combined.

In view of the fact that these savings banks were authorized to transact the business of life insurance on the distinct understanding that it was their function "not to induce working people to take insurance regardless of whether they really want it or not and can afford to carry it, but rather to supply insurance on proper terms to those who do want it and can pay for it," it would not seem possible that such a large rate of cancelled policies could exist, but, as the official report shows that the cancellations did occur, we are driven to the conclusion that the idealists must have fallen from their high estate to the level of the regular industrial insurance companies whose methods were so severely denounced.

When we look for a solution of this anomaly we find it in Mr. Hunter's paper:

> "Attractive circulars illustrating the benefits of savings bank insurance were inserted in the pay envelopes week after week. Meetings of employees were held at the noon hours, at which addresses were made on the subject of savings with particular reference to savings bank insurance. Labor unions were memorialized and urged to adopt the movement as part of their platforms. A corps of speakers were schooled in the fundamentals of life insurance and sent broadcast over the State to address labor unions, civic bodies and clubs of all kinds. In fact everything was done that ingenuity and energy could devise but the results were meagre."

> "At this juncture, there appeared upon the scene a prominent manufacturer in Massachusetts, who was very much interested in the success of the movement. Recognizing the greater driving power of the 'spoken' than the 'written' word, he introduced into his employment an industrial agent and gave him the run of his factories. In other words, at his own expense, he introduced the 'agency system' of life insurance companies into his own plant."

So it seems that the man who was the chief offender in industrial insurance and who was to be wholly eliminated in this ideal system had to be employed to save the scheme from utter failure. We are not told whether this industrial agent applied for and received an agent's license from the Insurance Commissioner. It would be interesting to know on whose behalf he would apply for a license. He certainly could not apply for one to work for the savings banks because that is prohibited; and it does not seem as if it would be lawful for him to apply to act as an agent for an individual, because an individual is not authorized to transact the business of life insurance. The fact remains, however, that an industrial agent was employed to solicit life insurance in violation of the clear intent of the law authorizing savings banks to transact this business.

In another part of Mr. Hunter's paper he says:

> "About eighty employers of labor were agencies for the banks and in most cases had employed an insurance agent to interview the men at their work."

It is probable that the word "solicit" would more correctly describe the work of the agent in this quotation than "interview."

It is quite evident from the above that the savings banks have at least learned their lesson that "over the counter" insurance is not a success.

Incidentally, in this connection, it may be mentioned that in the same year the Savings Bank Insurance Act was passed in Massachusetts there were two charters granted for companies to transact "over the counter" insurance. One was called the Mutual Direct Life Assurance Society, and the other took the title of Economic Life Assurance Society. The Mutual Direct Life Assurance Society was granted an extension to May 3, 1909, within which to organize, and an application for an extension of time on behalf of the Economic Life Assurance Society was also made. No complete organization was ever made of either Society.

Mr. Hunter states,

> "The financial condition of the insurance department of the banks was very satisfactory."

This is not to be wondered at when the State pays for the services of the Actuary, State Medical Director, and all policies, applications and blank forms and books used in the transaction of the business.

If this were all, there would be no occasion for criticism except by the advocates of the scheme, because the results would be so meagre as to discourage even its most ardent defenders, but when the agency through which alone business can be obtained is resorted to without expense to the savings banks, and which constitutes the largest item of expense for industrial companies, it would be surprising if their financial condition were not satisfactory. The effort to keep down expenses is carried to such an extreme that the People's Savings Bank, in its report for the last two years, did not charge out any disbursement for rent, although the law governing these institutions provides "expenses pertaining to the conduct of both the savings department and insurance department, such as office rent and salaries of general officers, shall be apportioned by the trustees equitably between the two departments."

Allusion has already been made to the large average amount per policy with the statement that this large average amount shows conclusively that it is not industrial insurance.

It is curious to note with what insistence comparisons are made with industrial premium rates. For instance, on page 207 in the paragraph immediately above the table, it is stated, "the monthly rates of the savings banks are transposed to weekly payments by simple proportion." Of course they are. But a monthly premium is more naturally transposed into, and compared with, a quarterly premium than a weekly premium, and therefore a comparison with a quarterly premium for a $500 policy such as is issued by industrial companies would be a fairer one than that taken. Industrial companies do not issue policies for monthly premiums nor do the savings banks issue policies for weekly premiums, and a comparison of benefits given on the basis of a monthly premium with those given on the basis of a weekly premium, is purposely used to create the impression of an excessive charge on the weekly premium basis. As a matter of fact, the savings banks realized, soon after they entered the field for life insurance, that the industrial companies were offering insurance cheaper to the working people on a quarter annual premium basis under $500 intermediate policies than the savings banks were offering under their monthly premium scheme. The savings banks through the State Actuary thereupon issued a table for $500 insurance with premiums payable annually, semi-annually or quarterly; but we have no means of knowing what proportion of their business is on that basis as the figures are not published. But this we do know, that even the savings banks acknowledge that a higher relative premium ought to be paid on a monthly basis than on a quarterly basis, for, according to their own tables, a man aged 25 who makes application for $500 whole life insurance on a monthly premium basis (premiums ceasing at age of 75) is required to pay $1.00 a month or $12.00 a year, while the same man applying for a $500 whole life policy on the quarterly premium plan is required to pay only $2.46 quarterly or $9.84 a year; and

even if this latter premium were computed on the assumption that the policy would become fully paid up at age 75, so as to correspond exactly with the plan of the monthly premium policy, the premium would be but $9.92 a year. In other words, the insured pays an addition of $2.08 a year or 21 per cent. on the amount of quarterly premiums for the privilege of paying monthly. With this recognition in their own publications of the increased cost with the increased frequency in premium payments, they are hardly warranted in comparing monthly premiums with weekly premiums on an equal basis for the purpose of showing the increased cost of the latter.

At least one industrial company is offering $500 intermediate whole life policies at lower premiums than those charged by the savings banks, as shown by the following table.

ANNUAL PREMIUM FOR $500 WHOLE LIFE INSURANCE CHARGED BY THE SAVINGS BANKS AND BY AN INDUSTRIAL COMPANY.

Age.	Savings Banks.	Industrial Co.	Age.	Savings Banks.	Industrial Co.
20	$ 8.20	$ 7.72	45	$19.23	$18.12
25	9.45	8.90	50	23.75	22.34
30	11.05	10.42	55	29.72	27.94
35	13.12	12.38	60	37.72	35.36
40	15.77	14.88			

Mr. Hunter states that:

> "The opponents of the measure replied * * * that insurance would not be granted by the savings banks any cheaper than the industrial companies."

It should be stated here that the industrial companies never opposed the legislation which granted the savings banks the privilege of writing life insurance. What they did object to was the attack made on industrial insurance as the basis for this legislation. The industrial companies however do believe now, as they always have believed, that on the basis of "a square deal" they can furnish insurance to the working people of Massachusetts at a lower cost than the savings banks.

It is a fact worthy of note that when an effort was made two years ago to permit industrial companies to insure groups of not less than one hundred at rates lower than the regular published industrial rates, it was opposed by the men who were upholding the savings bank insurance scheme, and they contributed their share towards the defeat of the legislation which was then proposed. They attacked industrial insurance because of the relatively high premiums charged and opposed legislation intended to reduce the cost.

Mr. Hunter says:

> "Owing to the technical nature of life insurance, the value

> of the sentiment of the general public may be doubted, but the sincerity of the sentiment could not be questioned, and it was on account of this sentiment that the measure finally became law in June of 1907."

This, of course, constitutes a part of the early history of the measure, and to those conversant with the subject who observed the methods employed by its advocates and attended the meetings of the committee of the legislature before which the hearings were held, the impression made was that the sentiment in large measure was not sincere, but manufactured; and the results, especially when taken in connection with the advertised membership of 110,000 in the Massachusetts Savings Insurance League, would seem to confirm that impression.

The fundamental difference in theory between the savings bank plan of insurance and industrial insurance is that the expense of a corps of agents is eliminated in the former, but this difference is only theoretical because as a matter of fact there is an agency expense attached to the savings bank insurance which somebody pays.

A corps of speakers "schooled in the fundamentals of life insurance and sent broadcast over the State to address labor unions, civic bodies and clubs of all kinds" could not be secured without money. Employment of industrial insurance agents engaged by somebody to prosecute the work also costs money, and the question arises whether it is not more honorable to openly pay the necessary expenses for securing applicants for insurance and to report the same where they can be seen and read by all men, than to be paying these expenses in such a manner that they do not appear in the published reports, and then to advertise the business as if no expense had been incurred in securing it.

The industrial companies claim that the business cannot be written to any extent without the work of agents and that the agents are entitled to a living wage for their work. The expense attached to the employment of agents has been the most flagrant charge against the companies. It was not charged that they were paying exorbitant commissions to agents but, on the contrary, the quotation from the report of the Armstrong Committee states the agents were underpaid. It was simply held that this item of expense was unnecessary and could be eliminated, thus reducing the cost of insurance "to those who do want it and can pay for it." The claim of the industrial companies having been justified by the experience of the savings banks it would seem as if there were no further cause for controversy.

In the Massachusetts report for 1863 by Elizur Wright and Geo. W. Sargent, Insurance Commissioners, appears the following:

> "A very large, and sometimes much the largest part of the expense, however, goes to support canvassing agents and to

> publish documents tending to increase the business. And among the honorable workers in the civilized world, to whom the public as well as the insured will die indebted, we give faithful and successful life insurance agents a high place. It is hardly possible to believe that a life insurance agent can achieve any long-continued success without bringing into action some of the noblest qualities of a sterling man, and no field that we know of is more inviting to an ambition that would devote the best of talents to the benefit of society at large and individuals in particular.

These words are just as true today as they were in 1863.
Mr. Hunter says:

> "Whether savings bank insurance can be carried to a successful issue is of little importance compared with the influence it must have on future plans of state interference in insurance."

Savings bank insurance cannot be carried to a successful issue on its own merits. This has been proven by Mr. Hunter. If its operations had been limited within the lines advocated by its sponsor and within the spirit of the law it would have died a natural death just as the two companies did that received charters to transact business essentially in the same way, but presumably for larger amounts. Its influence will have to be judged on this basis and not on the published reports which do not disclose the expense of procuring the business irrespective of who pays it.

MR. ROBERTSON G. HUNTER:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

Mr. Craig states that the comparison made by me between the premiums of the savings banks and those of the industrial companies is unfair for two reasons:

(1) The average policy in the savings banks is $420, whereas in the industrial companies it is $140.
(2) The premiums of the savings banks are payable monthly, whereas the premiums of the industrial companies are payable weekly.

In computing the average policy in the savings banks Mr. Craig has included the "intermediate" policies of the banks. With such policies excluded, it would be found that the average of each monthly premium policy on October 31, 1909, was $355 and on May 1, 1912, $255. This is the average policy for each adult life, as no policies were issued by the savings banks on the lives of children. The average policy given by Mr. Craig for the industrial companies would be increased materially if the numerous small policies on children were excluded. Moreover, the average policy of the industrial companies seems to be increasing in recent years, as I find that the average policy issued by one of the industrial companies in 1910 was $170.

Bearing these facts in mind and the further fact that the same unit of premiums per month in the savings bank purchases 25 per cent. more insurance than in the industrial companies, I think the comparison made by me is not so far out of joint as Mr. Craig's figures would appear to show.

Regarding Mr. Craig's second objection to the comparison, I agree with him that it costs more to collect premiums 52 times than 12 times a year, but I do not think that monthly premium insurance should on this account be considered as "intermediate" insurance. In its broadest sense industrial insurance is insurance of the masses and not weekly premium insurance. If it is proven that the industrial classes cannot purchase insurance by monthly premiums, then the comparisons I have made are unsound, but my experience in Massachusetts did not convince me that such was the case, although, such experience being necessarily limited during the first few years of the experiment to the better paid members of the industrial classes, I am not justified in making any sweeping assertion.

Mr. Craig seems to be of the opinion that the most significant lesson to be learned from the operation of savings bank insurance is the inability of an industrial company to exist and prosper without the services of the agent. I cannot agree with him. We knew that before savings bank insurance was suggested. It seems to me that the most significant lesson is found in the manner of collecting premiums through the voluntary co-operation of the manufacturer. This at least points to a tendency on the part of the manufacturer to a realization of his responsibility to his employes in providing for their future, and this tendency gives rise to the hope that in the future we may find a more economical method of collecting premiums than from house to house.

METHOD FOR HANDLING THE GAIN AND LOSS EXHIBITS—HENRY N. KAUFMAN.

VOL. XII, PAGE 213.

WRITTEN DISCUSSION.

MR. ARTHUR B. WOOD:

Mr. Kaufman's paper deals with a subject of practical interest especially to the actuaries of companies operating in the United States, for whatever our individual opinions may be regarding the value of the Gain and Loss Exhibit, or the wisdom of its publication, it has now become an annual requirement of the various Insurance Departments and must, therefore, be prepared.

There are different methods in vogue for calculating the several theoretical elements which enter into its composition, and the degree of accuracy aimed at is apparently determined by each company according to the value it attaches to the results. That an exact balance is not necessarily expected by the Insurance Departments is apparent from the item "Gain or loss unaccounted." From the published reports it would appear as if many companies obtain an exact balance as they report nothing under this heading, but it is more probable that the amount unaccounted for has been disposed of by adjusting some one or more of the items, the view being that fairly close approximations are sufficient for all practical purposes. Some actuaries, however, will take the position that absolute theoretical accuracy is to be sought for, if possible.

The object of Mr. Kaufman's paper is to show that these several items can be correctly ascertained without much practical difficulty and furthermore that the calculations in the main can be reduced to a matter of mere routine, the actuary being thus relieved of much personal supervision. In this connection the "Gain and Loss Analysis" which he has prepared will undoubtedly be found exceedingly useful. The supervision by the actuary cannot of course be entirely dispensed with, because special items which may affect the surplus will require his attention, such, for instance, as a change in the valuation basis of any section of the business, fluctuations in the values of foreign silver currencies, and other matters.

The basis of the method under consideration is the fundamental relationship existing between the several items as shown by the formula ${}_nI(1+i) - {}_nC = {}_nT$. The Gain and Loss figures are thus prepared simultaneously with the reserve calculations and the valuations are continuous from year to year. This method, therefore, has the double advantage that it checks the reserve liability and that the items when entered in the Gain and Loss Exhibit will balance with the surplus shown by the financial statements, when

due allowance is made for any special items peculiar to the individual company. The one difficulty which presents itself is that the items obtained directly from the reserve calculations as ordinarily made are not absolutely correct, although if they were used in their original shape, a balance would, nevertheless, be obtained (see line 11 of summary, p. 224). The net premiums would not correspond exactly with the actual gross premium revenue and the other items would be correspondingly incorrect, but the several errors would counterbalance one another. Account must, therefore, be taken of those cancellations under which the premiums were either overpaid or underpaid with reference to the policy anniversaries in the current year. His method of dealing with these adjustments forms a special feature of Mr. Kaufman's paper.

The cancellations with even years' premiums paid are divided into three main groups, "A," "B" and "C," consisting of cases paid to the anniversaries in the current year, and those one year overpaid and one year underpaid respectively. Fractionally paid cases are disposed of in the manner outlined by offsetting the net premiums overpaid against those underpaid on the assumption that the interest and cost affected by a unit of premium is the same for the same year of issue regardless of age or plan. The illustration based on six months' cancellations shows that the error involved is insignificant as compared with the total cancellations. It should be observed that this particular part of the work is not self checking as is the case with the main calculations and if errors are made they will escape notice. Great care must therefore be exercised in handling the cancellations to insure absolute accuracy. The fractionally paid cases are thus finally distributed among the three main groups as illustrated in the table on page 218. I would here call attention to an error in the section of the table headed "Before Adjustment." The figure 7 should fall under group "B" as these cases are all deaths and must therefore be regarded as fully paid. The figures 4 and 1 apparently belong under "Partially paid after Anniversary."

The three groups being valued separately, allowance can readily be made for the adjustments corresponding to the premiums overpaid and underpaid respectively.

Group C cancellations may be regarded as consisting of policies under which the annual premiums or portions thereof were outstanding or deferred at 31st December, and were not subsequently paid. A half year's interest and cost was included in the preceding year's figures just as if these premiums had been paid. The effect of deducting a full year's interest and cost in the final adjustment is to make allowance in the current year to offset the amounts so included in the preceding year. Errors due to lack of knowledge as to the final payment of outstanding premiums are thus automatically corrected from year to year.

In the case of Group B cancellations, which will be chiefly deaths,

a full year's interest and cost is added to correspond with the full year's premiums overpaid. This is in strict accordance with the assumptions involved throughout the calculations, namely that the cancellations all take place exactly at the policy anniversaries, that the reserve released is in every case the terminal reserve and that a balance must be preserved between the initial reserve, a full year's interest and cost and the terminal reserve. As compared with actual conditions in practice, however, the effect is to understate the profit from interest and to correspondingly overstate the profit from mortality. Mr. Kaufman has however adhered to the strict interpretation of the Gain and Loss Blank and the results obtained may, I think, be regarded as practically exact according to these assumptions.

The impression one receives from reading the paper, however, is that this method of handling the cancellations must entail a considerable amount of labor, probably more than some of us would feel warranted in expending, if we look at the matter solely from the standpoint of practical utility. It would undoubtedly be much more troublesome for some companies than for others, according to the particular nature of the business.

The general plan of preparing the Gain and Loss figures simultaneously with the reserve calculations, thus keeping them in balance is, however, strongly to be commended, but it may well be considered whether for practical purposes it would not be quite satisfactory to allow for the adjustments corresponding to the premiums of the B and C groups by some approximate method, and thus avoid much of the detail work.

The objection urged against the use of approximations is that large counterbalancing errors may occur and pass unnoticed, in which case the profit or loss from the several sources might not be even approximately correct; but if the approximation were applied only to the adjustments corresponding to the premiums overpaid and underpaid this objection would not apply. The net effect of the adjustments when accurately made according to the method described, will be to alter the amounts of the several elements as originally ascertained by probably not more than 1 per cent. The percentage is slightly larger in the illustration given at the close of the paper, but as this is based on old issues Group C is unusually small. If, therefore, the difference between the net premiums corresponding to the B and C groups were accurately ascertained in whatever manner might be considered most practicable, the loading would be correct, and it would appear that an approximation to the corresponding interest, cost and terminal reserve would give results for the total business very nearly exact, certainly close enough for any practical use.

The work would in any event be much simplified if the fractionally overpaid and underpaid cases were regarded as offsetting one another, and were all thrown into Group A. We would then have

to deal separately with the original B and C cancellations only. An error would be involved as Mr. Kaufman points out, but it would be comparatively small. From the table on page 218, it is seen that the effect would be merely to alter the percentages of the year's cancellations placed in the A and C groups from 79 and 14, the correct percentages, to 80 and 13 respectively.

It may not be out of place to mention that, even if the figures entered in the gain and loss blank are theoretically accurate according to the assumptions made, the profit or loss from each of the several sources will not be absolutely correct. For instance the blank presumably does not contemplate any allowance being made for the loss of interest due to the immediate payment of claims. To obtain the correct profit or loss from interest and mortality respectively we should however deduct the estimated loss of interest from the interest required to maintain the reserve and also from the cost of insurance, thus charging the loss against mortality rather than interest. It may of course be answered that this is a matter apart from the question at issue and that such allowance can in any event be made if desired, after the correct theoretical figures have been ascertained.

Notwithstanding the comments I have made, I am ready to acknowledge that Mr. Kaufman has successfully accomplished the object he has had in view and I feel that his work is deserving of all praise.

Whether or not others may consider it advisable to adopt this method in its entirety, it must be admitted that his paper contains many valuable suggestions which should prove useful to all who have the handling of this somewhat troublesome statement.

MR. SHEPPARD:

One of the first questions that suggests itself to a student of the Gain and Loss Exhibit is: On what does the balancing of the Exhibit depend? On examination it will be seen that a number of items can be excluded from consideration altogether, such as due and unpaid receipts considered as assets, or due and unpaid disbursements considered as liabilities, as they only affect the distribution of such receipts or disbursements between successive calendar years. Let us simplify the question by excluding instalment policies, dividends left to accumulate and annuities. The balancing of the Gain and Loss Exhibit will then be found to depend on the accuracy of the equation—

Reserves on all policies in force at the beginning of the year
+ net premiums received during the year
+ interest required to maintain reserve
= cost of insurance
+ reserves released on all policies cancelled during the year
+ reserves on all policies in force at the end of the year.

It should be borne in mind, that, as, according to the present con-

vention blank, net premiums due and unreported or deferred are taken credit for in "Non-Ledger Assets" while the mean reserves appear under "Liabilities," the result so far as the surplus is concerned is to put the reserve on to a cash paid basis. This can be seen at once by considering net due and unreported or deferred premiums as taken out of assets and used as a reduction of net reserve.

For the present, however, let us assume all premiums payable annually and that no premiums are due and unreported at the end of the year, also that all cancellations, except death losses, take place on the anniversary of the policy. Also let us assume that the company values for mean reserves and mean cost of insurance on policies in force at the beginning of the year, and makes corrections for changes in business in force during the year. Using Mr. Kaufman's notation, from formula (1) on page 223 may be deduced the following two formulas:

$$ {}_{n-1}M + \frac{i}{2+i}\left({}_{n-1}M + \frac{{}^{n-1}C}{2}\right) = \frac{{}^{n-1}C}{2} + {}_{n-1}T, \qquad (A) $$

and

$$ {}_{n}I + \frac{i}{2+i}\left({}_{n}M + \frac{{}^{n}C}{2}\right) = \frac{{}^{n}C}{2} + {}_{n}M. \qquad (B) $$

On adding together (A) and (B) and remembering that

$$ {}_{n}I = P + {}_{n-1}T, $$

we have the standard equation:

$$ {}_{n-1}M + P + \frac{i}{2+i}({}_{n-1}M + {}_{n}M + K) = {}_{n}M + K, \qquad (C) $$

where

$$ K = \tfrac{1}{2}({}_{n-1}C + {}_{n}C) $$

and is equal to the cost of insurance for the year; while the interest required to maintain the reserve is obtained from the formula given in Vol. II of the *Transactions*, p. 348. Equation (C), must necessarily hold for all policies in force both at the beginning and end of year, and satisfies the criterion above given for the balancing of the Gain and Loss Exhibit. An examination of (A) and (B) separately suggests the correct method of treating the cases of policies cancelled, changed or reinstated during the year, in order to enable the Gain and Loss Exhibit to balance as far as these policies are concerned.

Where the policy is cancelled on the anniversary equation (A) must hold. The policy has been included for valuation in ${}_{n-1}M$ but not in ${}_{n}M$. It has been included for valuation in $\frac{1}{2}({}_{n-1}C + {}_{n}C)$; it

is therefore necessary to value cancellations for the second half year's cost of insurance and subtract from the total obtained by valuing the whole year's cost of insurance on the business in force at the beginning of the year. The contributions of this policy to the items of mean reserve at the beginning of the year and interest required to maintain reserve now balance with the cost of the insurance and reserve released.

Where a policy not in force at the beginning of the year comes into effect during the year, equation (B) must hold, and the policy should be valued for the second half year's cost only. A change from one plan to another can be considered as a cancellation on the original plan and a reinstatement on the changed plan. The following figures will serve as a practical illustration:

Policy issued on American Experience, 3 per cent. basis, ordinary life, age at issue 40, sum assured $1,000, changed at end of 10 years to the 20 year endowment plan,

Equation (A).

Mean reserve of 10th year on original plan	$179.86
Interest required to maintain reserve $= \frac{.03}{2.03}(179.86 + 5.39)$	2.74
	$182.60

Mean cost of insurance of 10th and 11th years	10.92	
Less second half year's cost	5.53	
First half year's cost		5.39
Reserve at end of 10 years, original plan		177.21
		$182.60

Equation (B).

Initial reserve of 11th year on changed plan	$450.99
Interest required to maintain reserve $\frac{.03}{2.03}(454.02 + 3.74)$	6.77
	$457.76
Second half year's cost	3.74
Mean reserve of 11th year on changed plan	454.02
	$457.76

Equation (C) combines the two as follows:

Mean reserve 10th year original plan	$179.86	Cost of insurance for the year $= 10.92 - 5.53 + 3.74$	9.13
Interest required to maintain reserve $\frac{.03}{2.03}(179.86+454.02 + 10.92 - 5.53 + 3.74)$	9.51	Mean reserve 11th year changed plan	454.02
Net premium $= 450.99 - 177.21$	273.78		
	$463.15		$463.15

The above figures show that when a policy is changed the correct net premium for the purpose of the Gain and Loss Exhibit is the difference between the reserves on the two plans at the time of change.

The case of death losses needs to be specially treated, as the full premium is always paid for the year in which death takes place. It is assumed that the reserve entered in the cancellation register and used for the Gain and Loss Exhibit is obtained by interpolating by first differences between the initial and terminal reserves of the current policy year for the time the policy had been in force since the last policy anniversary to the date of receipt of notice of death. Divide the death losses into the two classes of death before and death after policy anniversary in current calendar year.

Let ${}_{n-1}V$ be the reserve released in the first case and ${}_nV$ be the reserve released in the second case, and the interval since last anniversary be $1/m$th of a year.

Then omitting suffixes for convenience, since

$$mV = (m-1)I + T$$

and

$$2M = I + T.$$

It can be proved that

$$M - V = \frac{2-m}{m}(M - T)$$

and therefore equation (A) may be modified as follows:

$${}_{n-1}M + \frac{i}{2+i} \cdot \frac{2-m}{m}\left({}_{n-1}M + \frac{{}_{n-1}C}{2}\right) = \frac{2-m}{m} \cdot \frac{{}_{n-1}C}{2} + {}_{n-1}V.$$

The policy, being in force at the beginning but not at the end of the calendar year, contributes ${}_{n-1}M$ to the reserve and $\frac{1}{2}({}_{n-1}C + {}_nC)$ to the cost of insurance. A correction should therefore be made, as in the case above shown, in order to exclude the cost of insurance of the second half year, and a further correction of a proportionate part of the first half year's cost of insurance to the end of the current policy year in which death takes place. A correction in the interest required to maintain the reserve should also be made, of the same proportion of

$$\frac{i}{2+i}\left({}_{n-1}M + \frac{{}_{n-1}C}{2}\right).$$

To illustrate this, take a policy issued September 1, 1902, at age 40, on the ordinary life plan, for $1,000, date of death February 1, 1912, American 3 per cent. basis, duration of policy 9 years, 5 months—we have;

Initial reserve 10th year	$182.51
Terminal reserve 10th year	177.21
Mean reserve 10th year	179.86
Reserve at time of death	180.30
Cost of insurance for 10th year	10.78
Interest required to maintain reserve for one-half year	2.74

and working from the mean reserve, the equation necessary to balance the Gain and Loss Exhibit is:

Mean reserve	$179.86	Reserve at time of death	$180.30
Less interest for 1 month	.46	Less cost for 1 month	.90
	$179.40		$179.40

or writing it in the way the transaction goes through the books:

Mean reserve	$179.86	Cost of ins. for one-half year	5.39
+ interest for one-half year	2.72	Less proportionate part for 7 months	6.29
	$182.60		—.90
Less proportionate part for 7 months	3.20	Reserve at time of death	180.30
	$179.40		$179.40

The case where death takes place after the anniversary is more difficult. Equation (A) holds, but the policy not being in force at the end of the year, though the net premium was paid, an adjustment will have to be made. Equation (B) modified for a policy in force during $1/m$th of a year after the anniversary can be shown to be

$$ {}_nI + \frac{2}{m} \cdot \frac{i}{2+i} \left({}_nM + \frac{{}^nC}{2} \right) = {}_nV + \frac{2}{m} \cdot \frac{{}^nC}{2}. $$

This indicates the nature of the correction in the cost of insurance, and interest required to maintain reserve as follows:

Consider these policies in force at the end of the year and calculate the two functions accordingly, make correction $(2 - m)/m$ of second half year's cost and interest as before.

Example: Ordinary life age 40, $1,000: issued February 1, 1902, date of death September 1, 1912, American 3 per cent. basis; policy in force 10 years, 7 mos.

1st half year—mean reserve	179.86	Cost of insurance	5.39
Interest	2.74	Terminal reserve	177.21
	$182.60		$182.60
2d half year—initial reserve	$201.96	Cost of insurance	5.54
Interest	3.03	Correction for cost of insurance	+.92
Correction for interest	+.50	Reserve at death	199.03
	$205.49		$205.49
Combining two half years:		Cost of Insurance	10.93
Mean reserve at beginning of year	179.86	Correction cost of insurance second half-year	.92
Net premium received	24.75	Reserve at time of death	199.03
Interest for year	5.77		
Correction for interest second half-year	.50		
	$210.88		$210.88

The above examples are given as showing some of the various questions that arise in connection with the Gain and Loss Exhibit and do not nearly cover the ground. I cannot refrain, however, from referring to one or two others on which as far as I have been able to learn, opinion among actuaries is divided. The first is whether, so far as the Gain and Loss Exhibit is concerned, a special correction should be made on account of the loss due to prompt payment of claims. In spite of the arguments submitted in favor of such a correction, the writer remains unconvinced. The above equations seem to show that a balance is struck, even if the net premiums and reserves, and the values of the cost of insurance and interest required derived from them, are insufficient to pay fully the claims as they arise. Another argument and, in the writer's opinion, a more cogent one, is as follows: Assuming death claims paid on the average one month after death and the average valuation rate of the company as $3\frac{3}{4}$ per cent., then the net premiums and reserves would be sufficient to cover about \$985 out of every \$1,000 insured. There is, therefore, a loss in surplus under every death claim, but it is concealed in the Gain and Loss Exhibit on account of the fact that the instructions are to base the gross loss under the mortality section of the Exhibit on the death claims paid. The writer claims that the more correct way would be to divide each \$1,000 paid out into two parts, \$985 to be entered in the mortality section as "death losses incurred which are covered by premiums charged and reserves held" and \$15 to be entered in miscellaneous losses as "loss due to earlier payment of claims than are provided by premiums charged and reserves held."

If, as some hold, \$15 should be added to the interest required to maintain reserve, the correction is made twice over.

Another disputable point arises out of new issues that were unsettled at the beginning and are in force at the end of the year. It is claimed that eighteen months' cost of insurance should be taken into account though the policy has not been in force for more than twelve months. Assuming such policies to be settled at the very beginning of the year, a study of equations (A) and (B) suggests to the writer the correct way to treat these cases, assuming, as throughout this discussion, that there are no unreported or deferred premiums to be taken into account, so that two premiums must be paid during the year under consideration. Assuming the first net premium paid at the beginning of the year, the first year's mean reserve is immediately held as a liability and the excess of the net premium over the reserve is immediately effective as a profit to the company. Hence the rule to meet these cases:

List policies unsettled at the beginning and in force at the end of the year. For cost of insurance and interest required to maintain reserve, add first year's mean reserve on these policies to previous year's figures. In miscellaneous profit, enter item "excess of net premiums over first year's mean reserves on policies unsettled at beginning of year in force at end of year."

When it is considered that what has been written above is only meant to cover the very simplest cases, it can be easily understood how difficult it is to make a perfect balance of the Gain and Loss Exhibit, when we take into consideration the complications arising from the payment of premiums otherwise than yearly, the case of dated back policies, policies issued and cancelled in the same year, etc. Mr. Kaufman's paper offers a good opportunity to discuss such points and the above lines are written with a view to furthering such a discussion.

MR. J. D. CRAIG:

Now that the Gain and Loss Exhibit has received a permanent place in the Annual Statement with Wisconsin requiring Ordinary, Industrial, Participating and Non-Participating Gain and Loss Exhibits, it becomes necessary for the companies to so arrange their accounts that, at the end of the year, the necessary data will be available, and Mr. Kaufman's system which permits of having it compiled in about five hours by a clerk in the office, is most welcome to those actuaries who have labored with this problem.

There is nothing in this paper that we can criticize, although Mr. Kaufman is not quite explicit enough where, on the top of page 214, he says, "The actual figures are taken from the items as entered in the first four pages of the Annual Statement, and such items are indicated by the page and line of the Convention Blank used December 31, 1910." Not only are the figures from the first four pages of the current Annual Statement taken, but also the figures on the third and fourth pages of the previous year's Annual Statement. In fact every single item in the "Non-Ledger Assets" and "Assets not Admitted" of the two successive years is used as is also every liability item with the exception of the reserve. There is no danger of any misunderstanding occurring on this point, however, as in the Analysis reference is made to the corresponding page and item of the previous year's Annual Statement.

ACTUAL FIGURES.

		Statement Figures.		Statement Figures Adjusted.
Income 1911		$98,135,273.71		$98,135,273.71
Disbursements 1911		60,025,695.77		60,025,695.77
		38,109,577.94		38,109,577.94
Non-Ledger 1911		8,983,585.77	+ $2,057,556.78	11,041,142.55
		47,093,163.71		49,150,720.49
Liabilities 1911	$352,785,890.36			
Reserve Included	306,442,065.00	46,343,825.36	+ 2,057,556.78	48,401,382.14
		749,338.35		749,338.35
Non-Ledger 1910		8,295,607.35	+ 1,892,671.15	10,188,278.50
		−7,546,269.00		−9,438,940.15
Liabilities 1910	$313,988,334.00			
Reserve Included	270,380,929.00	43,607,405.00	+ 1,892,671.15	45,500,076.15
		36,061,136.00		36,061,136.00
Reserve 1911	$306,442,065.00			
Reserve 1910	270,380,929.00	36,061,136.00		36,061,136.00

It might interest the members to have the "actual" figures presented, and the "theoretical" figures analyzed. There are also one or two points which can be emphasized with advantage.

By computing the actual figures according to the method used in my own company, the final results may be compared with the identification numbers given in Mr. Kaufman's Gain and Loss Analysis, and the intent of the various combinations realized.

The first step is to arrange the totals of the six pages of the Annual Statement in such a way as to produce the increase in the reserve, and, in order to make the matter clear, I am presenting this arrangement.

The adjustment in the last column is obtained by adding the loading on the due and unpaid and deferred premiums to the "Non-Ledger Assets" at the end of 1910 and 1911 and also to the "Liabilities" for the same years, as the Gain and Loss Exhibit requires the gross premiums to be reported as unpaid and the loading to be reported as outstanding expenses, and by adding these amounts to the total of both "Non-Ledger Assets" and "Liabilities" the net result is not changed.

With these totals as a guide, we itemize each one of the six pages into the various details required, after which, by being taken across the schedule, they may be entered directly in the blank with the knowledge that, if the theoretical figures are made correctly, the Gain and Loss Exhibit will balance. At the risk of taking too much time and space, I have drawn off these itemized figures from the Annual Statements of my own company for the benefit of those who may desire to follow the subject (p. 145).

The secret of making the Gain and Loss Exhibit balance, is to pivot both the actual and the theoretical figures around the increase in the reserve and then make a few slight modifications, such as Mr. Kaufman suggests in reference to adding the balance of the actual installment figures to the theoretical interest figures.

Passing to the theoretical figures, it is interesting to understand how those given on page 224 combine to produce the increase in the reserve. The simplest equation for the theoretical figures is "net premium," plus "interest necessary," minus "cost of insurance," minus "total reserve cancelled," equals "the increase in reserve"; and the totals as given by Mr. Kaufman on the bottom of page 224 when arranged in this form are,

Premiums				$989,648.00
Interest necessary				635,839.00
Total				$1,625,487.00
Cost of insurance			$411,925.00	
Reserve released		$706,015.00		
Reserve transferred from		22,130.00		
Reserve matured end.		248,266.00		
		$976,411.00		
Reserve trans. to	$28,245			
Reserve rev.	1,865	$30,110,00	$946,301.00	$1,358,226.00
Increase in reserve				$267,261.00

ACTUAL FIGURES, December 31, 1911.

	Income.	Assets 1911.	Liabilities 1910.	Total.	Disbursements.	Liabilities 1911.	Assets 1910.	Total.	Balance.
Premiums	82,339,846.04	8,850,109.97	1,352,377.00	92,542,333.01		1,604,698.52	8,099,871.29	9,704,569.81	82,837,763.20
Expenses	141.00		3,171,948.20	3,172,089.20	23,882,202.51	3,709,505.90		27,591,708.41	—24,419,619.21
Interest	15,648,593.63	4,435,649.76	45,675.98	20,129,919.37	97,635.89	52,508.30	3,857,777.72	4,007,921.91	16,121,997.46
Death claims			643,245.58	643,245.58	22,895,365.93	693,934.49		23,589,300.42	—22,946,054.84
Annuities					129,691.30			129,691.30	— 129,691.30
Sur. values			100,356.05	100,356.05	2,733,639.09	123,945.06		2,857,584.15	— 2,757,228.10
Div. to S. H.					140,000.00			140,000.00	— 140,000.00
" cash					1,561,491.05			1,561,491.05	— 1,561,491.05
" ren. prem.					5,490,822.74			5,490,822.74	— 5,490,822.74
" pdup. adds.					177,570.29			177,570.29	— 177,570.29
" apportioned			8,403,453.99	8,403,453.99		8,432,975.30		8,432,975.30	— 29,521.31
Spec. funds			458,330.55	458,330.55		500,000.00		500,000.00	— 41,669.45
Profit acct.	50,202.75			50,202.75					50,202.75
Loss "					24,502.28			24,502.28	— 24,502.28
Decrease in book value R. E.					746,032.15			746,032.15	— 746,032.15
Profit Sals B. & S.	5,916.84			5,916.84					5,916.84
Loss " "					534.74			534.74	— 534.74
Book over market B. & S.		—1,902,983.56		—1,902,983.56			—1,422,000.48	—1,422,000.48	— 480,983.08
Other unadmitted assets		— 343,058.62		— 343,058.62			— 348,695.03	— 348,695.03	5,636.41
Sick benefit					150.50			150.50	— 150.50
Health and welfare work					590,054.19			590,054.19	— 590,054.19
Surplus			28,742,083.64	28.742,083.64		30,711,882.87		30,711,882.87	— 1,969,799.23
Mat. endts.					1,432,839.59	17,476.00		1,450,315.59	— 1,450,315.59
Sup. contracts	19,944.30	1,425.00	129,160.74	150,530.04	11,877.61	141,668.04	1,325.00	154,870.65	— 4,340.61
Spec. ledger liabil.	70,629.15		453,444.42	524,073.57	111,285.91	412,787.66		524,073.57	
Capital stock			2,000,000.00	2,000.000.00		2,000,000.00		2,000,000.00	
Total	98,135,273.71	11,041,142.55	45,500,076.15	154,676,492.41	60,025,695.77	48,401,382.14	10,188,278.50	118,615,356.41	36,061,136.00

If the net premiums as given in lines 12 and 13 are available or can be derived with reasonable accuracy, a close approximation to these theoretical figures may be obtained from the data given in lines 1, 2 and 9 without making the detailed classification of groups A, B, C, etc., by assuming that all of the policies in force at the beginning of the year will remain in force throughout the year, and then making the necessary corrections for the cancelled.

The full year's data on the policies in force at the beginning of the year may be obtained as follows:

M	$15,652,491.00
One-half years's interest (2% of $15,550,521)	311,010.00
Total	$15,963,501.00
One-half year's cost	209,025.00
Difference, V	$15,754,476.00
Net premium	1,020,349.00
Total	$16,774,825.00
One-half year's interest (2%)	335,497.00
Total	$17,110,322.00
One-half year's cost	208,671.00
M	$16,901,651.00

Combining the two half year's interest as well as the two half year's cost and letting P = the "net premium," I = the "interest necessary," C = "cost of insurance," we have

P	$+$	I	$-$	C	= increase in reserve.
$1,020,349	+	$646,507	—	$417,696	= $1,249160.

The corrections necessary on account of policies cancelled may be made by deducting all the factors included above, from the anniversary of the policies until the end of the year, irrespective of whether the cancellation occurred before or after the current anniversary, and then making special provision for the additions or deductions necessary when lapse occurred other than on the anniversary date.

The deductions from the anniversary to the end of the year on the policies cancelled may be obtained by deducting the corresponding figures on the policies in force at the end of the year as deduced from line 9 from those just obtained; but, in deriving them, it is necessary to start at the end of the year and work back to the anniversary by adding the "cost" and deducting the "interest necessary" and the "net premium." Thus the mean reserve on the policies in force at the end of the year is $15,919,752 and this plus one-half of the "cost" ($195,702) less the discount ($315,989) and less "net premium" ($978,318) produces the previous year's terminal reserve on the policies in force at the end of the year. Reversing these transactions and subtracting from the corresponding items for the policies in force at the beginning of the year, the half year's data are obtained on the policies that are cancelled. Thus,

	On Policies in Force Beginning of Year.	Of Policies in Force End of Year.	Difference.
Difference, V	$15,754,476	$14,821,147	$933,329
Net premium	1,020,349	978,318	42,031
Total	$16,774,825	$15,799,465	$975,360
One-half year's interest (2%)	335,497	315,989	19,508
Total	$17,110,322	$16,115,454	$994,868
One-half year's cost	208,671	195,702	12,969
M	$16,901,651	$15,919,752	$981,899

This gives the deductions to be made from the figures previously obtained for the entire year,

$$P + I - C - V = \text{Increase in reserve,}$$
$$\$42{,}031 + \$19{,}508 - \$12{,}969 - \$933{,}329 = \$981{,}899,$$

and shows also that approximately a year's data, or rather the current year's data, on the cancelled policies is

$$P + I - C - \text{Increase in reserve} = 0$$
$$\$42{,}031 + \$39{,}016 - \$25{,}938 - \$55{,}109 = 0.$$

So that if we know that the policies cancelled paid $11,330 of net premium beyond their anniversary, we may obtain the ratio of the year's premiums on the cancelled and apply to the other items thereby producing,

$$P + I - C - \text{Increase in reserve} = 0$$
$$\$11{,}330 + \$10{,}519 - \$6{,}993 - \$14{,}856 = 0.$$

Summarizing these results we have	P		I		C		V	= Increase in reserve
Full year's data on policies in force beginning of year	$1,020,349	+	$646,507	−	$417,696			= $1,249,160
Data for last half of year on policies cancelled	42,031	+	19,508	−	12,969	+	$933,329	= 981,899
Difference	978,318	+	626,999	−	404,727	−	933,329	= 267,261
Data after anniversary on policies cancelled	11,330	+	10,519	−	6,993	−	14,856	= 0
	$989,648	+	$637,518	−	$411,720	−	$948,185	= $267,261
Mr. Kaufman's results	989,648	+	635,839	−	411,925	−	946,301	= 267,261
Difference			1,679		205		1,884	0

These differences range from one-half of 1/10th of 1 per cent. to not quite 3/10ths of 1 per cent. and have the advantage of being easily approximated.

Mr. Kaufman refers to the treatment of revivals, and it is probable that he intends policies unplaced at the beginning of the year to be included with them, but it seems wise to call attention to this fact and mention that they require special treatment. All such policies in force at the end of the current year are charged the

second year's reserve, obtained by considering them as being in force a year and a half, and the corresponding factors must therefore be included as the policies are considered in the current year's issue.

Mr. Kaufman makes no reference to the calculation of the reserve expected to be released or the expected payments on annuities, and it might be well to submit an illustration referring to single life annuities in force both at the beginning and at the end of the year. Special calculations must be made on other forms of annuities as well as on those that are issued or cancelled during the year; but the single life annuities in force both at the beginning and end of the year probably constitute a large percentage of the total business, and the calculation of the theoretical figures thereon will serve as a basis when deriving the functions of the more complex annuities.

In order to carry one year's mean reserve forward to the next year's mean reserve, it is necessary to add one-half a year's interest on one initial reserve to one half a year's interest on the succeeding initial reserve, which is equivalent to a full year's interest on the average initial reserve. This average initial reserve is just $.50 less than the mean reserve where the annuity is for $1.00; so that it is only necessary to deduct the $.50 from the mean reserve and multiply by the rate per cent. in order to obtain the interest necessary to maintain the reserve. Where the annuity is for an amount different from $1.00, or where a group of annuities is used, one half of the yearly amount payable should be deducted.

Deducting the mean reserve at the end of the year from the mean reserve at the beginning of the year increased by interest, the difference will be the excess of the expected payments to annuitants over the reserve expected to be released, and these two items may be sub-divided by means of the equation, p times the yearly amount payable, minus q times the average reserve, equals the excess. As q equals 1 minus p, and as the average reserve equals the mean reserve at the end of the year less one-half of the yearly payments, the equation can be solved for p, after which the reserve expected to be released and payments expected to be made are easily ascertained.

In one small class in our company, where the annuities were based upon 4 per cent. interest, the 1910 mean reserve was $82,791, the 1911 mean reserve, $79,516, and the yearly payments, $13,513, on the annuities in force both at the beginning and end of the year. With this data we calculated the theoretical figures as follows: The reserve December, 1910, plus the "interest necessary" ($92,791 plus $3,041) was $6,316 in excess of the December, 1911, mean reserve, and this amount is the excess of the expected payments over the reserves expected to be released. Therefore the yearly payments multiplied by the probability of living ($13,515 p), minus the probability of dying $(1-p)$ multiplied by the average reserve (mean reserve at the end of the year minus one-half of the yearly payments) equals $63,616. Simplified as an equation this becomes,

$\$13{,}513p - (1-p)(\$72{,}760) = \$63{,}616$, from which p equals .916579, the expected payments, \$12,396, and the reserve expected to be released, \$6,070. These figures will be found to be sufficiently accurate, while at the same time they must balance.

MR. KAUFMAN:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

I thank Mr. Wood for calling attention to the error in the table on page 218. There is a typographical error in formula 6, page 223, where the prefix n is omitted before the last C and T, also in the sixth line from the bottom of page 220 $p_{x+\frac{1}{2}}$ should read $p_{x-\frac{1}{2}}$. Incidentally I would add that the paragraph at the bottom of page 220 deals with the expected payments and reserves to be released on annuities, to which Mr. Craig states I made no reference. From casually examining Mr. Craig's method of handling the annuity portion it appears to me it is practically the same as on page 220, with the exception that he uses p_x instead of $p_{x-\frac{1}{2}}$.

From the discussion of Mr. Wood and Mr. Craig and from opinions of actuaries expressed since writing my paper, I find that approximate figures for the Gain and Loss Exhibit are the rule rather than the exception. To understand this we should consider the principal reasons for compiling, or the purposes of these figures when obtained. They apear to me as follows:

1. For the Convention Blank as required by the State Departments.

2. For the company's own use.

If (1) is the only purpose of the Exhibit there probably will seem no urgent necessity for obtaining anything but approximate figures, because, as mentioned by Mr. Wood, the insertion of the item "Gain or loss unaccounted for" in the Convention Blank presupposes and allows the use of such figures.

On the other hand, if (2) is the purpose of the Exhibit we may then assume that the company considers it of value, or we might go a step farther and say of enough value to desire accurate figures. While there may be differences of opinion as to how much effort should be expended to obtain absolutely accurate results, yet in my mind there is no question that we should indicate some method of obtaining accurate figures, regardless of labor involved, for the benefit of any company caring for accurate results, and in order that others desiring only approximate results might have some standard for measuring such approximations (making exact calculations only periodically). When it is stated that certain methods will give approximate results, I doubt very much if we know how near such results are to the true figures. The fact that the Gain and Loss Exhibit is in balance does not mean that the various items are correct. For instance, line 11, page 224, is in perfect

balance, but we cannot stop there because our "B" and "C" extra values affect the preceding items materially without disturbing the final balance.

I therefore will suggest several corrections in order to make the theoretical results correspond to those of actual conditions. The first correction is the result of Mr. Wood's statement that one-half a year's cost and interest of the "C" group were included in the preceding year's figures. The adjustments for these, however, can easily be made as shown in the following illustration, using the original figures of the "C" group on page 218 and giving the adjusted figures below.

EXTRA VALUES "C" GROUP.

	Net Premiums.	Interest.	Mean Cost.	Reserves Released.
Original	443	203	133	513
Adjusted	443	101	66	478 (5,112—4,634)

It may be noted from the above that the full year's net premiums are taken in order to get the correct loading, but only one-half year's interest and mean cost; also that the corresponding reserve released is the difference between the mean reserve as of December 31, 1909 ($5,112) and the 1909 terminal reserve ($4,634). Thus it can be seen that the adjusted figures are still in balance and at the same time conform to actual conditions and can as readily be obtained as the original figures. At first thought it seemed as if a similar correction should be made for the "B" group, but on further consideration this group is found to be made up almost entirely of death claims, and, therefore, the full year's interest and cost should be included. I do not agree with Mr. Sheppard in regard to the necessity for handling the death claims separately and dividing the year into m parts, as it is generally conceded that no matter when the death occurs we are entitled to the full year's cost for that policy year. Inasmuch as we are dealing with calendar years and not with policy years, let us look at the matter on that basis. Taking the whole year's deaths during the calendar year on the supposition of a uniform distribution of deaths during the year (which is in accord with mean reserve valuations) we can assume that one-half will die before the anniversary and one-half after the anniversary. In the plan of releasing the reserve as of the time to which the premium is paid, in order to make our balance we use a half year's cost on those deaths occurring before the anniversary and eighteen months' cost on those occurring after the anniversary. The sum of these costs will, on the average, be equivalent to a calendar year's cost on all the deaths. This, of course, has no reference to the correction for the immediate payment of death claims. For that correction we can deduct the loss entailed from the interest required to maintain the reserve and also from the cost of insurance, as suggested by Mr. Wood.

The correction for issues that are unsettled at the beginning and

are in force at the end of the year should be made somewhat similarly to that for the "C" group, using only one-half year's interest and cost and the mean reserves as of the preceding December 31, instead of the regular net premiums. In like manner treat the revivals not in force at the end of the preceding year. In this connection also see paragraph on page 220 in reference to revivals. This can be done most conveniently by keeping such issues and revivals separate and valuing them in groups by themselves, and there will then be no trouble in substituting the mean reserves for net premiums and also obtaining the half-year's interest and cost.

It should be borne in mind that all the corrections above referred to do not disturb the balance in any way, and, as the adjustments are made after the valuation figures have been completed, the true figures can be used as readily as the original ones.

The following correction should, however, be made when first handling the changed policies referred to in the last paragraph of page 219. It is necessary to have two general classes of "Transfers," one in which the change is made from a lower to a higher premium and the other in which the change is made from a higher to a lower premium. In the former case the company will receive extra gross premiums on account of such changes and the differences in reserves will be in the nature of net premiums. In the latter case the differences in reserves will be in the nature of reserves released. There should therefore be two distinct classes of "Transfers" in order that the differences in reserves may be transferred to the proper columns; namely, "Net Premiums" or "Reserves Released."

As the tendency seems to be for an increase in use of perforated cards, the following alternative method is suggested for handling the cancellations by the Hollerith system or by any other automatic card system available in the future. At first sight this may appear to involve considerable detail by reason of the various groups, but I can speak from experience that, as the work of sorting is mechanical, this method really involves less labor and care than any other which has come to my attention and is to be recommended wherever the Hollerith system is available. First, prepare a code similar to the following for punching the cards at time of cancellation, indicating the respective quarterly periods to which the premiums have been paid. The cards, having been punched according to this code, can all be thrown together and valued in the usual manner without any reference to the extra "B" and "C" groups. At the end of the year the termination cards can be sorted on the Hollerith machine, and all cards not paid to the anniversary in the current year may be valued in groups according to each quarter of a year paid before or after the anniversary. Therefore to obtain the extra figures of lines 12 and 13, page 224, we will require to make only one valuation at the end of the year for each class of termination as shown in the code (with the exception of code 5). For convenience, however, keep those with different rates of interest separate if this

will not automatically be done in making the classification by years for valuation.

CODE SHOWING DATES TO WHICH CANCELLATIONS ARE PAID.
CURRENT YEAR 1910.

	Code Number.
C —Paid to anniversary in 1909	1
C_1—1/4 1909 paid	2
C_2—2/4 1909 paid	3
C_3—3/4 1909 paid	4
A —Paid to anniversary in 1910	5
B_1—1/4 1910 paid	6
B_2—2/4 1910 paid	7
B_3—3/4 1910 paid	8
B_4—Paid to anniversary in 1911	9

From the valuation figures the correct proportion of net premiums, interest, cost and reserves released should be used; namely, full year's values for C and B_4, three-quarters for C_1 and B_3, one-half for C_2 and B_2, and one-fourth for C_3 and B_1, except for the C and C_1 groups we should limit the interest and cost to that of a half year and use the corresponding adjusted reserves released as heretofore explained for corrections in the "C" group. In using these valuations in this manner we obviate the detail of offsetting the fractionally paid groups.

The discussion of this paper indicates that some way of estimating these "B" and "C" groups is desirable. Methods may be found for obtaining close approximations of the interest, reserve released and cost, *provided the net premiums are accurately calculated.* We are indebted to Mr. Craig for showing us some remarkably close approximations to the true figures on this supposition. If as reliable an estimate of the net premiums could also be made, the most troublesome part of our work would be eliminated. If, however, they must be calculated, it is my contention that some plan of grouping and valuing the policies in question similar to the preceding method involves less difficulty and detail and will probably prove more accurate than recording the net premiums seriatim. It is hoped that a further study of this matter will enable us to eliminate details of this kind to a larger extent.

A DETERMINATION OF THE CONSTANTS IN MAKEHAM'S FORMULA BY THE METHOD OF LEAST SQUARES—ILLUSTRATED BY GRADUATIONS OF THE AMERICAN EXPERIENCE TABLE—
JOHN S. THOMPSON.

VOL. XII, PAGE 225.

WRITTEN DISCUSSION.

MR. HENDERSON:

Mr. Thompson's paper is particularly valuable for its detailed presentation of Professor Pearson's method of graduating a mortality table. His exposition of the method brings out clearly the fact that Professor Pearson has not succeeded in entirely eliminating the effect of arbitrary judgment from his method. In fact, any method which only uses part of the experience, or which places greater weight on one part of the experience than on another, will necessarily produce results which will vary according to the part of the experience that is taken into account and with the relative weights assigned to the different sections. In fact, the different values of the constants which are derived by the various methods taken up in this paper are somewhat startling, but, when we reach the table of net premiums on page 238, we find that the difference is less than might have been feared.

In fact, if we take Mr. Thompson's graduation by the methods of moments, in which the value of log c is less than .044, and compare it with his graduation by least squares grouped, where its value is greater than .046, we find that the net premiums are substantially the same from 35 to 50 inclusive and that the difference is less than 1 per cent. of the premium even up to age 60. In this connection it may be worth pointing out that there is evidently a clerical error in the net premium at age 55 according to the table graduated by the method of moments. These facts would seem to indicate that a determination of the value of log c to two significant figures is sufficiently close for all practical purposes. This is especially the case when we consider that any graduated table necessarily partakes to a certain extent of the inaccuracies and fluctuations of the ungraduated table, and that, even on the assumption that Makeham's law is a true law of nature, the constants determined from any particular set of observations are not rigidly accurate, but are merely so determined as to render the probable departure a minimum. Now, it is one of the features of a minimum that ordinarily small variations in the constants in either direction from the values producing that minimum have relatively no effect on the value of the function under consideration. It follows

from this that extreme precision in the determination of the constants is not important for our purposes. This is more especially the case with respect to the value of log c.

MR. PORTCH:

The paper submitted by Mr. Thompson at the last meeting has a double value. It furnishes a good example of the mathematical processes involved in the determination of Makeham's constants by the methods of moments and of least squares. It also gives us three new graduations of the American Table that may be considered from a practical view point.

Regarding the mathematical work there seems little room for discussion or criticism. Perhaps we should note, however, that Mr. Thompson has shown the practicability of the method of least squares in determining the constants, at least in an adjustment of this kind. The data here used already formed a smooth curve before the regraduation was applied. But it would seem probable, especially by making suitable groups of observation equations as Mr. Thompson has done in his third graduation, that this method may be successfully applied to any data that can be satisfactorily graduated by Makeham's formula. It will be noted that by using grouped equations, or, rather, equations that are the means of selected groups, Mr. Thompson has obtained his best results. This is in accord with the results obtained by Messrs. King and Hardy (J. I. A., XXII, 200), and others.

The only practical purpose to be served by a Makehamized regraduation of the American Table is in the calculation of joint life benefits. We should doubtless prefer to use the original American Table for joint life calculations, because it is fixed as our standard for single life benefits, and the joint life calculations should correspond. But the work of making such calculations by the original table is so laborious that we have chosen even to modify our standard in order to make the calculations more conveniently. The saving of time in these calculations is a sufficient reason for this modification. But there has perhaps been an indirect result of greater economic value. If the figures for business written in United States and Canada since Mr. Hunter's table was published were available, they would probably show that a very marked increase in the proportion of joint insurance has been made by reason of the greater facility in quoting rates and values. As there is a large legitimate field for joint insurance, the service rendered by our companies has probably been materially increased by means of what might be regarded as merely a mechanical convenience.

Mr. Hunter's table has answered the purpose well. It will probably not be displaced in practical use so long as the American Table remains our standard. There is, however, one slight practical difficulty in its use and I was curious to see if Mr. Thompson's

third graduation, the closest to the original table, removed this difficulty.

It has been noticed that the net premiums for joint term insurances of five and ten years on two lives, calculated by Mr. Hunter's table, are at certain ages greater than the sum of the two net premiums for the single lives by the original American Table. I have calculated such net premiums by Mr. Hunter's and Mr. Thompson's third table for five and ten year periods and quinquennial ages covering the usual range. They show the following comparison:

Comparison of Term Insurance Annual Premiums for two Joint Lives of equal Age, by Hunter's Makehamized American Table and Mr. Thompson's third Table (least squares, grouped) with twice the corresponding One Life Premium by original American Table (Int. 3½ per cent.).

Equal Ages.	5 Year Term.					10 Year Term.				
	Twice the One Life Premium Amer.	Hunter Joint Life.	*Δ	Thompson Joint Life.	*Δ	Twice the One Life Premium Amer.	Hunter Joint Life.	*Δ	Thompson Joint Life.	*Δ
25	15.84	15.67	.17	15.57	.27	16.20	16.00	.20	15.91	.29
30	16.64	16.43	.21	16.35	.29	17.18	17.00	.18	16.91	.27
35	17.86	17.74	.12	17.64	.22	18.72	18.69	.03	18.60	.12
40	19.82	19.93	−.11	19.86	−.04	21.34	21.52	−.18	21.47	−.13
45	23.24	23.65	−.41	23.60	−.36	26.10	26.29	−.19	26.28	−.18
50	29.76	29.90	−.14	29.93	−.17	34.74	34.20	.54	34.33	.41

It will be seen that the peculiarity mentioned is common to both tables, although the undesirable factor is somewhat minimized in Mr. Thompson's graduation.

This difficulty is, however, a very minor one and perhaps inevitable at some period of the table. It necessitates only an adjustment of loading at certain ages. Its main importance is in connection with the reinsurance of joint life cases between companies where the contract provides reinsurance upon renewable term rates with a fixed percentage of loading. Of course, if it could be avoided in the range of the table say from ages 25 to 50 at entry, it would add an attractive feature to the regraduation.

Mr. Thompson's third table is remarkable for its closeness to the original data. His three graduations are individually successful and will stand as confirming our use of Mr. Hunter's table for practical purposes. He is especially to be congratulated upon the paper's mathematical value, which will cause it to be carefully studied by many persons outside of this Society.

MR. THOMPSON:

(AUTHOR'S REVIEW OF DISCUSSION.)

In connection with the foregoing discussions there is very little to be said. Perhaps the liberty may be taken of repeating that the object of the paper was not to present a more precise graduation of the American Experience by Makeham's formula; for practical purposes this has been accomplished in an eminently satisfactory manner by Mr. Hunter by the use of the time-tried method of Messrs. King and Hardy, and one of the results of this investigation is to show that as a practical application of Makeham's law it is difficult to improve upon it. The object has been rather to study the effect of applying the method of least squares to this problem in the hope that it might result in furnishing a new and more accurate means of determining the constants in Makeham's formula, just as it has been used for that purpose in other branches of science. The extent to which this has been realized (or not) may be observed from the experiments, the results of which are set forth in the paper and in the discussions presented above.

It was anticipated that a serious objection would be raised that the smallness of the mean error (regardless of sign) in $\log l_x$ and in l_x was due to the fact that only seventy terms were used in the final graduation as compared with eighty in that of Mr. Hunter, the advantage being gained in the avoidance of the severe variations which necessarily set in at the higher ages. Even if it were not granted that the longer range furnished the best possible group of terms, this criticism if sustained would have some weight. Accordingly, a trial graduation was made employing the data from ages 10 to 89 inclusive in eight groups of ten each. The value of $\log_{10} c$ was taken as .0445, that being the one which out of those tested appeared to promise the best results. When the process described in the paper was completed, the mean error in $\log l_x$ was .0023736 compared to .0025264 in Mr. Hunter's graduation. This is a very slight advantage, if, indeed, it is any; at the same time it is probable that a better solution than that furnished by $\log_{10} c = .0445$ could be obtained.

In conclusion I would like to thank those who have taken the trouble to discuss the paper in so painstaking a manner.

NET PREMIUMS AND RESERVES FOR POLICIES GIVING INSTALMENT DISABILITY BENEFITS—EDWARD B. FACKLER.

VOL. XII, PAGE 241.

WRITTEN DISCUSSION.

MR. MEAD:

In Mr. Fackler's paper we have presented to our attention for the first time an extended study of the question of net premiums and reserves for instalment disability benefits. Mr. Fackler's paper speaks for itself, and I shall limit my comments to a brief consideration of the relative merits of the two methods of computation which he employs and of the increased cost resulting from the option of waiver of premium alone in addition to the privilege of receiving the sum insured in instalments.

In my opinion the first method is the more theoretically correct as being more in accord with the actual conditions. On the other hand the alternative method is the more conservative, since, according to it, the value of the premiums payable is greater, as might have been expected from *a priori* reasoning. The conditions as to the payment of premiums and the receiving of the benefits are the same up to age 60 by the two methods, but the alternative assumes that the mortality of the active that survive age 60 is the same as under the basic table, i. e., no allowance is taken for the improved mortality amongst the active after age 60 due to the elimination of those lives disabled prior to age 60 but who survive that age. The value of the premiums according to the first method is less than that of the regular basic premiums, because the amount saved through the payment of the instalments in case of disability more than counterbalances the value of the premiums lost and the value of the anticipation of payments upon the sum insured.

It will be observed that the ordinary life reserves for the combined benefits are greater than the regular reserves. This is due to the smaller premium that is paid after age 60. On the other hand the limited payment life reserves by this method are less than the regular reserves, where there are no premiums payable after age 60, owing to the fact that the cost of insurance after age 60 is assumed to be less. Where the limited payment life premiums extend beyond age 60 the reserves are greater than the regular reserves, just as in the case of the ordinary life plan and for the same reason.

Mr. Fackler calls attention to the fact that the reserves by the alternative method closely approximate the regular reserves. As the premiums are on the side of safety in the case of the alternative method and no special provision for reserves would be required according to it, it is the method to be recommended for practical use.

Mr. Fackler's calculations assume that the instalment option is the only one available to the insured upon the occurrence of disability. As many of the disability benefits that are now being granted offer the option either of the instalments or of waiver of premiums, it will be worth while to investigate the additional cost resulting from the two alternative options. It would seem that at least as many would select the waiver of premiums as would die within the first policy year succeeding that in which disability occurs. Accordingly I have made an approximation of the premiums for the alternative options resulting from the assumption that 30 per cent. on the average of those who become disabled select the waiver of premium. I find the increased cost on this assumption over the premiums of Mr. Fackler's by his first method to be on the ordinary life and twenty payment life plans as follows:

Age.	Ordinary Life.	Twenty Payment Life.
20	$0.06	$0.10
35	.09	.12
50	.18	.18

The extra premium previous to age 60 in excess of the regular American Experience premiums are as follows:

Age.	Ordinary Life.	Twenty Payment Life.
20	$0.12	$0.06
35	.28	.07
50	.98	.83

CONCERNING THE AMERICAN EXPERIENCE TABLE OF MORTALITY—S. A. JOFFE.

VOL. XII, PAGE 253.

WRITTEN DISCUSSION.

MR. D. P. FACKLER:

We must all feel obligated to Mr. Joffe for his interesting account of his researches into the origin of the American Experience Table, and I, personally, feel particularly so.

It was very interesting to follow Mr. Joffe's explorations in the antiquities of American life insurance for which he had better opportunities than I had in 1908 when preparing the paper to which he so kindly refers. I was compelled to rely mainly on my recollections, while he, being in the office of the Mutual Life, had direct access to the books and papers referring to many of the things about which he writes.

Mr. Joffe refers to the mode of adjustment followed by Mr. Homans, and it may be well to quote Mr. Homans' own words in 1889 in his address at the first dinner of the Actuarial Society; he said: "After I had collated the experience of the Mutual Life I drew a curve representing the approximate rates of mortality at different ages and then found by a simple method of adjustment the rates of mortality of what is now called the American Experience Table." He also stated that as none of the records of experience in different countries, Great Britain, France, Germany and this country, showed any case of an individual attaining the age of one hundred years, he made the table terminate at the age of ninety-six. (See Vol. 1, *Transactions.*)

After Mr. Homans had made the adjusted rate of mortality above mentioned, he proceeded to obtain the columns of living and dying corresponding thereto as nearly as practicable, starting with a base of 100,000 lives at the age 10. As the living and dying were stated in whole numbers, neglecting the fractions that would result from the use of the basic rate of mortality, it follows of necessity that the rates of mortality corresponding to the actual ds and ls of the American Table do not exactly correspond to the basic rates, although Mr. Homans in the above quotation speaks somewhat as though they did. The original basic rates of mortality as adjusted by Mr. Homans are printed at page 180 of the proceedings of the First Session of the National Insurance Convention of Insurance Commissioners in 1871, but I fear that there are some misprints in it.

Mr. Joffe has referred to Mr. Charles Gill as having been the actuary of the Mutual Life from October, 1849, until 1855, but I have found that, though he may have done some work for that company in 1849, he could hardly have been its actuary until after

July 15, 1850, for he was employed by the Mutual Benefit almost continuously from April 16, 1849, to July 15, 1850, as appears from the minutes of the Board of Directors of that Company, which Mr. Amzi Dodd, its venerable ex-president, kindly showed me.

MR. JOFFE:

(AUTHOR'S REVIEW OF DISCUSSION.)

Mr. D. P. Fackler's favorable comments on my paper are greatly appreciated by me, as it is no mean honor for a writer on the American Experience Table to be complimented by one of the oldest and most respected members of this Society and the only survivor of those who were associated with Homans at the time of the construction of that table.

As there is no bibliography of the subject of the American Experience Table, it is very easy to overlook articles of greater or less importance, and I am happy to have been enriched through Mr. Fackler's quotations with two new sources which had escaped my attention. One would hardly look to after-dinner speeches for valuable information on an abstruse subject, and it never occurred to me to read the address made by Homans in 1889 when called upon by Mr. Fackler, the toast-master, to respond to the first toast: "The Time-tried System of Life Insurance, sound in principle and correct in practice; may it last until Death is no more." I have now read this article, and am sure that I shall refer to it again more than once before I have done with the subject. It contains points of interest on the history of the table, on its construction, on the limiting age, on the effect of selection, on its advantages over the Actuaries' Table, etc. As a matter of fact, I am almost tempted to reproduce now a considerable part of the address but for the fact that most of our members have the first volume of the *Transactions* in their possession and the others can easily refer to it. I will therefore only repeat the reference made to it by Mr. Fackler, and would recommend that whoever has not read the address and desires to increase his knowledge of the American Experience Table by information coming from its author himself, should make a careful study of pages 32 to 35 of the first volume of the *Transactions.*

By the way, I have noticed in this address an interesting error on page 34, tenth line from top. The years "1859 or 1860" are referred to, while it is evident that Homans wished to quote probably 1866 or 1867, the year he went with Barnes, the Superintendent of Insurance of New York State, to see Elizur Wright, the Insurance Commissioner of Massachusetts. It seems that while speaking of this trip, he was thinking at the same time of the period when he was constructing the American Experience Table, and unconsciously quoted the latter period. We may thus be justified in taking this slip of the tongue as a corroboration by Homans of the surmise that the American Experience Table was constructed in 1859 or 1860.

Of special interest also are the four or five lines preceding the line just quoted, where Homans says that, "The American Table was gotten out simply as a study . . . it was never intended by me to be offered as a standard of valuation, or as a basis for the transactions of a company. . . ." Personally I have been of the same opinion for several years, but was reluctant to advance it as it appeared to me too bold to make such a statement without something to substantiate it. It may now perhaps be à propos to state briefly that the first striking instance which aroused in my mind the suspicion that the table was rather a theoretical table, presented itself when I noticed that the rates of mortality for the ages 91, 92, 93 and 94 formed almost exactly the four geometric means between the corresponding rates for the ages 90 and 95. (This peculiarity may also be expressed in the following manner: The rate of mortality q_x, or its reciprocal r_x, for ages 90 to 95, is an exponential function of $95 - x$.)

With the little time at my disposal I have not yet had an opportunity to examine Mr. Fackler's "Remarks before the Committee on Valuation" and his "Graduation of the American Experience Table" in the Report of the 1871 Convention, but I expect to do so shortly and hope that his figures of the "beautifully graduated rate of mortality," which I understand were furnished by Homans himself, may help settle the question whether the primary function in the construction of the American Experience Table was the rate of mortality q_x, as pointed out by Mr. Fackler, or its reciprocal r_x, as I have concluded, perhaps erroneously, from my study of Gill's Table.*

Finally, with regard to the question of the date on which Gill entered the office of Actuary of The Mutual Life, I must say that I have gathered my data on this point from the "Historical Sketch,"** which I quoted in my recent paper on "Gill's Mortality Table," and that, as the date "October, 1849," appears in the address in which the late President Winston of The Mutual Life announced Gill's death to the Board of Trustees, such information should, on the face of it, appear sufficiently authentic. As Mr. Fackler states that Gill was employed in the Mutual Benefit from April 16, 1849, to July 15, 1850, it appears that he was employed by both companies at the same time, for I find from The Mutual Life's records that he was appointed Actuary of the Company on October 31, 1849, and filled that position until his death, October 22, 1855. The same records reveal the fact that, prior to becoming Actuary of The Mutual Life, Homans was made Assistant Actuary November 14, 1855, and still earlier, even in 1854, was employed in the Actuary's Department.

* I wish to avail myself of this opportunity to correct an error in my paper "Concerning the American Experience Table of Mortality," *Transactions*, Vol. XII, p. 256, 16th line from top, where the title of Gill's work is printed by mistake as "Assurance Premiums" instead of "Assurance Tables."

** J. I. A., VI, 216–227.

REPORT ON MORTALITY EXPERIENCED AMONG ANNUITANTS RESIDENT IN THE UNITED STATES AND CANADA—ARTHUR HUNTER.

VOL. XII, PAGE 261.

MR. J. K. GORE:

The later investigation made by Mr. Hunter confirms generally the results of that made in 1905. In particular, the deduction drawn from the earlier figures that the rate of mortality among annuitants in the United States and Canada is lower than that of any standard table may now be regarded as established. The newer experience does not show quite as low a rate of mortality as the earlier, but it is still low enough to raise a doubt as to whether any actual profit has been made on these transactions in the past.

As Mr. McClintock prophesied in 1909, if any profit has arisen or a loss has been avoided, it is only because of the excess of actual over assumed interest earnings. It may be of value to discover how much of the excess interest earnings are swallowed up by the very low mortality. In the present report the total exposed to risk is not given, but in the earlier paper we have the following figures:

Sex.	Exposed to Risk.	Expected Deaths British Offices.	Actual Deaths.	Expected less Actual Deaths.
Males	16,043	746	569	177
Females	21,141	943	756	187
Total	37,184	1,689	1,325	364

From the above it appears that had a constant of .01 been added to the mortality rate experienced at all ages the actual deaths would have been $1,325 + 372 = 1,707$, practically agreeing with the expected by the British Offices' select tables. In other words, up to 1904 net annuity rates based upon the British Offices' tables, properly loaded for expenses, and with an assumed rate of interest 1 per cent. lower than that carried, would have caused neither a loss nor profit. For the present experience we may make a rough estimate as follows:

	Experience to 1904.			Experience to 1910.		
	Expected Deaths.	Actual Deaths.	Per Cent. Actual of Expected.	Expected Deaths.	Actual Deaths.	Per Cent. Actual of Expected.
Males	746	569	76	1,519	1,295	85
Females ..	943	756	80	1,890	1,585	84
Total	1,689	1,325	78	3,409	2,880	84½
Deficiency in actual deaths			22	Deficiency in actual deaths.		15½

As a deficiency of 22 per cent. in the actual deaths in the earlier

experience was found to approximate in its effects a constant diminution of .01 in the rate of mortality, we may take a deficiency of 15½ per cent. in the deaths as approximately equal to a constant diminution of .007 in the rate of mortality. We, therefore, arrive at the conclusion that extra interest earnings of two-thirds of 1 per cent. are required to cover the difference between the actual rate of mortality according to the latest experience and that shown by the British Offices' tables.

The Prudential annuity experience is so small that very little reliance can be placed on the results. So far as it goes it confirms the larger experience for females, as will be seen from the following figures:

Policies.

	Expected Terminations by Death.	Actual Terminations by Death.	Per Cent. Actual of Expected.
Males	31.74	32	101
Females	28.90	23	80

Amounts.

Males	$10,193	$9,922	97
Females	8,570	5,479	64

The experience has not been taken out by lives. As illustrating the doubtful value of meager statistics, it may be mentioned that had the observations been carried to policy anniversaries in 1910, instead of 1911, the ratios for males would have been 68 per cent. by policies and 56 per cent. by amounts, instead of 101 per cent. and 97 per cent., respectively.

The statistics show some indication of a higher mortality among the holders of the larger annuities, but it is impossible as yet to say whether this feature is characteristic or accidental. It is possible that among the larger annuities are some which are purchased to save the annuitant the trouble of managing an estate, in which case the actual condition of the annuitant's health is not quite so closely scanned as where, say, an "own life" annuity is being purchased by an individual out of the savings of years.

Touching the experience of the first five annuity years, the relatively high mortality rate for males during the period subsequent to 1904 is doubtless partly accidental, as is the very low rate for the period ending in 1904. Probably the aggregate ratio for the full period, 86 per cent., is a much better measure of the actual forces at work than the low one of 71 per cent. or the high one of 112 per cent.

Whether or not Mr. Hunter has hit upon the true cause of the rise in the ratios of actual to expected mortality cannot as yet be determined, but if we exclude an accidental explanation of the phenomenon, it is clearly due to the action of the companies in some direction, as there is practically no change in the mortality for policy years after the fifth, these policy years not being affected

by any change by the companies in the treatment of applications effected since 1905.

I observe that Mr. Hunter declines to give the suggested explanation the weight it would have as his own opinion. At the best it would scarcely seem to be a sufficient explanation of so considerable a change in the mortality, even assuming that some of the change is due to accidental variations. Mr. J. F. Little, of the Prudential office, has offered another suggestion, arrived at as follows: Mr. Hunter, in a previous paper, has pointed out that in the case of female mortality, when the applicants approached the voluntary type, the company had an unsatisfactory experience. By making special efforts to obtain business on the lives of women, however, a fresh and better stratum was opened up, and the relatively bad mortality of the earlier type of female applicants was submerged in the bulk of business on better types so secured.

Something of the same kind, though in a minor degree, has perhaps happened in connection with the annuity business. While annuities were granted only to those who sought out the companies, we may be very sure that minimum mortality rates would prevail, and just in proportion as annuities were issued as the result of active canvass and persuasion on the part of agents, so would the mortality be affected by the inclusion of annuitants of a somewhat inferior vitality. The figures of annuity receipts by the companies for the last twenty years of issue included in the present experience are given below:

Year of Issue.	Premiums received for Annuities.	Number of Years' Contribution to Experience of First Five Policy Years, Period, 1905–1910.
1890	$3,246,193	
1891	2,914,373	
1892	2,577,931	
1893	1,974,966	
1894	2,584,453	
1895	3,577,584	
1896	5,038,846	
1897	6,064,135	
1898	5,027,385	
1899	6,400,834	
1890–1899	$39,406,700	
1900	6,301,666	1
1901	8,985,099	2
1902	10,333,550	3
1903	8,809,865	4
1904	8,947,950	5
1905	8,125,880	5
1906	5,055,801	4
1907	4,725,175	3
1908	3,848,581	2
1909	4,939,114	1
1900–1909	$70,072,681	

The third column shows the number of years' experience contributed by the year of issue in question to the section constituting the experience of the first five policy years for the period 1905–1910. From these figures it may fairly be assumed that, if we suppose the annuitants divided into two classes, those who voluntarily purchase annuities and those who do so as the result of pressure from agents, the exposures in the first five policy years for the older period, ending in 1904, will contain a much larger proportion of voluntary purchasers than those for the period 1905–1910.

There has been a heavy fall in the premiums received for annuities since 1905, possibly due in part to the somewhat disconcerting results of Mr. Hunter's earlier investigations, published in that year. Part of the falling off, however, was undoubtedly due to the temporary loss of confidence in the companies on the part of the public, and this may well have affected voluntary purchasers of annuities fully as much as those buying under the persuasion of agents. If this is a correct view, the falling off of the last few years will not have so serious a financial effect on the companies as the figures might otherwise indicate.

MR. HENDERSON:

Mr. Hunter is entitled to the thanks of the actuarial profession and of the life insurance community in general for the work which he has done in investigating the mortality experience on annuitants resident in the United States and Canada. The results of his most recent investigation which appear in the last number of the *Transactions* fully justify the action of the Society in authorizing an investigation. They also justify its action in waiting until the volume of the new experience was at least equal to that of the former experience. The marked difference in the experience during the early years of the contract is hard to explain. If the cause is, as suggested by Mr. Hunter, an increase in the care taken to determine the true ages of the applicants for annuities, it would seem to indicate a very high percentage of over statement in the previous experience. This over statement would affect the mortality for all durations, but, of course, the increased rigidity of selection would not yet have had a chance to show its effects beyond the first five years. It is difficult to believe, however, that this cause alone would count for such a difference as that between 71 per cent. of the expected and 112 per cent., as shown in the male life experience. Probably a part of the difference is due to a change in the conditions under which the business is secured, which, for the time at least, reduced the effect of self selection. It remains to be seen whether this condition will be permanent and to what extent it will affect the longer durations.

I think the suggestion is a very good one that the experience on annuities should be reviewed at frequent intervals in order that a close watch may be kept on this subject. This is more especially

true in view of the fact that the annuity business on this continent will probably show a considerable increase in the future.

MR. THOMPSON:

The president asked us for a discussion on the subject of mortality among American annuitants probably because he had understood that The Mutual Life Insurance Company had undertaken an investigation, separate from that of the Society, of its entire annuity experience. Whilst we are not in a position to report fully on this matter and hope to make in the near future a complete statement of the case which will furnish some instructive comparison of domestic and foreign annuitants, it may be that the following table, which contains all the data on hand at present, will be of interest:

Sex	Entrants.	Years of Life.	Actual Deaths	Expected Deaths.			
				By McClintock's Table.	Ratio of Actual to Expected.	By British Office's Annuity Exp. Ultimate Table.	Ratio of Actual to Expected.
Domestic Annuitants.							
Male,	976	8,025	341	417.69	81.64%	424.87	80.26%
Female,	1,467	13,020	446	562.44	79.30%	626.06	71.24%
Foreign Annuitants.							
Male,	2,993	25,125	1,137	1,137.67	99.94%	1,161.63	97.88%
Female,	1,868	18,410	561	605.91	92.59%	681.91	82.27%
Domestic and Foreign Annuitants—Combined Experience.							
Male,	3,969	33,150	1,478	1,555.36	95.03%	1,586.50	93.16%
Female,	3,335	31,430	1,007	1,168.35	86.19%	1,307.97	76.99%

The most interesting deduction from these data is the closeness with which the McClintock Table has estimated the mortality among annuitants since its general application to the problem of annuity valuation. This adherence to actual experience is particularly marked, strange though it may seem, in the case of foreign annuitants and applies better to male lives than to female. This peculiarity is doubtless accounted for by the fact that the McClintock Annuity Table was framed some years ago at a time when the domestic business formed an even smaller proportion of the total annuity business of the company than it does at the present time. And the divergence herein noted need not be regarded as serious when it is considered that there are more than double as many foreign annuities as domestic in this particular company's business, and that there are about 20 per cent. more contracts on male lives than on female lives.

It should be noted that in the case of the foreign annuity experience only immediate single life annuities were included if issued for the whole of life and paid for in cash.

MR. ARTHUR HUNTER:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

The comments of Mr. Gore are very interesting and, as usual, he has given us a number of valuable suggestions. In my opinion, the greater care in determining the age of the annuitants is not sufficient to account for the increase in the mortality. Mr. Gore is probably right in assuming that a considerable part of the increased mortality is due to a smaller proportion of the annuities being sought by the applicants, and a larger proportion sold through the persuasion of the agent.

In a previous discussion on annuities, it was stated that the mortality among foreign annuitants was distinctly higher than among American and Canadian annuitants. Mr. Thompson's figures prove this point, and we shall look forward with interest to his giving us further information at a future date.

I do not think we should prepare any tables of reserves based on our present statistics. I think we ought to wait a few more years. In the past all the improvement in mortality, or the greater part of improvement in mortality, has come at the younger ages; the scientists have been working on infantile diseases very largely and diseases of young life, have been trying to do away with diphtheria, cut down the mortality from typhoid fever, which is more a young person's disease, from consumption, etc. At the present time, if you will notice, the scientists are working on the old age diseases. They are working on Bright's Disease and apoplexy: men like Metchnikoff are attacking the problem of the intestinal tract, etc. All of this will tend to decrease the mortality at the older ages, and that is the worst thing that we could look to have from the standpoint of an annuity experience though the best thing which we could hope for from a humane standpoint.

It would not be expedient at this time to prepare reserves based on the data in our possession. Five years from now I should be glad to make a further investigation if the companies are willing to supply the necessary data.

Book Notices.

Interest and Bond Values. By M. A. Mackenzie, M.A., F.I.A., A.A.S. Toronto, University Press, 1912. Pp. 105.

A small book explaining the usual forms of interest tables and tables of bond values without attempting to go into the more advanced theory of the subject. It is written in an exceedingly simple style and furnishes many problems and illustrations which help in mastering what comes within its scope. For the persons outside of the actuarial profession who have to deal with bond values and simple interest problems it will be valuable in giving a comprehension of the theory together with a good working knowledge of the tables which they use. To the actuarial beginner it will prove useful as an introduction to the more difficult and extensive books upon the subject of interest, and it will at the same time give him a practical knowledge of bond valuation which the more advanced theoretical works do not give. Some of the methods used are new and ingenious.

Journal of the Institute of Actuaries Students' Society. Volume 1, No. 1. London, Sanders, Phillips & Co. 1911.

This is the first number of the *Transactions* of the new Students' Society which has recently been organized among the junior members of the Institute. Those qualified for ordinary membership in the Society are Fellows under thirty years of age or within two years of graduation, and Associates or Students of the Institute. The Society meets weekly during the winter months for the discussion of matters of special interest to the junior members, more especially in connection with the examinations of the Institute. This number includes, besides the rules of the Society, a paper by Mr. A. E. King on the "Graduation of the British Offices' Annuitants' Experience Tables," a paper by Mr. E. W. Phillips on "Indian Railway Securities" and a lecture by Mr. Steuart Macnaghten on "Life Assurance Book-keeping." The contents of this number give great promise for the future value of the Society itself and of its Journal.

The Elements of Statistical Method. By Willford I. King, M.A. New York, The Macmillan Company, 1912. Pp. xvi + 250.

The subject of this book, as its title indicates, is the methods to be followed in statistical work rather than the mathematical theory. The subject is taken up from the practical standpoint rather than the theoretical, and the book is intended primarily for the use of those interested in sociology, political economy or administration, although the general principles set forth are applicable to every variety of statistical data. No pretense is made in this work of presenting any but the most simple of the mathematical theorems upon which statistical method is based. The first chapter deals with the historical development of the science which is defined in the second chapter as follows:

"The science of statistics is the method of judging collective natural or social phenomena from the results obtained by the analysis of an enumeration or collection of estimates."

Purists would probably object to defining a science as a method, but the scope of the science is very well indicated by the definition. Another chapter is devoted to a discussion of the uses and characteristics of statistics. The second part of the book is devoted to the subject of the gathering of material, taking up such points as the planning of the collection of data and approximation and accuracy. The third part is devoted to the subject of the analysis of the material collected, taking up under this heading tabulation, frequency tables and graphs, averages, dispersion and skewness, the final chapter being devoted to historical statistics which is the name used in the book for statistics in which time is an important element. The fourth part is devoted to the subject of comparison of variables taking up the various methods which have been used including correlation. The whole book forms a very valuable introduction to the subject.

Old Age Dependency in the United States (A Complete Survey of the Pension Movement). By Lee Welling Squier. New York, The Macmillan Co., 1912. Pp. xii + 361.

This book is an economic rather than an actuarial work, but in view of the great importance of the question of old age pensions, it is noticed here. It is divided into four parts. The first is "Dependent Superannuation" and treats of the extent and the cost to

the community. The second is "Causes of Old Age Dependency." The third is "Efforts at Relief"; under this are considered the efforts of labor organizations and fraternal benefit societies and the pension schemes of industrial establishments and transportation companies; in this part are also included teachers retirement funds, municipal pensions and government pensions. The fourth takes up "Plans for Prevention," under which the chief headings are "Thrift," "Pensions by Purchase" and "Pensions for Service."

The Business of Insurance. Edited by Howard P. Dunham. New York, The Ronald Press Co., 1912. 3 Vols. Pp. 1500.

This book is not yet issued, but is in press and the first volume is promised in a few days. It is a general work taking up in separate parts the various kinds of insurance. Each kind of insurance is again divided into subjects and to each subject a chapter is given. The different chapters are written by different persons, the author of each chapter being one engaged in the branch with which that chapter deals.

Abstract from the Minutes of the Annual Meeting of the Actuarial Society of America, held in New York City on Thursday and Friday, May 16th and 17th, 1912.

Hotel Astor, New York City.
May 16th, 1912.

FIRST DAY.

The meeting was called to order by the President, Mr. Welch, at eleven o'clock A.M. The following members were present:—

FELLOWS.

Alsop,
Cammack,
Carpenter,
Cole,
Craig, J. D.,
Craig, J. M.,
Crawford,
Dawson,
Dow,
Fackler, D. P.
Fackler, E. B.,
File,
Flynn,
Gaylord,
Gore, J. K.,
Graham, W. J.,
Hall, S. S.,
Hardcastle,
Henderson,
Hunter, A.,
Hunter, R. G.,
Huntington,
Hutcheson,
Ireland,
Joffe,
Johnston,
Kilgour,
Laird,
Linzmeyer,
McKechnie,
Macdonald,
Marshall, E. P.,
Mead,
Moir,
Morris, E. B.,
Nichols,
Papps,
Paterson,
Peiler,
Perrin,
Pipe,
Portch,
Rhodes,
Roche,
Rose,
Salter,
Sheppard,
Stilwell,
Strong, W. M.,
Thompson,
Torrey,
Watt,
Welch,
Wells,
Wood, A. B.,
Wood, W. A. P.,
Woodward, G. B.,
Woodward, J. H.,
Young.

ASSOCIATES.

Allison,
Baber,
Blehl,
Bliss,
Breiby,
Brown,
Bulkley,
Dark,
Davenport, J. S.,
Dickenson,
Fitzgerald, C. R.,
Forster,
Franks,
Gamwell,
Gould,
Hammond,
Hughes,
Kaufman,
Kime,
King,
Linton,
Macfarlane, W.,
Maclean, A. T.,
Maclean, J. B.,

MacPhail,	Phillips,	Thomas,
Moodie,	Reid,	Vineberg,
Muckle,	Ryan,	Walker,
Murphy,	Stanley,	Washburne, A. C.,
Parker, J. G.,	Strong, A. W.,	Whitney.

After the calling of the roll, the President read his Address to the Society, which, on motion, was ordered to be laid before the Society for discussion at the next meeting.

The minutes of the semi-annual meeting held in Hartford, October 12th and 13th, 1911, were approved as printed in the *Transactions*.

The Secretary reported that the Committee on the Fackler prizes had found no paper which seemed to them worthy of the first prize, and had unanimously awarded the second prize to Mr. H. N. Kaufman for his paper on "Some Uses for the Hollerith Machine," and that their report had been approved by the Council. Also that Council had decided not to prepare tables of annuity reserves based on the experience on annuitants in the United States and Canada until additional data had been obtained; also that it had been decided to print a slip containing offers of books for sale or for exchange or desired by members of the Society and by companies with which they are connected. This slip is to be issued with the *Transactions* and no charge is to be made for the printing; also that the report of the committee on the proposed new mortality table to take the place of the Actuaries' and American tables had been approved and ordered printed.

The report of the Treasurer was then read and on motion approved. The Society then proceeded to the election of officers and members of council, with the following result:

President	William C. Macdonald.
Vice-Presidents	James M. Craig. Arthur Hunter.
Secretary	Robert Henderson.
Treasurer	David G. Alsop.
Editor of the Transactions	Wendell M. Strong.
Members of Council for three years	Frederick H. Johnston. Percy C. H. Papps. Wm. Young.

The Society adjourned at 1 o'clock.

Afternoon Session.

After the Society had reassembled at 2:30 P.M. the reading of papers prepared for this meeting was proceeded with.

After the reading of the papers was completed the Society took up the discussion of the papers presented at the meeting in October,

1911. At 5 o'clock the Society adjourned to meet at dinner at 7 o'clock and to reassemble for business on the following morning at 10 o'clock.

SECOND DAY.

FRIDAY, *May 17th*, 1912.

The meeting was called to order by the President. The President reported for the Council that Mr. D. P. Fackler had not renewed his offer of Fackler Prizes and that the Council recommended that the Society offer biennially a prize of $100 to the Associate, or to the Fellow of not more than two years standing at the time of closing of contest, who shall present to the Society the best paper on some subject to be suggested by the Committee on Papers or approved by it.

It was unanimously voted to thank Mr. D. P. Fackler for his kindness in granting prizes to Associates for three successive triennial periods.

On motion the recommendation of the Council regarding the biennial prize was adopted. The President also reported that they had adopted tentatively a system of disability notation and ordered it printed and sent to all the Fellows of the Society in order that they might have an opportunity to make suggestions before the notation was included in the *Transactions*. He also reported that the invitation of the Fifteenth International Congress on Hygiene and Demography to send a representative to attend the meeting in September at Washington had been accepted and that Mr. A. A. Welch had been appointed a delegate from the Society with power to appoint a substitute. He also reported that the question of deciding the place for holding the Fall Meeting had been referred to the President, Vice Presidents and Secretary.

The discussion of papers read at the meeting in October, 1911, was then resumed and completed. A discussion was also held on the preliminary report of the joint Committee on the Medico-Actuarial Mortality Investigation.

At 1 o'clock the Society finally adjourned.

ROBERT GEORGE HANN.

By the death of Robert George Hann on the 18th of November last, the Society lost one of its older Fellows, he having been elected to membership in the Actuarial Society in October, 1890.

Mr. Hann was a man of wide attainments in actuarial and scientific affairs, and one whose writings were always notable. Of a quiet and retiring disposition, he was perhaps not so well known to the younger members of the Society as some others of the elders have been. In recent days his health had interfered with his active participation in the Society's meetings.

He was born in London, July 28, 1841, studied at Westminster and also at Cambridge University, giving especial attention to railway engineering. For nearly forty years he had been professionally engaged in life insurance in this country. He was for four years in the Actuarial Department of the New York Life; in 1883 he was First Actuarial Clerk of the Equitable Life Assurance Society and remained connected with that department of the company up to the time of his death, having been Assistant Actuary and Associate Actuary before his final appointment as Consulting Actuary.

Mr. Hann was always clear and forceful in his expression of professional views and sympathized with the older school of actuarial thought, not accepting fully the modern ideas that have been put forward in connecting with life insurance methods. Among his writings are a "Sketch of Certain Methods of Distribution Pursued in Great Britain," and a paper on the "Mortality Experience among Lives accepted at ages over Sixty by the Equitable Life Assurance Society of the United States," these appearing in the *Transactions* of the Actuarial Society. He revised and corrected the first edition of Willey's "Principles and Practice of Life Insurance," and contributed to the Fourth International Congress of Actuaries a paper on "War Mortality in the United States."

Examinations of the Society

Held on June 5 and 6, 1912.

Examination Committee.

For Fellowship.

Joseph H. Woodward (Chairman),
Albert G. Portch,
Edward E. Hardcastle.

For Associateship, Section B.

James D. Craig (Chairman),
John S. Thompson,
Louis Linzmeyer.

For Associateship, Section A.

Part I.

Oliver W. Perrin (Chairman),
Edward E. Cammack,
Franklin B. Mead.

Part II.

Sidney H. Pipe (Chairman),
William A. Watt,
Lorne K. File.

William Young, General Chairman.

EXAMINATION FOR ENROLMENT AS ASSOCIATE.

Section A—Part I.

1. (*a*) Simplify $\dfrac{1.9\dot{3}\dot{6} - .1\dot{3}9\dot{1}\dot{3} + .3\dot{7}4\dot{6}\dot{8} - .4\dot{8}1\dot{9}}{12\ 2/3 - 1\ 1/12 \div 1/2 + 1/16}$

and extract the cube root of your result to the nearest third decimal place.

(*b*) "A" dies leaving an estate of $30,000 in cash from which a tax of 1% is to be deducted; the balance is to be invested in 4% bonds quoted at 132 and the income is to be divided equally among three children. What will be the annual income of each of the children?

2. (*a*) For what value of "*k*" will the roots be equal in magnitude and opposite in sign in:

$$\frac{x^2 - bx}{ax - c} = \frac{k - 1}{k + 1}$$

(*b*) Two persons "A" and "B" entered into a speculation to which "B" subscribed $15.00 more than "A". After four

months "C" was admitted who added \$50.00 to the stock, and at the end of twelve months from "C's" admission they found that the total gain from the commencement was \$159.00. "A" then withdrew and received for principal and gain \$88.00. What did "A" originally subscribe?

3. (*a*) Find the factors of the expression:

$$y^2 z^2 (y - z) + x^2 z^2 (z - x) + x^2 y^2 (x - y)$$

(*b*) Solve for "*x*" and "*y*":

$$(x^2 + y^2) \frac{x}{y} = 6$$

$$(x^2 - y^2) \frac{y}{x} = 1$$

(*c*) Find the value of:

$$\frac{1 + x}{1 + \sqrt{(1 + x)}} + \frac{1 - x}{1 + \sqrt{(1 - x)}} \text{ when } x = \frac{\sqrt{3}}{2}$$

4. The price of a diamond varies as the square of its weight. Three rings of equal weight each composed of a diamond set in gold have values \$a, \$b and \$c, the diamonds in them weighing 3, 4 and 5 carats respectively. Show that the value of a diamond of one carat is \$ $(\frac{a+c}{2} - b)$ the cost of workmanship being the same for each ring.

5. (*a*) If *a, b, c* be in Arithmetic Progression, *b, c, d* in Geometric Progression and *c, d, e* in Harmonic Progression, show that *a, c, e* are in Geometric Progression.

(*b*) Find the sum of "*n*" terms of the series whose "*nth*" term is $3 (4^n + 2n^2) - 4n^3$.

6. (*a*) How many odd numbers without repeated digits are there between 3,000 and 8,000? How many of these are divisible by 5?

(*b*) In how many ways can "2*n*" different things be divided into "*n*" pairs? In how many ways can "2*n*" different things be divided among "*n*" persons, 2 to each.

7. (*a*) How many different games of lawn tennis, each side consisting of an American and a Canadian, may be arranged

from a party consisting of nine Americans and five Canadians? If two of the Canadians are left handed and two of the Americans are left handed, all of the others being right handed, and if two left handed players may not be partners, how many games may be similarly arranged?

(*b*) What limitations are there, if any, to the general proof (due to Euler) of the Binomial Theorem for any index? When the two series

$$f(m) = 1 + mx + \frac{m(m-1)}{1.2}x^2 + \frac{m(m-1)(m-2)}{1.2.3}x^3 + \dots$$

and $f(n) = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + \dots$

are infinite, under what conditions will their product be convergent?

8. (*a*) Find the first negative term in the expansion of $(1+\frac{4}{3}x)^{\frac{27}{4}}$ and the greatest term in the expansion of $(1-\frac{2}{3}x)^{-2}$ when $x = 3/4$.

(*b*) If C_o, C_1, C_2, $C_3 \dots$, C_n denote the coefficients in the expansion of $(1+x)^n$, n being a positive integer, prove that

$$1 \times C_2 + C_1 \times C_3 + C_2 \times C_4 + \dots + C_{n-2} \times C_n = \frac{\lfloor 2n}{\lfloor n+2 \ \lfloor n-2}$$

9. (*a*) Write an expression for the value of "*e*", the base of the Natural System of Logarithms. State why this system is called the "Natural System" and compare its uses with those of the "Common System". How would you obtain logarithms in the latter system from those in the former?

(*b*) It is said that in England each year the births are in the proportion of 1 to 33, and the deaths in the proportion of 1 to 46 of the population. If there were no emigration nor immigration, in how many years would the population double itself at this rate?

Given, log 2 = .30103, log 3 = .47712
log 253 = 2.40312, log 1531 = 3.18498.

(*c*) Solve for "*x*" in terms of "*y*":

$$y = \log_e \frac{e^x + \sqrt{e^{2x} - 4}}{2}$$

10.(*a*) Define the terms "Recurring Series", "Scale of Relation", "Generating Function", and illustrate your definition in each case.

(*b*) State and prove the Exponential Theorem and thence find the sum of the infinite series:

$$1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \cdots.$$

(*c*) Write down the Logarithmic Series and test it for convergency or divergency, stating definitely and fully the results of your tests.

11. Five balls are in a bag, and it is not known how many are white. One is drawn and found to be white, and without replacing the ball drawn, another is drawn and found to be white. Find the probability that all are white. If the first ball drawn had been replaced before the second trial was made, what would the probability have been that all were white?

Assuming that both the first and second balls drawn are not replaced, what would be the probability that a third drawing would give a ball of another color?

Explain what is meant by "Inverse Probability" and point out clearly the various fundamental principles of Probability that are involved in the foregoing example.

12. (*a*) A baseball player has a batting average of 25% (i. e., he makes twenty-five hits out of every one hundred times "at the bat"). What is his chance of making at least three hits out of five times "at the bat"?

(*b*) A purse contains three dollars and three quarters and a person draws out three coins from it at random. He then puts three dimes in the purse and again draws out three coins at random. Show that he may just lay 8 to 3 with advantage to himself against the three latter coins being all of different denominations. What is the "expectation" of this second drawing?

(*c*) There are two flat disks: each disk is marked "*o*" on one face and "1" on the other; these two disks are tossed five times. What is the chance of obtaining a total of five?

(Answer only two of the following questions, Nos. 13, 14, and 15).

13. (*a*) Explain the nature of a logical proof and apply your explanation to your discussion of the following theorem:

If from a point on the base of an isosceles triangle perpendiculars are drawn to the two equal sides, their sum is equal to a perpendicular drawn from either extremity of the base to the opposite side.

(*b*) Through the points of intersection of two circles two parallel secants are drawn, terminating in the curves. Prove the secants equal.

14. (*a*) Show that in any right angled triangle the square on the side subtending the right angle is equal to the sum of the squares on the sides containing the right angle.

(*b*) Construct a square whose area shall be twice the area of a given square. Hence show how to represent geometrically $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, etc.

15. (*a*) State in detail at least three methods of attacking a geometrical proposition.

(*b*) Find the locus of the middle point of a chord of a given length that can be drawn in a given circle.

(*c*) If a straight line touch a circle and from the point of contact a straight line be drawn cutting the circle, the angles made by this line with the line touching the circle shall be equal to the angles in the alternate segments.

SECTION A—PART II.

1. (*a*) Show that, with a certain reservation, the operators D and $\triangle$, where $u_n = D^n u_o$, satisfy the ordinary fundamental laws for algebraic quantities and hence, that they can be combined with one another as if they were symbols of quantity.

(*b*) Prove that

$$u_o + u_1 x + \frac{u_2 x^2}{1.2} + \frac{u_3 x^3}{1.2.3} + \cdots$$

$$= e^x \left[u_o + x \triangle u_o + \frac{x^2}{1.2} \triangle^2 u_o + \cdots \right]$$

2. (*a*) In a series of $(n + r - 1)$ equidistant terms, of which r terms are missing, what steps are necessary in order to interpolate the missing terms and what assumptions must be made?

(*b*) Given $u_0 = 1864$
$u_1 = 2003$
$u_3 = 2718$
$u_5 = 3769$

find u_2 and u_4.

3. (*a*) Sum the series

$$u_a + u_{a+1} + u_{a+2} + \ldots\ldots u_{a+n-1}$$

so as to obtain an expression in terms of u_a and its successive differences. What assumptions must be made?

(*b*) Sum to 18 terms, the series
336, 504, 720, 990, 1320, 1716, etc.

(*c*) Sum to n terms $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots\ldots$ etc.

4. Given $$\frac{x}{(1+x)^t - 1} = \frac{1}{t} - \frac{t-1}{2t}\,x + \frac{t^2-1}{12t}\,x^2$$
$$- \frac{t^2-1}{24t}\,x^3 + \frac{(t^2-1)(19-t^2)}{720t}\,x^4 - \frac{(t^2-1)(9-t^2)}{480t}\,x^5 \text{ etc.}$$

obtain formula for the approximate sum of u_0, u_t, u_{2t}, etc., up to u_{nt} in terms of u_0, u_1, u_2, etc., assuming $nt = 1$ and $u_m = 0$.

5. (*a*) Define "equated time of payment" for a number of sums due at different times.

(*b*) Deduce the approximate value customarily used.

(*c*) Prove that this approximate value is greater than the true equated time.

6. (*a*) How would you construct tables showing the prices of bonds bearing various rates of interest and redeemable at par in any number of years, such that they will yield various rates of interest per annum, from 2½% up to 6% increasing by 1/20 of 1%?

(*b*) How would you use such tables to obtain the present value of an annuity of 1 per annum for n years at 4.15% per annum?

7. (*a*) Deduce from first principles a formula for the amount of an annuity certain for n years, payable by instalments k times a year, interest at the rate x per annum, being convertible m times a year?

(*b*) Given that $a_{\overline{n}|} = x$ and $a_{\overline{2n}|} = y$ determine i in terms of x and y.

8. (*a*) Describe the principal books of account of a life office, their uses and their relation to one another.

(*b*) Under a commission contract with an agent, an insurance company makes a cash advance of \$300 against commissions to be earned on paid-for business. During the following three months his commissions on business amounted to \$400. How would this transaction be dealt with in the company's books?

9. Obtain the differential coefficient of y with respect to x in the following:

(*a*) $y = \dfrac{x^2 + 2}{\sqrt{x^2 + 1}}$

(*b*) $y = x^{\frac{2}{3}} (x^2 - a^2)^{-\frac{1}{3}}$

(*c*) $y = \dfrac{(1 + 2x^2)\ \sqrt{x^2 - 1}}{x^3}$

(*d*) $(1 - x^2)\, y = (1 - x)\, y^2 - x^3$

10. Evaluate

(*a*) $\displaystyle\int \frac{(x - 8)\, dx}{x^3 - 4x^2 + 4x}$

(*b*) $\displaystyle\int \frac{dx}{\sqrt{x^2 + 4x + 8}}$

(*c*) $\displaystyle\int \frac{(x + 2)\, dx}{x\sqrt{x}}$

(*d*) $\displaystyle\int \frac{(2x^3 - 5x + 1)\, dx}{x^4 - 5x^2 + 2x - 7}$

11. (*a*) If $x = t^2 - 2t + 4$
and $y = t^4 - 3t^2 + 7$

find $\frac{dy}{dx}$ and determine its values when $t = 0, 2, 5$.

(*b*) Evaluate $(1 + x)^{\frac{1}{x}}$ when $x = 0$.

(*c*) If $u = \frac{e^{xy}}{e^x + e^y}$ show that

$$\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = (x + y - 1)\, u$$

12. A loan of x dollars is to be repaid by the end of 20 years by quinquennial instalments. The rate of interest on the loan is 4% payable annually, and y dollars are set aside annually for its repayment. Determine the equation for finding the rate of interest at which the annual sum so set aside must accumulate in order that the quinquennial instalments will repay the loan by the end of 20 years.

13. A property is mortgaged for \$30,000 at 5% per annum. \$2,000 per annum is repaid semi-annually on the 1st May and 1st October, and interest is paid on the outstanding principal at the same time. The principal and interest having been paid regularly for five years, the mortgagee sells the mortgage to yield the purchaser a nominal rate of 6% per annum convertible half-yearly. What was the purchase price?

$$a_{\overline{20}|} \text{ at } 3\% = 14.87748.$$

14. (*a*) The funds of a life assurance company increased from A at the beginning of the year to B at the end of the year, and the net interest earned during the year was I. Obtain formulæ for the approximate effective rate of interest and the force of interest earned on the funds during the year.

(*b*) Determine from these formulæ the relationship between i and δ.

15. (*a*) Draft a form of "Cash Statement" and "Balance Sheet" embodying all the items usually met with in life insurance practice and describe how each item arises.

(*b*) Explain as clearly as you can the difference between a "Cash Statement" and a "Revenue Statement". Which of the two better evidences the progress of a company? Give reasons.

(*c*) From a company's annual statements, how would you determine the amount of surplus earned during the year?

SECTION B—PART I.

1. (*a*) Give symbols for the probability that of 2 lives, (x) and (y),

(i) Both will survive n years.
(ii) At least 1 will survive n years.
(iii) Exactly 1 will survive n years.
(iv) Both will die in the nth year from the present time.
(v) The second death will happen in the nth year.
(vi) (x) will survive n years and (y) $n-1$ years.
(vii) The first death will happen in the nth year.

(*b*) Interpret verbally the following symbols and obtain expressions for their values on the assumption that deaths are uniformly distributed in each year of age:

$$ {}_{n-1}|q^1_{x\,y}\,; \quad {}_{n+t}|\,q^1_{x:\overline{y\,(\overline{t})}} $$

(*c*) Express $Q^2_{x\,y\,z}$; $Q^3_{x\,y\,z}$; $Q^1_{x\,\overline{y\,z}}$; $Q^{\frac{1}{1}}_{x\,y:z}$ in terms of probabilities determining at the first death.

2. (*a*) Assuming that the deaths in each year are uniformly distributed and supposing that two persons (x) and (y) both die in the same year, prove that the chance of (x) dying before (y) is 1/2.

(*b*) Derive an expression for the probability that (x) will die within t years after the death of (y).

3. Distinguish between "Mean after-lifetime", "Most Probable after-lifetime" and "Vie Probable". Calculate all three for age 90 from following data:

x	l_x	x	l_x	x	l_x
90	1273	94	222	98	19
91	871	95	129	99	9
92	575	96	71	100	4
93	366	97	37	101	1

Prove analytically and by general reasoning that

$$e_x = p_x (1 + e_{x+1})$$

4. (*a*) Six persons, A, B, C, D, E and F are of the same age, the rate of mortality at that age being 1 per cent. What is the probability (to 5 decimal places), that within a year,

(i) They will die in an assigned order?

(ii) A will die first and F last, the remaining order not being fixed?

(iii) A will die in the first year and be the first to die?

(*b*) An office has \$10,000 assured at age 40 by ten policies of \$1,000 each. Given that $q_{40} = .02$, find the probability that the claims will not reach \$5,000 in the year.

5. A thirty-five year pure endowment is taken at age x under the following conditions: No return of any kind if insured die in first 5 years, return of Net Premiums without interest if death occur in next 20 years, return of Gross Premiums with compound interest at rate j if insured die in last 10 years. Find Net Annual Premium if computed at rate i, Gross Annual Premium being obtained therefrom by loading with percentage and constant.

6. (*a*) Prove $A_{\overline{x\,x\,x}} = A_{x\,x\,x} - 3\ (A_{x\,x} - A_x)$

Does same relation hold among annual premiums? Prove answer.

(*b*) Prove the formula for a contingent insurance

$$A^1_{x\,y} = \tfrac{1}{2}\left(A_{x\,y} + \frac{a_{x-1:y}}{p_{x-1}} - \frac{a_{x:y-1}}{p_{y-1}}\right)$$

What supposition regarding deaths is made in obtaining this formula?

7. Prove algebraically and by general reasoning that

$$\mathrm{a}_x = \Sigma_{n=1}^{n=\infty} \left\{ v^n (1+i) \frac{(1+i)^n - 1}{i} \Big|_{n-1} q_x \right\}$$

8. How would you proceed to calculate commutation columns for the purpose of finding the net single premium for an assur-

ance which provides for the payment of U_r if death occur in the $(r+1)th$ year, U_r being a rational integral function of the third degree in r?

9. Deduce a formula for the net single premium for a life annuity with the provision that in case of the death of the annuitant before the total annuity payments equal the single premium paid to the Company, the excess shall be returned. Ignore the question of loading.

10. (a) Prove $P^1_{x:\overline{n}|} = \dfrac{D_x - D_{x+n}}{N_x - N_{x+n}} - d$ and give verbal interpretation.

(b) Express in terms of D and N columns and the rate of discount, the annual premium for (i) an endowment assurance to mature in n years, (ii) a whole life assurance premium limited to n payments. Subtract (ii) from (i) and give a verbal interpretation of the result.

11. Prove that on the assumption of a uniform distribution of deaths $\mathring{a}^{(m)}_x = a^{(m)}_x + A_x \dfrac{i\,(j_{(m)} - \delta)}{j_{(m)}\,\delta^2}$

12. (a) Deduce a value for ${}_{\frac{1}{t}|}a_x$. Using a first approximation for this benefit and for $a^{(m)}_x$ find how long a quarterly annuity must be deferred in order that its value may be the same as that of a monthly annuity.

(b) Prove that $a^{(2)}_{x:\overline{n}|} = \frac{1}{4}\left\{3\, a_{x\,\overline{n}|} + a_{x\,\overline{n}|}\right\}$ approximately.

(c) How would you approximate to ${}_{\frac{1}{6}|}a^{(4)}_{40:30:\overline{11\frac{3}{4}}|}$

13. (a) Deduce a formula for the annual premium to provide for \$1,000 payable at the end of 20 years with the proviso that if the insured now age 2, dies in the meantime the premiums will be returned together with compound interest at 3½%, and with the further proviso that if the party who pays the premiums and is now age 40 should die, the policy would become paid-up.

(b) An increasing n-year Endowment Policy is issued to a person aged x, the premiums and amount insured for successive years both to be in the ratio 1. 2. 3. . . . n, while the endowment is for n dollars. What is the net premium for the first year?

14 (*a*) Describe how to calculate a complete table of net premiums and reserves for endowment policies maturing at quinquennial ages 55 to 75 inclusive, given (i) an arithmometer (ii) a complete table of temporary annuities for all ages and durations. Explain how results can be verified at various stages.

(*b*) What are advantages and disadvantages of the "continuous" process of constructing tables?

(*c*) At 5% interest A is .90476 when $a = 1$. What is the value of A when $a = 3.1$; when $a = 7.3$? Explain your answer.

15. (*a*) In a stationary community supported by 5,000 annual births, and by immigration at age 15 equal to 10% of the population at that age, each member on attaining the age 21, makes payment of \$100 and contributes \$10 at the end of each succeeding year, until and inclusive of the 50th birthday, receiving thereafter an annuity of \$100 payable at the end of each year.

In respect of each contributor who dies before receiving the first payment of \$100 a payment of \$50 is made. Find expressions for

(i) Number of contributors;
(ii) Annual receipts;
(iii) Total yearly annuity payments;
(iv) Annual death-claims.

(*b*) Criticise the statement that the mortality under A (a newly settled mining district) was 12 per 1,000 and therefore more favorable than B (a middle Western United States farming section) at 13 per 1,000, and still more favorable than that of C (an Eastern United States city of over 1,000,000 inhabitants) at 15 per 1,000.

16. If a mortality table were such that the numbers living formed a series in geometrical progression, what would be the nature of the tables giving the chance of dying in a year, and the expectation at any age?

17. (*a*) Name (i) four Population mortality tables which have been constructed, and (ii) four Office Experience mortality

tables which have been constructed, stating for what purpose the several tables have been used.

(*b*) State briefly at least five of the recommendations of the Armstrong Committee of the New York Legislature that were adopted.

(*c*) When was the first valuation of the affairs of a life office ever made and why?

SECTION B—PART II.

1. (*a*) Prove that $m_x = -\frac{d \log_e L_x}{dx}$ and deduce therefrom the relation which subsists between m and μ when deaths are assumed to be uniformly distributed.

(*b*) What do T_x and Y_x represent? Give two meanings for each symbol. Express T_x and Y_x as definite integrals.

2. (*a*) Express as definite integrals, explaining briefly the significance of each: ${}_n|q^1_{x\,y}$; ${}_{|m+n}q^1_{x:\overline{y\,(\overline{n|})}}$; $\bar{A}^4_{w\,\frac{x}{3}\,\frac{y}{2}\,\frac{z}{1}}$

(*b*) If the force of mortality be constant (c), prove by direct integration that $\bar{A}_x = \frac{c}{c+\delta}$.

3. (*a*) Show by integrating $\frac{1}{D_x}\int_0^\infty D_{x+t}\,dt$ by parts that $\bar{a}_x$ may be expressed in terms of a continuously increasing assurance, a continuously increasing annuity and the force of discount. Find the total amount payable under the increasing annuity in the nth year.

(*b*) Define $\bar{N}_x$ and $\bar{M}_x$ and prove by integration that

$$\mathring{a}_x = a_x + \tfrac{1}{2}\,\bar{A}_x - \frac{\mu_x}{12}$$

4. Show that upon Gompertz's hypothesis as to the law of mortality, the relation ${}_{|t}q^1_{yz} = Q^1_{yz} \times {}_{|t}q_{yz}$ is accurately true.

5. State Makeham's law of mortality and demonstrate the forms assumed by the functions μ_x and l_x. Prove that if this law holds, $c^y \frac{d}{dx} a_{x\,y} = c^x \frac{d}{dy} a_{y\,x}$

6. (*a*) Deduce a formula for ${}_nV_x$ in terms of single premiums for Whole Life Insurance.

Give verbal interpretation of formula.

(*b*) Forty-four policies of 1 each all effected at age 40, have been in force 1, 2, 3, etc., up to 44 years respectively. The sum of their values is 17.52789. Having given $A_{40} = .379434$, find the value of the future net premiums.

7. Show that P_x may be transformed into

$$v\,q_{x+n}\,(1-{}_nV_x) + v\,p_{x+n}\,({}_{n+1}V_x - {}_nV_x) - (1-v)\,{}_nV_x$$

and explain the significance of the transformation.

8. Deduce a formula to find G_x, the present value at the date of issue of the assumed mortality gains on a Whole Life Insurance of \$1 at age x.

9. Give a clear definition of reserve. Explain the prospective and retrospective methods. Show for an Ordinary Life Policy that the one is algebraically the equivalent of the other.

10. Prove ${}_nV^1_{x\,\overline{t}|} = \dfrac{a_{x:\overline{t}|}}{a_{x:\overline{t}|}}\,a_{x+n:\overline{t-n}|} - a_{x+n:\overline{t-n}|}$

and ${}_nV_{x:\overline{t}|}^{\,1} = v^{t-n}\,{}_{t-n}p_{x+n}\,\dfrac{a_{x:\overline{n}|}}{a_{x:\overline{t}|}}$

and that their sum $= {}_nV_{x\,\overline{t}|}$

11. (*a*) Prove algebraically and by general reasoning, that the difference between the net annual premium for a 20-Year Endowment insurance and the net annual premium for a 10-Year Term insurance, is equal to the net annual premium for a 10-Year Pure Endowment, for the amount of the reserve at the end of the 10th year on the 20-Year Endowment insurance.

(*b*) A claim under a policy was settled by issuing a supplementary contract, providing for the payment to the beneficiary of a certain amount annually in advance for 20 years, the annual payment to be continued as long after the 20-year period as the beneficiary lives. The supplementary contract has been in force exactly 10 years, an instalment being now due. Give formula for the reserve value.

12. Given a standard table of mortality, show clearly how to construct a hypothetical table which, with the same rate of interest, will produce the same reserve values as the standard table on Ordinary Life policies, the rate of mortality at age 20 in the hypothetical table being greater than that of the standard table by .01. Will it give the same values on Limited Payment Life and Endowment policies? Explain your answer.

13. With a table which follows Makeham's law show that the value of a contingent assurance subject to a continuous premium is equal to the value of a Joint Life policy of a reduced amount which is independent of the duration of the assurance.

14. Deduce either Woolhouse's or Lubbock's formula of approximate summation and apply such formula to the computation of the value of a continuous annuity from the value of a yearly annuity.

15. What are the main interrogatories (other than those of the medical examiner) contained in an application for insurance? State the advantage of making the application a part of the policy contract.

16. (*a*) What optional modes of settlement at Death or Maturity would you incorporate in an Endowment policy?

(*b*) Draft (i) a clause providing for annual dividends, (ii) a provision for applying the dividends to accelerate the maturity of the policy as an Endowment or to shorten the premium-paying period.

EXAMINATION FOR ADMISSION AS FELLOW.

PART I.

*1-A. (*a*) Assuming the relation $l_x \bar{A}_x = -\int v^t \, d\, l_{x+t}$.

prove $\bar{A}_x = \mu_x \bar{a}_x - \dfrac{d\,\bar{a}_x}{dx}$

*Candidates who have passed the Associateship Examinations prior to the year 1910 will omit Questions 1-B and 2-B and will take Questions 1-A and 2-A.

Other candidates will omit Questions 1-A and 2-A and will take Questions 1-B and 2-B.

The remaining questions are for all candidates.

(*b*) State, and indicate the method of deriving, either Lubbock's or Woolhouse's formula for approximate summation.

*1-B. (*a*) State briefly what is known concerning the origin of the American Experience Table of Mortality and discuss, both theoretically and practically, the usefulness of this table as a measure of mortality among American insured lives at the present day.

(*b*) Under what conditions must a census be conducted and what data must be collected by the enumerators if a reliable table of mortality among the general population is to be compiled? Would you advise that the sexes be considered separately in constructing such a table? Indicate the data available in any government census report with which you are familiar.

*2-A. (*a*) Assuming that for an annuity on n joint lives of *different* ages we may substitute an annuity on m joint lives of *equal* ages, deduce the law of mortality involved.

(*b*) Complete the equations

(1) $\bar{A}^{3}_{x\,y\,z\,\overline{1}} = \frac{1}{l_{x\,y\,z}} \int$

(2) $\bar{a}_{y\,z}{}^{1}\big|_{wx} = \frac{1}{l_{w\,x\,y\,z}} \int$

(3) $\bar{A}^{2}_{w\,x\,y\,z\,\overline{1}} = \frac{1}{l_{w\,x\,y\,z}} \int$

*2-B. (*a*) What choice of methods exists when it is desired to determine the constants in Makeham's formula for any given set of mortality statistics?

(*b*) Describe the method employed in recording the data in the Medico-Actuarial Mortality Investigation as regards (1) age at issue; (2) duration; (3) existing at close of observations; (4) hazardous occupations; (5) medical impairments; (6) build. Give a formula suitable for determining the "exposed to risk" in this investigation.

*Candidates who have passed the Associateship Examinations prior to the year 1910 will omit Questions 1-B and 2-B and will take Questions 1-A and 2-A.

Other candidates will omit Questions 1-A and 2-A and will take Questions 1-B and 2-B.

The remaining questions are for all candidates.

3. (*a*) Cite any statistics with which you are familiar indicating that a higher rate of mortality prevails among persons insured for large amounts than among persons insured for small amounts. Is there any reason why, under the usual methods followed in investigations of a particular company's experience, such statistics may be presumed to understate the real difference between the two classes? What special points would you consider in passing on an application for a very large amount of insurance?

(*b*) Name and describe any published experiences regarding mortality among

(1) Abstainers as compared with non-abstainers.
(2) Reformed intemperates.
(3) Insured under term policies.
(4) Insured giving personal history of appendicitis.

4. (*a*) Describe in detail the methods of treating data in regard to age at purchase, duration and the tabulation of the fundamental columns for both select and aggregate tables, used in the British Offices Annuity Experience, 1893. Give the formula for the exposed to risk in each case.

(*b*) Explain the various steps in the graduation of an unadjusted l_x column by G. F. Hardy's method of applying Woolhouse's formula. What advantages has this process over Woolhouse's original process?

5. (*a*) What are the practical advantages of a system of loading premiums by a percentage of the net premium plus a percentage of the ordinary life net premium at the same age?

(*b*) Describe the general methods of loading premiums usual in either (1) Great Britain or (2) France.

6. (*a*) It is desired to construct a table of gross ordinary life premiums based on a given mortality table and loaded with a percentage and a constant in such a way as to correspond as closely as possible with a given table of gross premiums of which the method of construction is not known. How would you determine the percentage and constant?

(*b*) What bases of mortality and interest would you use and how would you load the premiums for the following classes of contracts?

(1) Single-premium whole-life, participating.
(2) Single-premium whole-life, non-participating.
(3) Single premium 20-year endowment, participating.
(4) Single-premium 20-year endowment, non-participating.
(5) Ten-year term, convertible without medical examination within seven years, non-participating.
(6) Weekly-premium life (industrial).
(7) Reversionary annuities.

7. (*a*) Discuss both theoretically and practically the question of issuing a policy for $10,000 at a premium loaded less in proportion than the premium for a similar $1,000 policy.

(*b*) Interests controlling a newly-organized stock life insurance corporation writing a non-participating business propose to form an auxiliary corporation which is to enter into an agreement with the insurance corporation whereby the latter is to pay over to the former the entire loadings on all premiums collected by it, and in consideration the auxiliary corporation agrees to pay all expense charges incurred by the insurance corporation. Discuss the feasibility and the advantages, if any, of this proposition.

8. (*a*) Compare the "New York Rule" with the "Massachusetts Rule" for valuing policies of industrial life insurance. Sketch the conditions which led to the adoption of these rules.

(*b*) You are advised by counsel: "A life insurance policy should be valued in accordance with its terms as expressed therein." Amplify and critically analyze this statement. Does it apply unconditionally and in all cases?

9. (*a*) The 1911 Convention Blank contains the following interrogatory: "Have there been included in this statement proper reserves to cover liabilities which may have been actually incurred on or before December 31, but of which no notice was received at the home office until subsequently?" Assuming that a company answers this question in the affirmative, state the

principal item or items of liabilities which will have been affected and the proper method of computing these items. Give your opinion as to the necessity of holding such reserves.

(*b*) A company valuing its policies at 3½% desires to sell a parcel of real estate. The best offers it can obtain are either (1) $500,000 in cash together with a short-term purchase-money mortgage for $400,000 at 5%; or (2) $450,000 in cash together with a purchase-money mortgage for $600,000, maturing in 20 years at 3%. At what value should the mortgage be entered in the assets if the first offer is accepted? If the second? Which offer would you advise accepting?

10. (*a*) Among the "assets not admitted" by the various States is the excess of premium notes and loans and of net uncollected and deferred premiums over the net values of the respective policies. Draft a form of schedule to show the excess of credits on account of any particular policy over the reserve thereon and discuss the question of whether mean reserves or terminal reserves should be used in the calculation.

(*b*) Draft a form of working sheet for calculating, in respect to any particular purchase of fixed term securities by a life insurance company, the effective rate of interest earned on the investment and the amortized value thereof at the end of the calendar year of purchase.

11. (*a*) You are employed by a large life insurance company to make an investigation into the affairs of a small life insurance company whose capital is reported to be impaired and about one-third of whose old business is on an assessment plan, for the purpose of determining the price at which the larger company can safely purchase the stock of the smaller preliminary to reinsurance. Outline the main points which you probably would have to investigate.

(*b*) A twenty-premium life policy, non-participating, provides for a specified increasing "guaranteed dividend" from year to year which the policy-holder may either draw in cash or apply to purchase paid-up insurance. A "net valuation" of this policy is required by statute. How would you compute the reserve?

12. (*a*) Enumerate a considerable number of the chief causes of impairment in a life risk and classify them according as their intensity is likely to

(1) Increase with the duration of the insurance;
(2) Decrease with the duration of the insurance;
(3) Remain constant for a long period.

(*b*) Do you consider it advisable to demand a reading of the blood-pressure in examinations for life insurance? What conditions may be indicated by abnormal pressure?

13. (*a*) With what restrictions and under what conditions would you accept risks in the following general classes:

(1) Showing tubercular family history;
(2) Showing personal history of albuminuria;
(3) Overweights;
(4) Employed as linemen;
(5) Employed on railroad trains.

(*b*) It is proposed to establish a life insurance company to deal exclusively with under-average risks. What general methods would you recommend for establishing and conducting the business of such a company?

14. (*a*) A company desires to allow in all policies a privilege to change at any time to a policy of the same amount and date on any plan requiring a higher premium, upon payment of the difference in premiums with interest. State what you would recommend in such a case, with reasons.

(*b*) Request is made to change a 20-payment life policy payable in twenty instalments which has been in force five years, to the continuous instalment plan on the payment by the insured of the accrued extra premiums with interest. Would you grant this request? If not, what alternative proposition would you make?

15. (*a*) What general principles should govern a company in passing upon requests for special forms of policies or special provisions in existing policies to meet individual requirements?

(*b*) What would you do with a request for an ordinary life policy on a man aged fifty-one in favor of his son aged seven, the policy to be written in sufficient amount to produce a life

income to the beneficiary of $500 per annum, subject to the provision, however, that if the insured should die before the beneficiary attains age thirty, the annuity is to be deferred until he reaches that age?

16. (*a*) A ten-premium life policy for $1,000 is written in January; in November the insured requests in lieu thereof a new policy with current date at his attained age (which has changed in the interval) for $5,000 on the ordinary life plan, provided the company will allow credit on the new policy for the premium paid on the original policy less the term rate from January to November. Would you agree to this proposition; and if so, how would you write the new policy and what adjustments of commission would you make? Would you agree to a similar proposition if made in November of the second policy year, the insured to be credited with the first two premiums on the old policy, less the term rate for one year and ten months?

(*b*) A man dies leaving $100,000 invested in securities yielding 4½%. The income from this investment is to go to his wife aged 65 for life, with the remainder to his wife's daughter aged 40 if she be alive at the wife's death; otherwise, the remainder is to go to the daughter's children. State the formula you would use in calculating the value of the daughter's interest and indicate the table of mortality and rate of interest prescribed for taxation purposes by the laws of some particular state or country.

PART II

1. (*a*) Give a brief historical account of the development of the Homans "contribution plan" of apportioning surplus from its inception in 1863 to the present day.

(*b*) A company that has been paying the first annual dividend on its policies at the end of the second year, decides that on future issues it will pay the first dividend at the end of the first year. What considerations would you take into account in determining the modifications you would make in the dividend basis?

2. (*a*) Describe in detail Weeks's "Asset-Share" system of apportioning surplus.

(*b*) Where the business of a company written previous to 1907 is nearly all upon deferred dividend plans while the recent

business is wholly on the annual dividend plan, what method would you adopt for allocating against the deferred dividend business its equitable share of the company's expenses?

3. (*a*) You are required to draft a report to the board of directors on the subject of what the new business of your company is actually costing. What points would you be careful to explain?

(*b*) A new company wishes to do an annual dividend business. Draft a dividend clause for the policies. What general considerations would influence you in determining a scale of dividends for use during the early years?

(*c*) Under what circumstances is a uniform reversionary bonus system of surplus distribution equitable? What advantages and disadvantages has it in practice?

4. (*a*) Criticise the practice of dating back policies of insurance and indicate how your arguments apply to the case of annuities. An annuity is issued in 1912 and dated back to 1911. Show how this transaction would appear in the annual statement and in the gain and loss exhibit.

(*b*) The insured under a twenty-payment life deferred-dividend policy for $10,000 issued March 1, 1905 at age 35, annual premium $380.00, fails to pay the premium due March 1, 1910, and the policy automatically goes under non-participating extended insurance. The insured writes the company on June 1, 1912, that he wishes to restore the policy to full force. He also furnishes proof that he overstated his age by five years in the original application and wishes his policy reissued on the basis of age 30, annual premium $340.00. The policy contains this clause: "If the age of the insured is misstated, the amount payable shall be the insurance which the premium paid would have purchased at the true age of the insured." Show in detail (1) how you would carry out the proposed restoration and (2) the necessary entries in the company's books and records upon the completion of his restoration.

5. (*a*) Explain the difference between the "general agency" and the "branch office" systems of agency organization and indicate their relative advantages.

(*b*) Describe any method with which you are familiar for conditioning the payment in whole or in part of renewal commis-

sions upon the efficiency of service of the agent or upon the amount and quality of the business renewed under his supervision. What are the practical limitations of such a method?

(*c*) To what extent and in what way does "competition" enter into the business of life insurance?

6. (*a*) Describe briefly the methods that you would follow in organizing the actuarial department of a life insurance company with special reference to the following points:

(1) Separation of the force into divisions;
(2) Uniform distribution of the work through the year, including provision for vacation season, and for extra work at end of year;
(3) Difference in organization depending on size of company;
(4) Preservation of due proportion between the number of clerks and volume of work;
(5) Mechanical equipment.

(*b*) A renewal premium of $500 due on January 25 is paid to the company's agent on the preceding December 24. On December 26 the agent mails a report showing the payment of the premium and taking credit for 7½% commission. This report is received at the home office on December 31. On December 28 the agent remits cash for the amount of the premium less commission and less discount at 3½% for prepayment of premium, which he had previously omitted to report. This remittance is received at the home office on January 3. Indicate the necessary entries on the books of the home office and show how the company's balance sheet for the year ended December 31 will be affected by the transaction.

7. (*a*) A life insurance company has heretofore rented its home office space in a convenient office building. What considerations would govern you in making a recommendation as to whether the company should buy land and erect a building of its own and whether such building, if erected, should be devoted entirely to present and prospective accommodation for the company's offices or should contain space to be permanently rented to other tenants.

(*b*) Should life insurance policies provide for loans upon demand?

(*c*) Discuss the contention that the assets of a life insurance company should be mainly invested in the territory from which the premiums are derived.

8. (*a*) From the following data, assumed to be taken from the books and records of a life insurance company, construct a gain and loss exhibit for the year 1911.

	1910	1911
1. Ledger assets December 31	$27,000,000	$29,500,000
2. Accrued interest	425,000	450,000
3. Gross uncollected and deferred premiums	460,000	475,000
4. Loading on above	97,000	100,000
5. Agents' balances	10,000	14,500
6. Admitted assets	27,778,000	30,310,500
7. Policy reserve (3½% basis)	24,500,000	26,700,000
8. Death claims outstanding	75,000	62,500
9. Unearned interest	83,000	90,000
10. Insurance expenses due or accrued.	50,000	65,000
11. Dividend funds	850,000	950,000
12. Unassigned funds	2,220,000	2,443,000
13. Premiums received		5,000,000
14. Interest and rents received (less amortization)		1,450,000
15. Death claims paid		1,500,000
16. Matured endowments		230,000
17. Surrender values paid		590,000
18. Dividends paid or applied		800,000
19. Taxes and expenses on real estate		30,000
20. Other expenses paid		800,000
21. Loading on premiums received during the year		1,000,000
22. Expected mortality on net amount at risk		1,150,000
23. Reserves released by death		663,000
24. Terminal reserves on policies surrendered for cash		625,000
25. Net gain on other surrenders, lapses and changes		40,000

(*b*) Using the same data, compute the gross and net rates of interest earned on the company's assets during 1911.

*9-A. You are retained to draft a bill to provide for the pensioning of the members of the police force of a large municipality. Retirement is to take place at age 65 or at earlier total and permanent disability. The annual pension is to equal one-half the average annual salary for the ten years immediately preceding retirement. One-third of the necessary contribution to the fund is to be paid by the municipality and the remaining two-thirds is to be derived from a voluntary percentage levy on the salaries of the men. It is desired to place the plan in full operation at once and to arrange as nearly as possible that current contributions from year to year shall meet current payments for pensions, thus avoiding the necessity of providing a large initial fund and the expense and risk of administering a large fund at any time. Criticise this plan and indicate what your practical recommendations would be. What methods would you suggest for treating (1) voluntary withdrawals from the force; (2) dismissals for cause; (3) cases of temporary disability. How would you proceed to estimate a scale of percentage contributions of the salaries which would be equitable?

*9-B. (*a*) A company whose present scale of surrender values is more liberal than that endorsed upon its older forms of policies desires to extend to its old policy-holders the benefit of the new scale. Under what conditions would such a course be feasible and equitable?

(*b*) Give the arguments for and against the allowing of changes from the deferred dividend class to the annual dividend class.

10. (*a*) An editorial writer in a financial paper complains that the currency system of the United States "lacks elasticity". Explain what is meant by this phrase and outline such measures

*Candidates who prior to the year 1910 passed Section A of the Fellowship Examination will omit Question 9-A and will take Question 9-B.

Candidates who passed Part I of the Fellowship Examination in 1910 or 1911 or who have not yet passed in Part I will omit Question 9-B and will take Question 9-A.

The remaining questions are for all candidates.

as you know of which have been proposed to correct this condition.

(*b*) A United States national bank decides to issue $100,000 of bank notes. It purchases $100,000 United States government 2's of 1930 at 102. Money is worth 5½% to the bank. The tax is ½ of 1% of the issue and a redemption fund of 5% is required to be maintained. Neglecting incidental expenses estimate the profit to the bank when all the notes are in circulation.

11. (*a*) Explain what is meant by the "Quantity Theory" of money, making reference in your explanation to the "velocity of circulation" and the volume of business transacted.

(*b*) What effect, if any, may a rapid increase in the world's supply of gold be expected to have on the value of (1) long-term bonds; (2) short-term bonds; (3) stocks; (4) real estate.

(*c*) When money rates go up do bonds sell higher or lower? Why? What class of bonds is an exception to this rule?

(*d*) In what way, if at all, should a life insurance company take account of fundamental business conditions in determining its investment policy?

12. (*a*) What provision is usually inserted in life insurance policies in regard to assignment? Give your estimate of the legal value of this provision.

(*b*) What is the legal status of an absolute assignment to a person without insurable interest in the life insured?

(*c*) What is the position of an assignee under a policy in the application for which a material misrepresentation has been made?

13. A life insurance policy contains the clause: "This policy, after two years, will be incontestable except for non-payment of premiums."

(*a*) Discuss the contention that if the insurance company can show that the policy was procured through fraud it should be relieved of its waiver of defense.

(*b*) Does the above incontestable clause bar an insurance

company from setting up as a defense (1) suicide; (2) death at the hands of justice?

14. (*a*) Assuming that the board of directors of your company has asked you to advise regarding the providing for a disability benefit in life policies, indicate what you would cover in your report under the following heads:

(1) Form of benefit to be allowed;
(2) Basis of premiums;
(3) Basis of valuation;
(4) Policies from which the benefit should be omitted.

(*b*) How should the extra premiums, claims and reserves in respect to disability benefits contained in life policies be entered in the accounts of the company?

15. (*a*) In preparing a combined mortality and disability table what relation must exist between the total deaths at each age, among active lives, disabled lives, and mixed lives? Explain clearly why the American Experience q_x could not properly be applied to the number of active lives.

(*b*) Discuss the question of whether an aggregate, a select or an ultimate table of mortality should be used in dealing with the death rate among the disabled for the purpose of calculating premiums or reserves.

16. (*a*) In a certain combined mortality and disability table the number of active lives at age 30 is 85,147. The number of disabled attaining age 30 is 294, while the number becoming disabled in the course of the year is 48. There are 31 deaths among the disabled and 689 deaths among the actives. Find

$$q_{30}^{\prime}, \; q_{30}^{aa}, \; r_{30}, \; l_{31}^{aa}, \; l_{31}^{ii}.$$

(*b*) Express the following symbols in terms of more elementary functions:

$$D_x^{aa}, \; D_x^{ii}, \; N_x^{aa}, \; N_x^{ii}.$$

(*c*) Describe the data you would require and the methods you would adopt in obtaining the rate of sickness of a large fraternal order doing business in the healthy districts of the United States and Canada.

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TRANSACTIONS

OCTOBER 17TH AND 18TH, 1912.

Address of the President, W. C. Macdonald.

Extended Insurance.

Gentlemen:

Permit me to first express to you my appreciation of the honor which you have conferred upon me in electing me to the office of president of the Actuarial Society. I desire also to extend to you all on behalf of the Ontario members a hearty welcome to the City of Toronto.

This is the third occasion upon which this city has been honored by the Society in the holding of its semi-annual meeting here, first in 1891, and again in 1907. When the Society first met here twenty-one years ago we had an attendance at our meeting of thirty-three. To-day we have eighty-two. Of the thirty-three nine have passed away, while several of the others have retired from active work.

Our president then was Mr. D. Parks Fackler, who I much regret is not present with us to-day. I was greatly interested recently in reading his address, in which he reviewed the earlier efforts which had been made to form a society, and which had proved futile, and in which he also traced the early organization of this Society, and remarked:

"Now that we are on a firm footing let us hope that our Society may not only continue a source of pleasure and advantage to ourselves, but may also become a means of benefiting and elevating the whole business of life insurance."

We have seen the membership of the Society grow from about sixty, which it then was, until to-day we have a total membership of two hundred and fifty-seven, one hundred and forty-five Fellows and one hundred and twelve Associate members.

The annual publications of the Society contain many valuable articles and discussions, and already form a valuable library of insurance literature and give evidence of the mental activity and industry of the members. The system of examinations for Fellows and Associates, which has been established and carried on through the efforts and disinterested self-denial of a number of the members, has been the means of encouraging and enabling many student members to qualify for the profession and fit themselves for positions of present and future usefulness. The Specialized Mortality Investigation undertaken some ten years ago, and the Medico-Actuarial Investigation, which is now being conducted in conjunction with the Medical Directors' Association, is research work of an original and an important nature which cannot but prove of the greatest value to the members of the Society and that of the insuring public. The work of the Society has been "of advantage to ourselves," while our social gatherings have ever continued to be "a source of pleasure" to all, and a beneficial influence has, we believe, been exercised on the business in which we are engaged.

Some progress has thus been made towards the accomplishment of the high ideals set by Mr. Fackler. The foundations of the Society have been well and truly laid, and the future is full of promise and opportunity for an increased measure of usefulness.

In his last annual address Mr. Welch submitted the experience of his company, the Phœnix Mutual Life Insurance Company, under its extended insurance. The practical nature of the subject, together with the valuable information given relating to this now almost universal system—on this continent at least—of non-forfeiture, resulted in a suggestion being made that his address should, though not in accordance with the usual custom, be open to general discussion at this meeting. For this reason it appealed to me that I could not perhaps submit anything which would prove of more interest than the similar experience of my own company.

The following data are submitted with considerable diffidence as they are very limited, and judged alone could not be regarded as of particular value. Viewed, however, in the light of and in conjunction with the broader experiences of the Mutual Benefit, as sub-

mitted some four years ago by Mr. Rhodes, and that of the Phœnix Mutual, as presented by Mr. Welch, some importance may attach thereto.

The automatic non-forfeiture system of extended term insurance was adopted by the Confederation Life in 1893, and provided that in default of any premium after the second the policy would be extended automatically as a term insurance for the full amount of the original policy. In 1900 this provision was amended, and extended insurance was granted in the event of default in the payment of any premium after the third. The full reserve value of the policy, according to the H^m Table of Mortality and $4\frac{1}{2}$ per cent. interest, which was the basis then employed by the company in making its valuations, was employed as a single premium in the purchase of extended insurance. The single premium rates were computed according to the same table of mortality and interest rate, and were loaded 20 per cent. plus $6 per thousand for all ages and durations.

In the event of the death of the insured during the first three years of the extended insurance all overdue and unpaid premiums together with other indebtedness, if any, were deductible with interest accrued thereon from the amount of the claim. The extended insurance was without participation in profits. It was not made retroactive and applicable to policies issued prior to 1893. The experience, therefore, deals with the business written since that date in Canada.

The data under observation being limited, the investigation was confined to amounts only. As the company first introduced the provision in 1893, and two years elapsed before any policy went on extended insurance, there was no actual experience till 1895, and as the investigation closed with the policy anniversary in 1911, the period under observation was sixteen years, the same as that of the Phœnix Mutual.

For the sake of comparison the results were summarized in the same way as those of the Phœnix Mutual, viz.:

1. By years of lapse of the original policy.
2. By years of exposure.
3. By age of the insured at the date of issue of the original policy.

The expected loss was calculated according to two tables of mortality, firstly, the $O^{M(5)}$, which is the table now employed by the company, and secondly, the American Experience.

The ages at the date of extension are nearest ages. In the case

of insurance terminated otherwise than by death the duration was taken to the nearest one tenth of a year. In reducing months to tenths of a year the same arrangement was used as that by Mr. Rhodes in the Mutual Benefit Investigation, viz., that three and nine months are equivalent to two and one half and seven and one half tenths respectively. Three months was considered as three tenths and nine months as seven tenths. For the months one to three the tenths are identical. From four to eight months the tenths are one less than the number of months, and from nine to eleven months two less. In the case of insurances falling in by death the duration was carried to the anniversary of the date of extension following the date of death. The policy year method was followed. In the case of policies extended, and which were revived and subsequently extended, the duration of the second extension was taken from the date that satisfactory evidence of good health was furnished the company.

TABLE I.

SUMMARY OF THE MORTALITY IN THE EXTENDED INSURANCE FUND BY YEARS OF LAPSE OF THE ORIGINAL POLICIES.

Extended in the Year.	Amount Exposed.	Actual Loss.	Expected Loss. $O^{M(5)}$	Per cent. Actual to Expected.	Expected Loss. Am. Exp.	Per cent. Actual to Expected.
2	$1,450,620	$13,000	$12,808.10	101.5	$13,535.80	96.0
3	2,781,170	16,500	25,222.30	65.4	26,346.90	62.6
4	1,692,930	14,000	16,436.80	85.2	16,854.70	83.1
5	1,029,440	5,000	9,609.60	52.0	9,986.20	50.1
2–5	6,954,160	48,500	64,076.80	75.7	66,723.60	72.7
6	609,070	2,000	6,106.10	32.8	6,171.10	32.4
7	445,300	4,000	4,796.50	83.4	4,804.30	83.3
8	357,750	3,000	3,886.60	77.2	3,847.00	78.0
9	222,220	10,000	2,825.40	353.9	2,693.10	371.3
10	136,080		2,058.40		1,939.50	
6–10	1,770,420	19,000	19,673.00	96.6	19,455.00	97.7
11	101,510	2,000	1,386.10	144.3	1,335.10	149.8
12	88,430		918.50		912.50	
13	40,890		492.30		473.50	
14	31,200		471.20		448.70	
15	24,800		306.20		284.30	
11–15	286,830	2,000	3,574.30	56.0	3,454.10	57.9
16	11,700		254.00		231.90	
Total . .	$9,023,110	$69,500	$87,578.10	79.4	$89,864.60	77.3

The duration from the date of issue of the original policy to that of extension was taken as two years, and in this group were included

all those cases where the policy was extended, revived and again extended within two years.

Table I shows the result of the investigation summarized according to the years of extension, that is the number of years elapsed between the date of issue of the original policy and the date of the extension.

It will be observed that the experience in the first group covering the business extended in the years two to five is very favorable, the percentage of actual to expected losses being 75.7 per cent. by the $O^{M(5)}$, and 72.7 per cent by the American Experience Table. The experience in the second year, which is 101.5 per cent. of the expected under the $O^{M(5)}$ Table and 96 per cent. under the American, would be more favorable if the revived policies which were extended for a second time were excluded.

For the second period, viz., the sixth to the tenth years, the percentage of actual to expected deaths is 96.6 per cent. by the $O^{M(5)}$ Table and 97.7 per cent. by the American.

The experience of the eleventh to the sixteenth years is too meagre to furnish any reliable information.

The total amount exposed to risk was $9,023,110; the actual losses $69,500, the expected loss being according to the $O^{M(5)}$ Table $87,578.10, and according to the American Experience $89,864.60; the ratios of actual to expected being 79.4 per cent. and 77.3 per cent., respectively.

The company's experience upon its total business is less than two thirds of the expected.

With a view of revealing, if possible, more clearly the effects of selection, Table Ia was deduced from Table I by summing the experience under policies which had been in force at date of extension two years and over, three years and over, etc.

The results show the most favorable experience under policies extended in the earlier years of their duration, the ratio of actual to expected losses in the case of policies extended after being in force over two years, according to the American Experience, being 77.3 per cent., and increasing with some irregularity in the succeeding years to 112 per cent. in the seventh year, 123.3 per cent. in the eighth year, and 144.3 per cent. in the ninth year.

As Mr. Welch states, it is but natural to expect a rate of mortality relatively higher under policies which lapse in the later years of duration and are further away from medical selection. The reasons usually assigned as operating in favor of the lapsing of

TABLE Ia.

SUMMARY OF THE MORTALITY IN THE EXTENDED INSURANCE FUND BY THE NUMBER OF YEARS IN FORCE OF THE ORIGINAL POLICY.

Extended after being in Force over	Amount Exposed.	Actual Loss.	Expected Loss. $O^{M(5)}$	Per cent. Actual to Expected.	Expected Loss. Am. Exp.	Per cent. Actual to Expected.
2	$9,023,110	$69,500	$87,578.10	79.4	$89,864.60	77.3
3	7,572,490	56,500	74,770.00	75.6	76,328.80	74.0
4	4,791,320	40,000	49,547.70	80.7	49,981.90	80.0
5	3,098,390	26,000	33,110.90	78.5	33,127.20	78.5
6	2,068,950	21,000	23,501.30	89.4	23,141.00	90.7
7	1,459,880	19,000	17,395.20	109.2	16,969.90	112.0
8	1,014,580	15,000	12,598.70	119.1	12,165.60	123.3
9	656,830	12,000	8,712.10	137.7	8,318.60	144.3
10	434,610	2,000	5,886.70	34.0	5,625.50	35.6
11	298,530	2,000	3,828.30	52.2	3,686.00	54.3
12	197,020		2,442.20		2,350.90	
13	108,590		1,523.70		1,438.40	
14	67,700		1,031.40		964.90	
15	36,500		560.20		516.20	
16 Years	11,700		254.00		231.90	

a policy and taking advantage of the term extension are: (1) knowledge on the part of the policyholder that he has not long to live and will be protected under the extended insurance; (2) financial inability to pay the premiums, and (3) carelessness. Carelessness will no doubt operate to a much greater extent under policies of the shorter durations. In fact, under policies which have been some years in force, this cause of lapse will be practically eliminated, leaving the first two causes only operative, with the result that a relatively larger proportion of the policies which lapse in the later years will doubtless be due to the first cause, viz., knowledge on the part of the policyholder of some impairment and that he has adequate protection under the extended insurance provision.

There is still another feature which may play some part in the matter of selection. Under the older policies the period of extension is usually relatively much greater and would, therefore, offer a greater measure of protection to persons in impaired health and consequently a greater inducement to take advantage of this provision than under the shorter term insurance granted in the early years of the policy.

Table II shows the mortality experience according to years of exposure. It will be noted that practically all the claims occur within the first five years and slightly more than 90 per cent. within

TABLE II.

SUMMARY OF THE MORTALITY IN THE EXTENDED INSURANCE FUND BY YEARS OF EXPOSURE.

Year of Exposure.	Amount Exposed.	Actual Loss.	Expected Loss. $O^{M(5)}$	Per Cent. Actual to Expected.	Expected Loss. Am. Exp.	Per Cent. Actual to Expected.
1	$2,781,570	$33,000	$26,022.20	126.8	$27,085.20	121.8
2	2,032,110	16,000	19,331.40	82.8	19,980.90	80.1
3	1,401,040	14,000	13,621.70	102.8	13,995.30	100.0
4	1,002,900		9,694.10		9,940.90	
5	687,890	5,500	6,849.70	80.3	6,952.40	79.1
1–5	7,905,510	68,500	75,519.10	90.7	77,954.70	87.9
6	395,170		4,137.60		4,139.20	
7	251,380		2,611.40		2,602.80	
8	191,350		2,091.90		2,054.60	
9	124,270		1,401.60		1,365.40	
10	77,260		925.90		887.20	
6–10	1,039,430		11,168.40		11,049.20	
11	42,770	1,000	480.00	208.3	464.60	215.2
12	24,210		307.10		290.10	
13	10,230		95.70		97.50	
14	960		7.80		8.50	
15						
11–15	78,170	1,000	890.60	112.3	860.70	116.2
Total ..	$9,023,110	$69,500	$87,578.10	79.4	$89,864.60	77.3

the first three years. There were thirteen deaths in the first year of exposure, and of these careful examination showed that seven probably knew that they had but a short time to live. The causes of death of the thirteen were as follows:

Typhoid fever	2	0 knew.
Heart disease	2	2 "
Brain disease	2	2 "
Kidney disease	2	2 "
Paralysis	1	1 "
Quinsey	1	0 "
Peritonitis	1	0 "
Pneumonia	1	0 "
Suicide	1	0 "
	13	7 "

The high death rate in the first year of extension, which corresponds precisely with the experience of the Phœnix Mutual and the Mutual Benefit, is no doubt due in a large measure to the cause assigned by Mr. Welch, viz., that credit has not been given for all the exposures to which this experience would rightly be entitled on

account of the lack of promptness in payment of premiums, etc. Nevertheless we are obliged to conclude with him—"That the policyholder does, both consciously and unconsciously, exercise the privilege of extension to the loss of the company."

TABLE III.

SUMMARY OF THE MORTALITY IN THE EXTENDED INSURANCE FUND BY ORIGINAL AGE OF ISSUE.

Original Age of Issue.	Amount Exposed.	Actual Loss.	Expected Loss $O^{M(5)}$	Per cent. Actual to Expected.	Expected Loss. Am. Exp.	Per cent. Actual to Expected.
Under 25	$2,039,940	$15,000	$14,333.80	104.6	$16,624.50	90.2
25–34	4,038,640	25,000	33,126.20	75.5	35,767.30	69.9
35–44	2,236,390	24,000	25,766.50	93.1	24,492.80	98.0
45–54	640,190	3,500	11,904.50	29.4	10,675.50	32.8
Over 55	67,950	2,000	2,447.10	81.7	2,304.50	86.8
Total ..	$9,023,110	$69,500	$87,578.10	79.4	$89,864.60	77.3

In this table the results are summarized according to the age of the insured at the date of issue of the policy. It does not, like the experience shown in Tables I and II, follow that of the Phœnix Mutual. This may, however, be due to the limited nature of the data under observation.

While no comparison can properly be made with the broader experiences of the Mutual Benefit and the Phœnix Mutual, it is, however, significant that the experience conforms in certain important respects with the broader experiences of the Phœnix Mutual and the Mutual Benefit, viz.: (1) in the higher mortality experienced in the early years of exposure, and (2) also under policies extended in the later years of their duration, and (3) that the rate of mortality experienced is substantially higher than under the company's experience on its total business, thus giving evidence of the operation of the same natural and psychological laws.

While I concur generally in the conclusions arrived at by Mr. Rhodes and Mr. Welch, I am rather of the opinion that the retention of the provision providing for the deduction of any overdue premium or other indebtedness in the event of death occurring within two or three years from the date the extended insurance is entered upon, is advisable. Under any circumstances it cannot operate otherwise than to the advantage of the company, which means the continuing policyholders whose interest should be regarded as paramount.

The effect on Select Tables of a Variation in the Rates of Mortality to which the Lives Involved are Subject.

BY

PERCY C. H. PAPPS.

In constructing a mortality table based upon the experience of a life insurance company extending over a number of years, it is interesting to study the effect on the resulting select mortality tables of a variation in the mortality experience of the entire body of lives, due, for example, to an improvement in the general health of the communities in which they live.

Now, let $d^{t/n}$ represent the deaths in the policy year beginning in the nth calendar year of the investigation arising out of the entrants in the tth year; $l^{t/n}$ the numbers living at the policy anniversaries in the nth year out of the entrants in the tth year; and

$$q^{t/n} = \frac{d^{t/n}}{l^{t/n}}.$$

Let q represent the probability of dying according to some standard table, and Q the probability of dying according to the table based upon the data under investigation.

Then,

$$Q_{[x]} = \frac{d^{1/1} + d^{2/2} + d^{3/3} + \cdots + d^{m/m}}{l^{1/1} + l^{2/2} + l^{3/3} + \cdots + l^{m/m}} \cdot [x],$$

where the suffix is understood to be affixed to each symbol, and where the investigation covers the experience and issues of m years.

Again,

$$Q_{[x]+1} = \frac{d^{1/2} + d^{2/3} + \cdots + d^{m-1/m}}{l^{1/2} + l^{2/3} + \cdots + l^{m-1/m}} ([x] + 1)$$

and

$$Q_{[x]+n} = \frac{d^{1/n+1} + d^{2/n+2} + \cdots + d^{m-n/m}}{l^{1/n+1} + l^{2/n+2} + \cdots + l^{m-n/m}} ([x] + n)$$

$$= \frac{l^{1/n+1} \times q^{1/n+1} + l^{2/n+2} \times q^{2/n+2} + \cdots + l^{m-n/m} \times q^{m-n/m}}{l^{1/n+1} + l^{2/n+2} + \cdots + l^{m-n/m}} \cdot ([x] + n).$$

If it be assumed that an equal amount of insurance, number of policies, or number of lives be placed on the books each year, and that the rate of cessation does not vary, then $l^{1/n+1} = l^{2/n+2} =$ etc.— and,

$$Q_{[x]+n} = \frac{1}{m-n}(q^{1/n+1} + q^{2/n+2} + \cdots + q^{m-n/m})_{([x]+n)}.$$

Now, if q represents the mortality in the first year of the investigation, rq that in the second, r^2q that in the third, etc., then

$$Q_{[x]+n} = \frac{1}{m-n}(r^n q + r^{n+1}q + \cdots + r^{m-1}q)_{([x]+n)}$$

$$= \frac{r^n(1-r^{m-n})}{(m-n)(1-r)} q_{[x]+n}.$$

If the experience covered 20 years, that is, if $m = 20$, then

$$Q_{[x]+n} = \frac{r^n(1-r^{20-n})}{(20-n)(1-r)} q_{[x]+n}.$$

Now, the effects of selection last so long as $Q_{[x]+n}$ is less than $Q_{[x-1]+n+1}$; or so long as $Q_{[x]+n}/Q_{[x-1]+n+1}$, which may be supposed equal to K_n, is less than unity.

$$\frac{Q_{[x]+n}}{Q_{[x-1]+n+1}} = \frac{M_n}{M_{n+1}} \times \frac{q_{[x]+n}}{q_{[x-1]+n+1}} \quad \text{where} \quad M_n = \frac{r^n(1-r^{20-n})}{(20-n)(1-r)}.$$

Now, K may be considered as the measure of the effects of selection, and the smaller the value of K the greater will be the effects of selection. If k_n be taken as equal to $q_{[x]+n}/q_{[x-1]+n+1}$ then

$$K_n = \frac{M_n}{M_{n+1}} \times k_n$$

and the effects of selection in the final table are greater than, equal to, or less than in the standard table, according as M_n/M_{n+1} is less than, equal to, or greater than unity.

The values of M_n or

$$\frac{r^n}{20-n} \cdot \frac{1-r^{(20-n)}}{1-r},$$

when r equals .99, .98 and .97, are as follows:

VALUES OF M_n.

n	$r=.99$	$r=.98$	$r=.97$
0	.910,465	.830,977	.760,345
1	.905,751	.822,083	.747,730
2	.901,071	.813,309	.735,383
3	.896,425	.804,656	.723,293
4	.891,805	.796,125	.711,456
5	.887,217	.787,707	.699,868
6	.882,664	.779,406	.688,521
7	.878,141	.771,220	.677,408
8	.873,647	.763,145	.666,526
9	.869,178	.755,179	.655,870
10	.864,752	.747,324	.645,433
11	.860,351	.739,574	.635,211
12	.855,971	.731,930	.625,201
13	.851,634	.724,387	.615,396
14	.847,317	.716,947	.605,787
15	.843,030	.709,611	.596,378
16	.838,771	.702,374	.587,164
17	.834,542	.695,231	.578,129
18	.830,341	.688,185	.569,281
19	.826,169	.681,233	.560,612

The question as to whether the effects of selection in the final table are greater than, equal to, or less than the effects in the standard table, may be ascertained, as above stated, by computing the ratios M_n/M_{n+1}. It will be interesting, however, to see some numerical examples.

As a standard table the O[M] Select Tables may be taken. These are well-graduated tables and show a period of selection running for 10 years. The portion of the table for ages at entry 30 to 45 inclusive will be sufficient for the purpose of illustration.

This portion of the O[M] Select Tables is given in Table A.

Now, it is assumed that this table represents the rates of mortality prevailing in the first year of the investigation. For example, the rate of mortality amongst lives aged 35 who entered in the first year is assumed to be .00361; while the rate of mortality amongst lives aged 35 who were in their fifth policy year in the first year of the investigation would be .00698.

TABLE A.

O[M].

Age at Entry $[x]$.	Years Elapsed Since Date of Entry.											Age Attained $x+10$.
	0	1	2	3	4	5	6	7	8	9	10 or More.	
	$q[x]$	$q[x]+1$	$q[x]+2$	$q[x]+3$	$q[x]+4$	$q[x]+5$	$q[x]+6$	$q[x]+7$	$q[x]+8$	$q[x]+9$	$q[x]+10$	
30	.00312	.00502	.00580	.00633	.00680	.00728	.00778	.00829	.00883	.00940	.00986	40
1	.00321	.00512	.00592	.00647	.00698	.00748	.00801	.00855	.00913	.00973	.01022	1
2	.00329	.00523	.00606	.00664	.00717	.00770	.00826	.00884	.00945	.01009	.01062	2
3	.00339	.00536	.00621	.00681	.00737	.00794	.00853	.00915	.00980	.01048	.01105	3
4	.00350	.00549	.00637	.00701	.00760	.00820	.00883	.00949	.01018	.01091	.01153	4
35	.00361	.00564	.00655	.00722	.00785	.00849	.00916	.00986	.01060	.01138	.01205	45
6	.00374	.00580	.00675	.00745	.00812	.00880	.00952	.01027	.01106	.01189	.01261	6
7	.00388	.00597	.00696	.00770	.00842	.00915	.00991	.01071	.01156	.01245	.01323	7
8	.00403	.00616	.00719	.00798	.00874	.00952	.01034	.01120	.01210	.01306	.01391	8
9	.00420	.00637	.00745	.00829	.00910	.00993	.01081	.01173	.01270	.01373	.01465	9
40	.00438	.00660	.00773	.00862	.00948	.01038	.01132	.01231	.01336	.01447	.01546	50
1	.00458	.00685	.00803	.00898	.00991	.01087	.01188	.01294	.01407	.01527	.01635	1
2	.00480	.00713	.00836	.00938	.01037	.01140	.01249	.01364	.01485	.01614	.01731	2
3	.00504	.00742	.00873	.00981	.01088	.01199	.01316	.01440	.01571	.01710	.01837	3
4	.00530	.00775	.00913	.01028	.01143	.01263	.01389	.01523	.01664	.01815	.01953	4
45	.00558	.00811	.00956	.01080	.01204	.01333	.01469	.01613	.01766	.01929	.02079	55

TABLE A[1].

$r = .99$.

Age at Entry $[x]$.	Years Elapsed Since Date of Entry.											Age Attained $[x]+10$.
	0	1	2	3	4	5	6	7	8	9	10	
	$Q_{[x]}$	$Q_{[x]+1}$	$Q_{[x]+2}$	$Q_{[x]+3}$	$Q_{[x]+4}$	$Q_{[x]+5}$	$Q_{[x]+6}$	$Q_{[x]+7}$	$Q_{[x]+8}$	$Q_{[x]+9}$	$Q_{[x]+10}$	
30	.00284	.00455	.00523	.00567	.00606	.00646	.00687	.00728	.00771	.00817	.00853	40
1	.00292	.00464	.00533	.00580	.00622	.00664	.00707	.00751	.00798	.00846	.00884	1
2	.00300	.00474	.00546	.00595	.00639	.00683	.00729	.00776	.00826	.00877	.00918	2
3	.00309	.00485	.00560	.00610	.00657	.00704	.00753	.00803	.00856	.00911	.00956	3
4	.00319	.00497	.00574	.00628	.00678	.00728	.00779	.00833	.00889	.00948	.00997	4
35	.00329	.00511	.00590	.00647	.00700	.00753	.00809	.00866	.00926	.00989	.01042	45
6	.00341	.00525	.00608	.00668	.00724	.00781	.00840	.00902	.00966	.01033	.01090	6
7	.00353	.00541	.00627	.00690	.00751	.00812	.00875	.00940	.01010	.01082	.01144	7
8	.00367	.00558	.00648	.00715	.00779	.00845	.00913	.00984	.01057	.01135	.01203	8
9	.00382	.00577	.00671	.00743	.00812	.00881	.00954	.01030	.01110	.01193	.01267	9
40	.00399	.00598	.00697	.00773	.00845	.00921	.00999	.01081	.01167	.01258	.01337	50
1	.00417	.00620	.00724	.00805	.00884	.00964	.01049	.01136	.01229	.01327	.01414	1
2	.00437	.00646	.00753	.00841	.00925	.01011	.01102	.01198	.01297	.01403	.01497	2
3	.00459	.00672	.00787	.00879	.00970	.01064	.01162	.01265	.01372	.01486	.01589	3
4	.00483	.00702	.00823	.00922	.01019	.01121	.01226	.01337	.01454	.01578	.01689	4
45	.00508	.00735	.00861	.00968	.01074	.01183	.01297	.01416	.01543	.01677	.01798	55

TABLE A².

$r = .98$.

Age at Entry $[x]$.	Years Elapsed Since Date of Entry.											Age Attained $[x] + 10$.
	0	1	2	3	4	5	6	7	8	9	10	
	$Q_{[x]}$	$Q_{[x]+1}$	$Q_{[x]+2}$	$Q_{[x]+3}$	$Q_{[x]+4}$	$Q_{[x]+5}$	$Q_{[x]+6}$	$Q_{[x]+7}$	$Q_{[x]+8}$	$Q_{[x]+9}$	$Q_{[x]+10}$	
30	.00259	.00413	.00472	.00509	.00541	.00573	.00606	.00639	.00674	.00710	.00737	40
1	.00267	.00421	.00481	.00521	.00556	.00589	.00624	.00659	.00697	.00735	.00764	1
2	.00273	.00430	.00493	.00534	.00571	.00607	.00644	.00682	.00721	.00762	.00794	2
3	.00282	.00441	.00505	.00548	.00587	.00625	.00665	.00706	.00748	.00791	.00826	3
4	.00291	.00451	.00518	.00564	.00605	.00646	.00688	.00732	.00777	.00824	.00862	4
35	.00300	.00464	.00533	.00581	.00625	.00669	.00714	.00760	.00809	.00859	.00901	45
6	.00311	.00477	.00549	.00599	.00646	.00693	.00742	.00792	.00844	.00898	.00942	6
7	.00322	.00491	.00566	.00620	.00670	.00721	.00772	.00826	.00882	.00940	.00989	7
8	.00335	.00506	.00585	.00642	.00696	.00750	.00806	.00864	.00923	.00986	.01040	8
9	.00349	.00524	.00606	.00667	.00724	.00782	.00843	.00905	.00969	.01037	.01095	9
40	.00364	.00543	.00629	.00694	.00755	.00818	.00882	.00949	.01020	.01093	.01155	50
1	.00381	.00563	.00653	.00723	.00789	.00856	.00926	.00998	.01074	.01153	.01222	1
2	.00399	.00586	.00680	.00755	.00826	.00898	.00973	.01052	.01133	.01219	.01294	2
3	.00419	.00610	.00710	.00789	.00866	.00944	.01026	.01111	.01199	.01291	.01373	3
4	.00440	.00637	.00743	.00827	.00910	.00995	.01083	.01175	.01270	.01371	.01460	4
45	.00464	.00667	.00778	.00869	.00959	.01050	.01145	.01244	.01348	.01457	.01554	55

TABLE A[8].

$r = .97$.

Age at Entry [x].	Years Elapsed Since Date of Entry.											Age Attained [x]+10.
	0	1	2	3	4	5	6	7	8	9	10	
	$Q_{[x]}$	$Q_{[x]+1}$	$Q_{[x]+2}$	$Q_{[x[+3}$	$Q_{[x]+4}$	$Q_{[x]+5}$	$Q_{[x]+6}$	$Q_{[x]+7}$	$Q_{[x]+8}$	$Q_{[x]+9}$	$Q_{[x]+10}$	
30	.00237	.00375	.00427	.00458	.00484	.00510	.00536	.00562	.00589	.00617	.00636	40
1	.00244	.00383	.00435	.00468	.00497	.00524	.00552	.00579	.00609	.00638	.00660	1
2	.00250	.00391	.00446	.00480	.00510	.00539	.00569	.00599	.00630	.00662	.00685	2
3	.00258	.00401	.00457	.00493	.00524	.00556	.00587	.00620	.00653	.00687	.00713	3
4	.00266	.00411	.00468	.00507	.00541	.00574	.00608	.00643	.00679	.00716	.00744	4
35	.00274	.00422	.00482	.00522	.00558	.00594	.00631	.00668	.00707	.00746	.00778	45
6	.00284	.00434	.00496	.00539	.00578	.00616	.00655	.00696	.00737	.00780	.00814	6
7	.00295	.00446	.00512	.00557	.00599	.00640	.00682	.00726	.00771	.00817	.00854	7
8	.00306	.00461	.00529	.00577	.00622	.00666	.00712	.00759	.00806	.00857	.00898	8
9	.00319	.00476	.00548	.00600	.00647	.00695	.00744	.00795	.00846	.00901	.00946	9
40	.00333	.00494	.00568	.00623	.00674	.00726	.00779	.00834	.00890	.00949	.00998	50
1	.00348	.00512	.00591	.00650	.00705	.00761	.00818	.00877	.00938	.01002	.01055	1
2	.00365	.00533	.00615	.00678	.00738	.00798	.00860	.00924	.00990	.01059	.01117	2
3	.00383	.00555	.00642	.00710	.00774	.00839	.00906	.00975	.01047	.01122	.01186	3
4	.00403	.00579	.00671	.00744	.00813	.00884	.00956	.01032	.01109	.01190	.01261	4
45	.00424	.00606	.00703	.00781	.00857	.00933	.01011	.01093	.01177	.01265	.01342	55

The final select mortality tables, based upon the supposition that the rates of mortality prevailing in the nth year of the investigation are r times the rates prevailing in the $(n-1)$th year of the investigation when $r = .99$, $.98$ and $.97$, are shown in tables A^1, A^2 and A^3 respectively. These three tables give the same results as would be arrived at by assuming certain amounts at risk subject to certain rates of mortality in each year of the investigation, and, after working out the expected deaths for each year, finally comparing the total expected deaths with the total amounts at risk for each age at issue and policy year, irrespective of the year of issue.

An examination of the above tables shows that the effect of a decreasing mortality generally prevailing amongst the lives whose history enters into a mortality investigation is to show smaller effects of selection than really exist. In table A^3 the rates in the 11th year are smaller than in the 10th, so that, whereas the effects of selection last for 10 years before reaching the ultimate in each of the calendar years experience entering into the final table, the effects of selection shown in the final table itself last for only 9 years before reaching what appears to be a maximum, if not an ultimate, rate.

It has been assumed that an equal amount of insurance has been issued each year. If the mortality is not varying, an increase in the amount of insurance issued each year does not affect the resulting select tables. For

$$Q_{[x]+n} = \frac{l^{1/n+1} \times q^{1/n+1} + l^{2/n+2} \times q^{2/n+2} + \cdots + l^{m-n/m} \times q^{m-n/m}}{l^{1/n+1} + l^{2/n+2} + \cdots + l^{m-n/m}} \cdot ([x]+n)$$

and if

$$l^{2/n+2} = s l^{1/n+1}, \quad l^{3/n+3} = s^2 l^{1/n+1}, \text{ etc.},$$

then

$$Q_{[x]+n} = \frac{l^{1/n+1} \times q^{1/n+1} + s l^{1/n+1} \times q^{2/n+2} + \cdots + s^{m-n-1} l^{1/n+1} \times q^{m-n/m}}{l^{1/n+1} + s l^{1/n+1} + \cdots + s^{m-n-1} l^{1/n+1}} \cdot ([x]+n),$$

which is equal to $q_{[x]+n}$ if $q^{1/n+1}_{[x]+n} = q^{2/n+2}_{[x]+n} = \text{etc.} = q^{m-n/m}_{[x]+n} = q_{[x]+n}$.

But if $q^{1/n+1}_{[x]+n} = r^n q_{[x]+n}$, $q^{2/n+2}_{[x]+n} = r^{n+1} q_{[x]+n}$, etc., as before, then,

$$Q_{[x]+n} = \frac{r^n + sr^{n+1} + \cdots + s^{m-n-1}r^{m-1}}{1 + s + \cdots + s^{m-n-1}} q_{[x]+n}.$$

When $m = 20$

$$Q_{[x]+n} = \frac{s-1}{rs-1} \cdot \frac{r^n(r^{20-n}s^{20-n} - 1)}{s^{20-n} - 1} q_{[x]+n}.$$

In this case

$$M_n = \frac{s-1}{rs-1} \cdot \frac{r^n(r^{20-n}s^{20-n} - 1)}{s^{20-n} - 1},$$

and the values are as follows:

VALUES OF M_n.

$r = .97$, $s = 1.1$.

n	Multiplier.	n	Multiplier.	n	Multiplier.	n	Multiplier.
0	.671,968	5	.643,777	10	.611,389	15	.575,146
1	.666,671	6	.637,628	11	.604,434	16	.567,485
2	.661,204	7	.631,313	12	.597,325	17	.559,702
3	.655,565	8	.624,833	13	.590,071	18	.551,802
4	.649,757	9	.618,191	14	.582,677	19	.543,793

It seems unnecessary to give any numerical example of the table which these multipliers would produce, for a comparison of the values of the ratio M_n/M_{n+1} derived from the above values of M_n with the values of the ratios derived from the multipliers used to compute tables A^1, A^2 and A^3 will show the effects of selection as compared with that illustrated in tables A^1, A^2 and A^3. The values of M_n/M_{n+1} are given in the table on the following page.

Since the values of M_n/M_{n+1} are in every case greater than unity, it follows that a decrease in the rates of mortality from year to year, there being no variation in the amounts of insurance at risk, produces a final table showing lower effects of selection than were assumed for each year of the investigation. For it may be mentioned that since the rates of mortality assumed for any year of the investigation bear a constant ratio to the standard table, irrespective of age at issue or duration, the effects of selection assumed are for each year of the investigation identical with the standard table.

VALUES OF M_n/M_{n+1}.

n	$r = .99$, $s = 1.00$.	$r = .98$, $s = 1.00$.	$r = .97$, $s = 1.00$.	$r = .97$, $s = 1.10$.
0	1.005,205	1.010,819	1.016,871	1.007,945
1	1.005,194	1.010,788	1.016,790	1.008,268
2	1.005,183	1.010,754	1.016,715	1.008,602
3	1.005,181	1.010,716	1.016,638	1.008,939
4	1.005,171	1.010,687	1.016,557	1.009,289
5	1.005,158	1.010,650	1.016,480	1.009,644
6	1.005,151	1.010,614	1.016,405	1.010,003
7	1.005,144	1.010,581	1.016,326	1.010,371
8	1.005,142	1.010,548	1.016,247	1.010,744
9	1.005,118	1.010,511	1.016,171	1.011,125
10	1.005,115	1.010,479	1.016,092	1.011,507
11	1.005,117	1.010,444	1.016,011	1.011,901
12	1.005,093	1.010,413	1.015,933	1.012,293
13	1.005,095	1.010,377	1.015,862	1.012,690
14	1.005,085	1.010,338	1.015,777	1.013,094
15	1.005,078	1.010,304	1.015,692	1.013,500
16	1.005,067	1.010,274	1.015,628	1.013,906
17	1.005,059	1.010,239	1.015,542	1.014,317
18	1.005,050	1.010,205	1.015,463	1.014,728

The effect of a constant increase in the amount of insurance at risk in each year of the investigation is to tend to offset, to some extent, the effect of a decreasing mortality. A comparison of the two right-hand columns of the above table also shows that the effect of the increasing amount of insurance seems to be most powerful at the shorter durations.

A Suggestion for the use of Statistics based upon European Experience with Workmen's Compensation in arriving at Premium Rates for Insurance Covering this Risk in the United States.

BY

A. H. MOWBRAY.

Probably the most potent cause of the difficulty, misunderstanding and distrust attending the change now in progress in this country from a basis of employers' liability in tort for negligence to one of workmen's compensation for providing for the victims of industrial accidents is the lack of a sure statistical basis for determining rates for insurance and the inability or fear to use the experience of European countries.

We have assumed that European experience cannot be safely used, for we are among the most backward of nations in safeguarding our workers and caring for them when injured. Yet the condition is being rapidly changed as our people are becoming awakened to the need. What can be done is shown by the work of several of our large corporations. To cite but one, the Remington Typewriter Works reports (John Calder, manager, in "Human Engineering," edited and published quarterly by Winthrop Talbot, M.D., Vol. II, No. 1, April, 1912, p. 55):

"During the five year period ending December 31, 1911, the number of employees *increased* 66 per cent., the number of machines *increased* 25 per cent. and accidents of all kinds per 1,000 employees *decreased* 46 per cent." "During the five years no fatal accident occurred and no limbs or eyes were lost."

How rapid our progress will be depends in large measure upon what solution we adopt in providing for insurance of compensation, and to what extent safety equipment and methods are recognized in fixing insurance rates. Though our annual fire loss is still far larger than it should be, yet such progress in fire prevention as has been made is in no small degree due to the recognition of efforts in that behalf in fire insurance rates.

In many, if not most, lines of human endeavor progress is most rapid and substantial where the ideal is kept clearly in view, and each step toward it is recognized and suitably rewarded. We have the ideal in Germany's splendid development of accident prevention. "Model Regulations for the Prevention of Accidents," as adopted by the German Accident Insurance Associations, are given in detail in the Twenty-Fourth Report of the United States Commissioner of Labor (pages 1042–52), and the opportunity to thus refer to them is valuable, but to keep the ideal before our people and stimulate the effort to reach it we must do much more. Messrs. Schwedtmann and Emery say ("Accident Prevention and Relief," published for the National Assn. of Manufacturers of the U. S. A., 1911, p. 126):

"Undoubtedly many prevention appliances in use in Germany might well be adopted here, but the real and important difference is not in prevention apparatus—it is in the *prevention spirit.* In Europe and especially in Germany, accident prevention is kept constantly before the public, before the legislatures, before the employers, and before the workers. . . . Insurance rates are gauged according to the state of accident prevention practiced in individual shops" and all "combine and concentrate their efforts upon an educational and practical accident prevention campaign."

It will be in accord with the principles of progress above noted if the basis or minimum rate is that for an ideal shop, and suitable additions, corresponding to the extent to which a given risk may fall short of the standard or ideal, are provided in a manual or schedule for use in determining rates on actual risks offered. This is somewhat analogous to the schedule rating system used in fire insurance. It is probably even more closely analogous to the system Mr. Hunter has developed for handling substandard lives in life insurance, since Mr. Hunter's basis of reference is an acceptable standard risk though not the ideal risk as here proposed.

The publication of rates upon the basis of an ideal conforming to the German "Model Regulations" will keep before the public that ideal, and a comparison, easily made, of the premium charged for it with that actually charged for a given establishment will lay before the proprietor a measure of his reward for attaining it. The possibility of using the accumulated experience of Germany as the foundation for the rate on the ideal standard will permit the determination of such basis or minimum rate under the compensation

scale in use in a given state upon a sure foundation. The knowledge of this, if the charges for departures can be shown to be as well founded, will strongly tend to inspire a confidence in such rates on the part of the public, supervising officials and other state officers similar to that with which life insurance computations are now regarded. That there is not at present any such confidence in the soundness of the foundation for present rates few will deny.

As in life insurance, the fact of physical and moral conformity to the ideal and the measure of departure therefrom must be determined as the result of an actual examination and, limited as well as may be by appropriate rules, must be a matter of personal judgment of the examiner or reviewer.

Even though European experience does furnish a sound statistical basis for determining the net premium for the standard or ideal risk, the assistance it can thus render in the practical problem of rating actual risks is dependent upon finding an equally sound basis for measuring the effect upon the cost of compensation of the deficiencies found upon examination. Restated in these terms, the problem seems at first sight no less, perhaps even more, complex; and the search for usable European statistics as hopeless as ever; but there are statistics, hitherto considered of little value to our profession though of great interest and importance to the safety engineer, which may serve a useful purpose in this regard,—statistics of accident causation.

A given agency as an active cause of accidental injury may affect the cost of compensation through (1) the frequency of accidents caused thereby, (2) the seriousness of results of such accidents, (3) the wage group most subject to such accidents. The importance of a given cause, for example "falls from ladders," in each of these regards varies with the character of the industrial process carried on. Theoretically, differentiation in this regard should be carried to the point of recognizing minute differences which may affect the cost. Practically, it may be necessary to use fairly large groupings in order to get a fair average free from accidental variation.

Recognition in observed or reported conditions of the presence in a given measure of particular causes, is a problem for the safety engineer similar in nature to that imposed upon the medical director passing upon substandard lives, and the assistance to be drawn from statistics is of the same kind.

The proposed solution of the problem herein suggested is as fol-

lows. First fix upon a standard or ideal type of risk in each industry class and determine the rate thereon. Next fix for each cause of accidental injury the measure of change in premium rate per unit change in destructive power of that cause as shown by the frequency of accident and seriousness of result. Both these steps are purely actuarial. Finally determine the extent of change in intensity of force with which each cause acts corresponding to a given change in observed conditions: this as already noted is primarily a problem for the safety engineer, perhaps even in some cases by experimentation and mechanical tests of devices and processes. So far as he may be assisted by the study of statistical data we may work with him.

How far can the statistical record of European (let us say German) experience be used in practically applying these principles?

We have as noted the "Model Regulations," setting up the ideal, and we have the experience data of accidents by industry groups, and perhaps smaller divisions—the writer has access only to the summaries in the publications of the United States Bureau of Labor and not the original German publications—with risks conforming reasonably to that ideal. It is true that these data are not at present in the form most useful for application to any American schedule, but a way will be presently shown for transforming them, which it is believed does not distort the facts. In this way the standard rate can be determined.

We also have tabulations for each industry group, and perhaps smaller units, of the percentage of accidents due to each of seventeen causes. (For list of causes see Table IV below, page 230.) Having the absolute rate for the group, this enables the absolute rate for the cause within the group to be worked out. From this we can find the change in the rate for a change in intensity of action of each or any cause producing, for example, 10 per cent. more accidents of the same seriousness.

We also have in the Austrian experience tabulations of the effect of each cause, but these are not within industry groups, and it is a patent fact that, for example, power driven working machinery is much more dangerous in some lines than others. Here there seems to be a serious difficulty as far as the statistics are summarized and presented in the Bureau of Labor publications. But an approximate measure is at hand which may do very well until a better one is available. On page 1135 of the Twenty-Fourth Report of the

United States Commissioner of Labor is given a table showing the German experience as to "Avearge Compensation paid per Accident in 1897, Classified by Industry and Cause" and on page 1160 the general average per "case of accident" for industry groups. In the original publications the data may be further subdivided. Assuming the latter figure as the unit, the relative seriousness of consequence (including physical consequence and wage group affected) of each cause to the aggregate of causes as found in the industry can be determined. The unit of reference in this case being again the standard or ideal risk of which the conditions are known, the derived index of seriousness may be taken as a standard basis, and a departure of a given percentage in the active power of the cause in this regard may be measured by a corresponding increase in this index. While no American compensation scale comes up to the high German standard, is does not necessarily follow that an error is introduced by this process, and the error, if any, would seem to be on the side of safety since the more serious accidents are compensated there with life pensions to the injured man or dependents, not the case under most of our laws or proposed acts. This would make the more serious accidents have even more weight. Further search and study of the original German publications would perhaps reveal statistics of the kind needed so that this approximation need not be resorted to. It seems probably safe then to say that the necessary data for the actuarial side of the problem is available.

The publication of such data as is contained in Table 12 (page 64) of Bulletin No. 92 of the U. S. Bureau of Labor and in the table on page 182 et seq. of the Twenty-Fourth Report, summarized by Mr. Watt and appearing in condensed form on pages 68 and 69 of Volume XIII of the *Transactions,* indicates that there is probably sufficient data for use of the safety engineer in his part of the problem, especially if this data is supplemented by experimental study of machinery and processes.

To illustrate and test the practical working of these theories the problem has been undertaken of computing the standard rates and rate departures for the woodworking industry—groups should be subdivided in practice when statistics in greater detail are available —on the basis of the German experience summarized in Bulletin No. 92 of the United States Bureau of Labor (pages 1–96) and the scale of compensation provided in the Illinois Act (see Appendix),

assuming that act allows compensation at the same rate for temporary partial disability as for permanent partial disability, a point as to which the writer has some doubts.

The table on page 8 of this Bulletin shows that for this industry there were, in 1907, 51.80 notices of accident per 1,000 full time workers and 13.28 cases resulting in disability of at least thirteen weeks' duration or in death.

The table on page 92 shows that 29.77 per cent. of cases of the former type ceased to receive compensation on account of recovery within one year from the granting of a pension. The cases of 1907 are not further followed in this publication, but reading back to injuries of preceding years the following percentages are given:

39.56 per cent. recovered within two years from injuries received in 1906; 44.01 per cent. within three years from injuries received in 1905; 45.68 per cent. within four years from injuries received in 1904.

From these the following rates of recovery may be deduced, 1st year after pension granted, 29.77 per cent., 2nd, 9.79 per cent., 3d, 4.45 per cent. and 4th year, 1.67 per cent., all percentages being of the original number pensioned.

The table on page 76 gives the distribution of those injured in 1904 and still disabled after the lapse of similar periods, according to the percentage loss of earning power produced by the disability. In working with these figures and those for 1907 and other years it was assumed that the higher rates of recovery shown for the latter years are due entirely to recoveries from among the group of smallest loss of earning power. Fuller statistics will show whether this is justified.

From these data Table I, following, has been made up on the basis of 1,000 full time workers by computing, according to the

TABLE I.

After.	Unrecovered.	Disability—Loss of Earning Power.				Deaths.
		0–25 Per Cent.	25–50 Per Cent.	50–75 Per Cent.	75–100 Per Cent.	
Notices.....	51.80					0.44
13 weeks.....	13.28					
1¼ yrs.....	9.33	5.98	2.14	0.50	0.19	0.52
2¼ yrs.....	8.03	5.26	1.72	0.41	0.12	0.52
3¼ yrs.....	7.44	4.92	1.48	0.38	0.12	0.54
4¼ yrs.....	7.21	4.86	1.34	0.35	0.12	0.54

recovery rates shown above, the number of the 13.28 disabled at thirteen weeks still disabled after successive annual periods, and distributing these according to the extent of disability by means of the rates given on page 76 with the assumption noted.

In order to provide a basis for dealing with cases of temporary disability not extending into the second year, these data were exteneded by the following method. The figures in the last five columns, or rather the percentages of 13.28 quoted in the original data from which they were derived, were exterpolated to find the values corresponding to thirteen weeks, and, the result not accounting for the full 100 per cent., the values relating to disability were increased in the ratio of 96.11 to 83.57 as a reasonable approximation to the true facts, the deaths reported accounting for 3.89 per cent. The same distribution as regards seriousness of disability was assumed for the 51.36 non-fatal cases for which notice was sent in. Further values were found by finite difference interpolation. It being assumed that the rapid rate of recovery during the first ninety days does not markedly show itself during the first week, the interpolation for this value was not made strictly by a finite difference formula, but the first difference alone was used and one week was considered one-fourth of thirty days. Table II gives the extended data, the full faced figures being those derived by exterpolation and modification, and the interpolated values being indicated by italics. It is believed that the distortion, if any, introduced by these processes is slight, and at worst it only affects the loss percentage distribution of cases of comparatively short temporary disability, so that even a considerable error would not greatly affect the derived premium rates.

Since the compensation in event of death is the same, both in aggregate and instalments, whether the death occurs immediately or after a period of total disability, the lapse of time between injury and death may be left out of account, and this part of the standard rate becomes at once, $0.54 \times$ present value of 4 yrs.' wages in instalments over 8 yrs. Using 4 per cent. compound discount and assuming monthly instalments, $0.54 \times 3.693 = 1.994$ years' wages per 1,000 full time workers.

For determining the cost of disability compensation the following average loss rates have been taken for each group, 0–25, 10 per cent.; 25–50, 35 per cent.; 50–75, 65 per cent.; 75–100, 90 per cent.; and the resulting compensation rates are 5 per cent., 17½ per cent., 32½ per cent., and 45 per cent.

TABLE II.

RATES OF INJURY AND RECOVERY PER 1,000 FULL TIME WORKERS ACCORDING TO EXTENT OF INCAPACITY. WOODWORKING ACCIDENTS IN GERMANY IN 1907. BASED ON DATA IN BULLETIN NO. 92, U. S. BUREAU OF LABOR.

Time Since Accident.	Killed and Disabled.	Disabled—Loss of Earning Power.				
		Under 25 Per Cent.	25–50 Per Cent.	50–75 Per Cent.	75–100 Per Cent.	Deaths.
Notices.....	51.80	33.39	13.10	3.20	1.67	0.44
1 week.....	46.33	29.80	11.71	2.88	1.50	*0.44*
30 days.....	*29.93*	*18.96*	*7.55*	*1.93*	*1.00*	*0.49*
60 days.....	*18.40*	*11.58*	*4.58*	*1.18*	*0.56*	*0.50*
90 days.....	13.28	8.30	3.26	0.79	0.42	0.51
6 mos.....	*11.90*	*7.48*	*2.87*	*0.70*	*0.34*	*0.51*
9 mos.....	*10.79*	*6.82*	*2.56*	*0.62*	*0.28*	*0.51*
1 year.....	*9.96*	*6.34*	*2.32*	*0.55*	*0.23*	*0.52*
1¼ years.....	9.33	5.98	2.14	0.50	0.19	0.52
2 years.....	*8.28*	*5.40*	*1.81*	*0.42*	*0.13*	*0.52*
2¼ years.....	8.03	5.26	1.72	0.41	0.12	0.52
3 years.....	*7.57*	*5.00*	*1.53*	*0.39*	*0.12*	*0.53*
3¼ years.....	7.44	4.92	1.48	0.38	0.12	0.54
4 years.....	*7.26*	*4.87*	*1.37*	*0.36*	*0.12*	*0.54*
4¼ years.....	7.21	4.86	1.34	0.35	0.12	0.54

Taking the mean of the number disabled at the beginning and end of each period as the number compensated during the period (the first thirty day period is based upon the number at one week and thirty days but compensation is considered paid for the entire month, this being the apparent intention of the act) the following table was derived.

TABLE III.

Year After Injury.	Average Number Disabled One Year.				Cost in Years' Wages per 1,000 Full Time Workers.
	0–25 Per Cent.	25–50 Per Cent.	50–75 Per Cent.	75 Per Cent. and Over.	
1st.....	9.54	3.69	1.00	0.45	1.651
2d.....	5.87	2.07	0.49	0.18	0.896
3d.....	5.20	1.67	0.40	0.12	0.736
4th.....	4.94	1.45	0.37	0.12	0.675

The deferred payments will be subject to discount, but no consideration need be given the mortality element as this is already covered in the basic data. Unless due to the accident, death terminates the compensation, and hence such deaths are to be classed with the recoveries. This must be the case since the tabulation on

pages 76 and 92 each account for 100 per cent., and the deaths reported subsequent to the first year are too few to represent all the deaths.

If after four years disabilities are considered permanent, then the mortality element must be considered in discounting the deferred payments beyond this time. But, since deaths due to the results of accident are intended to be compensated as such, only normal mortality is to be considered. Eight years being the limit of time for which compensation is payable, we may add to the item for the fourth year the present value of a four year temporary annuity of equal amount—on Actuaries' 4 per cent. basis at age 30, 3.554×0.675—and then discount each of these items at 4 per cent.; thus since payments are spread over each year in instalments,

$1.651 \times .98$	1.618
$0.896 \times .94$	.842
$0.736 \times .91$	.670
$0.675 \times 4.554 \times .89$	2.737
	5.867

The Illinois Act provides further a life pension of 32 per cent. of wages for the totally and permanently disabled. The present value of an 8-year deferred life annuity on such lives is difficult to estimate. Assuming, to be safe, that it will be ten times the annual purchase, the present value in a given case would be 3.20 years' wages.

The table on page 1152 of the Twenty-Fourth Report of the United States Commissioner of Labor indicates that for several years the average rate of total permanent disability in the woodworking group has been 0.02 per 1,000 full time workers. From this the cost of this feature would be 0.064 years' wages per 1,000 full time workers.

Hence the net premium, exclusive of medical attention and disfigurement indemnity (see 5*a* and *c*), for the standard or ideal risk is made up,

For death indemnity	1.994
For disability indemnity	5.867
For additional total permanent disability indemnity	0.064
	7.925

or 0.79¼ per cent. of the annual payroll or of such modification of it as the maximum and minimum limits of compensation require.

From the published statistics of German experience the following figures were obtained by the methods above described as a basis of rate differentials according to conditions.

TABLE IV.

Cause.	Per Cent. of All Accidents.	Number per M Full Time Workers.	Coefficient of Cost.	Rating Value (2)×(4).
(1)	(2)	(3)	(4)	(5)
I. Motors, etc. (prime movers)	0.55	0.073		0.467
II. Transmission apparatus....	1.76	0.234	0.85	1.496
III. Working machinery.....	55.61	7.385		47.269
IV. Elevators, hoists, etc.....	1.21	0.161	1.21	1.464
V. Steam boilers, etc.....	0.08	0.011	1.58	0.126
VI. Electric currents.....				
VII. Explosives.....	0.02	0.003	0.77	0.015
VIII. Inflammable hot, and corrosive substances.....	0.76	0.101	1.61	1.224
IX. Collapse, fall, etc., objects..	7.25	0.963	1.30	9.425
X. Falls of workers from stairs, ladders, on floor, etc.....	6.21	0.825	1.46	9.067
XI. Loading, unloading, etc.....	11.52	1.530	1.16	13.363
XII. Teaming, drayage, etc.....	4.87	0.647	1.30	6.331
XIII. Operation of railways.....	1.44	0.191	1.16	1.670
XIV. Shipping and water transportation.....	0.09	0.012	1.62	0.146
XV. Animals, bite, kick, push, etc.....	0.68	0.090	1.30	0.884
XVI. Tools, hand apparatus, etc.	3.58	0.475	0.95	3.401
XVII. Miscellaneous.....	4.37	0.580	1.26	5.506
	100.00	13.281		101.854

The slight discrepancy between the sum of column (2) and column (5) is probably due to a decrease in seriousness of some or all kinds of accidents between 1897 and 1907. Similar data to that for 1897 can probably be obtained from Germany and the whole computation put on the same basis, or better, data obtained showing the physical consequences from which the cost can be computed on our own scales of compensation.

The method of application of this table would be as follows.

If, according to the report of the inspector including a review of the accident record in the plant, conditions are found which, upon the basis developed by the safety engineers, will probably result in twice as many falls (Cause X) of such a kind that each one would cost on the average 150 per cent. of the cost of falls in the standard risk, but otherwise the risk be fully up to standard in all respects,

then since this cause will call for three times—200 per cent. excess —the outlay in compensation that it does in the standard risk, twice its cost value should be added to the standard rate, in this case 18.13 per cent. Each cause of accident would be treated in the same way.

It may be said in criticism of the proposed use of European experience with such a system of rating, that it places too heavy burdens on the inspection department. It is difficult to see how any system of individual rating such as is sought by the National Association of Manufacturers and others interested and forms the basis of most arguments in opposition to state insurance, can be devised which will possess the needed flexibility and not place the bulk of the burden on this department. As has been pointed out, there is already a considerable mass of statistical data available for the use of this department, and the use of that in this way would lay the foundation for a large accumulation of our own material. As is always the case, the psychological element will be the most difficult to measure, and it will be a factor in the seriousness, if not the frequency, of accidents from almost all the enumerated causes, A systematic plan of study of the records of the plant and of those conducting it will certainly be helpful.

Appendix.

SUMMARY OF INDEMNITIES PROVIDED BY WORKMEN'S COMPENSATION ACT OF ILLINOIS.

Death. Sec. 4. (*a*) To widow, children or parents (if deceased has contributed within five years to their support), four times average annual earnings ($1,500–$3,500) in instalments equal to one half average wages, at intervals corresponding to wage payments.

(*b*) To collateral heirs dependent upon earnings of deceased, a percentage of (*a*) corresponding to the proportion of earnings of deceased devoted to this purpose.

(*c*) If no dependents are left, burial expenses up to $150. (*Note.*—For safety it has been assumed that all killed will leave dependents to be compensated under (*a*). It has been also assumed that (*a*) and (*b*) are not cumulative, especially in view of the provisions of another subsection (*e*) as to how payments are to be made.)

Disability. Sec. 5. (*a*) Necessary first aid, medical, surgical and hospital services and medicines. Limited to eight weeks and

$200. (*Note.*—This not covered in above computations for lack of a basis.)

(*b*) "If the period of disability lasts for more than six working days, and such fact is determined by the physician or physicians, as provided" elsewhere in the Act, one half earnings (limits $5–$12 per week) "beginning on the eighth day of disability." Maximum limit the same as the death benefit. The physical examination is "for the purpose of determining the nature, extent and probable duration of the injury."

(*c*) Non-disabling disfigurement, "a reasonable amount" not to exceed one quarter of the death indemnity.

(*d*) Permanent partial disability—"one half of the difference between the average amount . . . earned before the accident, and the average amount which he is earning, or is able to earn in some suitable employment or business after the accident, if such employment is secured."

(*e*) Permanent total disability—first eight years the same as for temporary total disability, thereafter or after total payments equal death benefit life annuity of 8 per cent. of the death benefit payable monthly, not less than $10 per month. If death occurs before the total of payments made equals the amount payable as a death benefit and the deceased leaves a widow, children, parents or other lineal heirs then the difference between the compensation paid and the death benefit is to be paid them.

Except as above for permanent total disability, no compensation shall extend over a period of more than eight years and limits are fixed as $5 per week (lower) and $12 per week (upper).

The Act defines certain injuries as totally and permanently disabling but states that the enumeration of such injuries is not to be construed as excluding others. It further provides for commutation to lump sum payments in some cases, provides rules for determining the wages to serve as the basis for computing compensation, etc.

The Mortality Experience of The Mutual Benefit Life Insurance Company on Paid-up Policies Issued in Lieu of Surrendered Policies.

BY

E. E. RHODES.

The mortality experience of the Mutual Benefit under extended insurances was presented to the Actuarial Society in October, 1908. It is now supplemented by the company's experience under paid-up policies issued in lieu of surrendered policies. The lines of the former investigation have been followed as closely as possible, and the reader is referred thereto for such details as do not appear herein.

In investigating the mortality under paid-up insurances it was thought desirable to determine not only whether there had been any selection by the insured against the company, but to ascertain how the experience thereunder compared with the experience under extended insurances. Under the company's original nonforfeiture system of 1879 provision was made for automatic extended insurance in the event of lapse, or for a paid-up policy if the original policy was surrendered within three months from date of lapse. Prior to 1879 the insured who was unable to continue the payment of premiums had no choice but to take a paid-up policy for a smaller amount, or a cash value. In investigating the mortality experience under paid-up policies the experience under all such policies was first investigated, excluding such limited premium policies as by their terms stood good for paid-up insurance for such a proportion of the original policy as the number of premiums paid bore to the full number. A subsequent investigation was then made of the experience under paid-up policies issued since 1879, which, as stated, covered only those policies which were chosen by the insured in preference to the extended insurance.

The experience under *all* paid-up policies issued by the company in lieu of policies surrendered covered the following data:

Mode of Exit.	Year "0."		Other than Year "0."	
	Policies.	Amount.	Policies.	Amount.
Surrendered	160	$143,208	3,151	$2,527,160
Existing			2,643	1,731,732
Died			3,874	3,419,361
Total decrements	160	$143,208	9,668	$7,678,253
			160	143,208
	Total data		9,828	$7,821,461

The total exposed to risk was equivalent to 142,184 policies insuring $98,126,090.00 at risk for one year. The average duration of policies which entered into the investigation was 14.71 years by policies, and 12.78 years by amounts, showing a somewhat shorter duration on the policies for large amounts. It is interesting to notice that the average duration of exposure under extended insurances was 3.65 years by policies, and 3.62 years by amounts. This difference is partially accounted for by the fact that the experience under extensions covered the years 1879–1907, while under paid-up policies the experience covered the years 1868–1910.

A feature of the data to which attention may be called is the fact that a large percentage of the policies was terminated by death. The percentage for each mode of exit is as follows:

Mode of Exit.	Policies.	Amount.	Percentage by	
			Policies.	Amounts.
Surrendered	3,311	$2,670,368	33.69	34.14
Existing	2,643	1,731,732	26.89	22.14
Died	3,874	3,419,361	39.42	43.72
	9,828	$7,821,461	100.00	100.00

The data relating to paid-up policies issued since 1879 is as follows:

Mode of Exit.	Year "0."		Other than Year "0."	
	Policies.	Amount.	Policies.	Amount.
Surrendered	78	$71,145	1,180	$1,030,969
Existing			1,550	1,266,600
Died			1,602	1,694,583
Total decrements	78	$71,145	4,332	$3,992,152
			78	71,145
	Total data		4,410	$4,063,297

It will be noticed that the paid-up policies issued since 1879 amounted to about 45 per cent. of the total paid-up policies issued, but the amount insured under policies issued since 1879 accounted for nearly 52 per cent. of the total amount insured. Prior to 1879 there were 5,418 policies for $3,758,164.00, showing an average of $694.00. Since 1879 there were 4,410 policies for $4,063,297.00, showing an average of $921.00.

Under paid-up policies issued since 1879 the total exposed to risk was equivalent to 60,554 policies, insuring $46,908,095.00 at risk for one year. The average duration of these policies which entered into the investigation was 13.98 years by policies, and 11.75 years by amounts. It will be seen that the elimination of the paid-up policies issued prior to 1879 reduced the average duration by about three fourths of a year in the case of policies, and approximately one year in the case of amounts. Although the elimination of paid-up policies issued prior to 1879 reduced the number of policies entering into the investigation to less than one half the total and to a little more than one half the amount, yet the proportion of policies terminating by death in the experience under policies issued since 1879 is still large, and only slightly smaller than for all the paid-up policies. The percentage for each mode of exit is as follows:

Mode of Exit.	Policies.	Amount.	Percentage by	
			Policies.	Amounts.
Surrendered	1,258	$1,102,114	28.53	27.12
Existing	1,550	1,266,600	35.15	31.17
Died	1,602	1,694,583	36.32	41.71
	4,410	$4,063,297	100.00	100.00

The policies and amounts at risk, the actual deaths, and the percentage of actual to expected deaths according to the four tables used in the former investigation, were summarized for all ages combined. The results are shown in Table B, which gives the experience according to years elapsed between the dates of entry and the dates of issue of the paid-up policies. Owing to the fact that there were only five deaths occurring amongst paid-up policies issued one year after the date of issue of the original policies, the paid-up policies issued during the first and second year of the original policies have been combined. Tables B for paid-up policies

Year of Extension or Surrender.	Experience.	Percentage of Actual to Expected Deaths by							
		Mod. Eng.		O[M]		Comp. Prog.		American.	
		Pols.	Amts.	Pols.	Amts.	Pols.	Amts.	Pols.	Amts.
1–5	Extended	78.27	81.57	82.77	85.34	93.25	96.02	72.08	75.00
	Paid-up after 1879	91.04	97.45	83.22	88.58	89.73	95.31	84.10	90.01
	Paid-up total	86.69	100.88	78.15	90.87	81.51	95.37	78.61	92.10
6–10	Extended	99.02	104.42	90.41	94.84	103.84	108.70	93.67	98.70
	Paid-up after 1879	88.42	86.73	78.20	76.72	81.92	79.41	80.55	78.41
	Paid-up total	90.08	97.61	79.77	86.51	81.72	88.11	81.06	87.53
11–15	Extended	106.64	112.48	92.64	97.38	103.83	109.05	99.86	105.21
	Paid-up after 1879	92.15	93.92	81.39	83.22	81.77	82.63	81.92	82.86
	Paid-up total	92.47	94.18	81.65	83.33	81.61	82.79	81.81	83.05
16–20	Extended	116.2	110.2	99.6	94.4	108.3	102.5	107.6	101.9
	Paid-up after 1879	94.34	101.19	83.73	89.92	82.17	87.65	82.61	88.15
	Paid-up total	95.31	102.50	84.52	90.98	83.36	89.24	83.82	89.76
21–25	Extended	119.0	128.3	102.7	110.6	108.2	116.3	109.0	117.3
	Paid-up after 1879	102.02	105.92	91.08	94.49	88.98	92.10	89.55	92.70
	Paid-up total	106.90	111.47	95.36	99.38	93.37	97.10	93.99	97.76
26–end	Extended	116.3	120.6	102.3	106.1	103.1	107.0	104.3	108.3
	Paid-up after 1879	97.23	100.95	87.14	90.51	82.50	84.53	82.95	84.93
	Paid-up total	96.00	98.25	85.99	87.91	80.78	81.26	81.17	81.61
1–end	Extended	92.0	97.5	88.9	92.7	99.6	103.3	85.4	90.3
	Paid-up after 1879	92.69	97.35	82.73	86.69	84.18	85.95	82.97	85.69
	Paid-up total	91.09	98.98	81.05	87.99	82.26	87.89	81.40	87.58

issued since 1879, as well as for *all* paid-ups issued, are given in full in order that a comparison may be made with the similar table relating to extended insurances. The above comparative table shows the percentages of actual to expected deaths for extended insurances, paid-ups issued after 1879, and all paid-ups issued. The data in this comparison is grouped according to periods of five years.

It will be noticed that for paid-up policies the ratio of actual to expected deaths is generally higher by amounts than by policies, which was the case with the experience under extended insurances. It will be seen that there is very little difference between the experience on all paid-up policies issued and on those issued since 1879. In other words, the elimination of the earlier issues had but little effect on the experience. Furthermore, the result of the elimination was in some instances to decrease, and in some to increase the percentages, the tendency being to increase the percentages so far as the policies were concerned, and decrease the percentages in case of the amounts.

Comparing the experience under paid-up policies issued since 1879 with the experience under extended insurances, it will be seen that there is a remarkably close agreement for all years combined between the two experiences, as measured by the Modified English Table, the paid-up policies showing 92.69 per cent. and 97.35 per cent. for policies and amounts respectively, as compared with 92.0 per cent. and 97.5 per cent. for the extended insurances. Measured by the $O^{[M]}$ Table, the percentage of actual to expected was about 6 per cent. less than in the case of extensions. Measured by the Compound Progressive Table, the percentage of the paid-ups was from 15 per cent. to 17 per cent. less, and according to the American Table from $2\frac{1}{2}$ per cent. to 5 per cent. less, than in the case of extended insurances. When the comparison is made with reference to the years between the date of issue of the original policy and the issue of the paid-up policy or granting of the extension, as the case may be, it will be seen that the general tendency is for the experience under paid-up policies to be in excess of that under extended insurances, where the paid-up or the extension occurs within a few years of the original date of issue. Where the termination of the original policy occurs after it has been in force for a longer term of years, the tendency then appears to be for the experience under paid-up policies to be more favorable than under extended insurances.

Table B[1] was compiled by summing the data used in Table B for two years and over, three years and over, etc. A comparison of this table with the table on page 610 of Volume X of the *Transactions* will prove interesting.

In the extended insurance investigation the data was very much more extensive than the data entering into the experience under paid-up policies. In the former investigation Table C showed the percentages of actual to expected deaths with reference not only to the year of the extended insurance, but also with reference to the ages at date of extension. The data entering into the paid-up investigations did not seem to be sufficiently extensive to warrant any attempt at keeping the experience according to the years of exposure under the paid-up policy. Accordingly, the experience was taken off for the first twenty years' duration under the paid-up policies, and the figures are shown for ages at the date of paid-up, grouped in order to compare with the similar figures for the extended insurances. In order to compare readily the mortality according to the three experiences with reference to the age at the termination of the original policy, the table, page 239, has been prepared. Ages below twenty-six and over eighty-five have been omitted owing to paucity of data.

In the extended insurance investigation attention was drawn to the fact that there was a tendency to a higher mortality as the age at which the extended insurance was granted increased. In the experience under paid-up policies there is apparently the same tendency to an increase in mortality up to about age 65, but from that age on there appears to be a well marked tendency to a decrease in mortality compared with the expected.

The policies and amounts at risk, the actual deaths, and the percentages of actual to expected deaths for each year of exposure under paid-up policies are shown in Table D. Attention may be drawn to the fact that in the investigation of the extended insurances the total deaths numbered 1,069, and of this number 747 occurred within the first four years of extended insurance. In the experience covering all paid-up policies issued there were 3,874 deaths, and only 504 occurred within the first four years. Furthermore, 1,141 occurred after twenty years. In the experience under paid-up policies issued since 1879 there were 1,602 deaths, and of this

Age at Paid-up or Extension.	Experience.	Actual Deaths.		Percentage of Actual to Expected Deaths by					
				Mod. Eng.		O[M]		Comp. Prog.	
		Pols.	Amts.	Pols.	Amts.	Pols.	Amts.	Pols.	Amts.
26–35	Extended	207	$440,586	83.5	90.2	93.5	100.9	107.4	116.0
	Paid-up after 1879	37	17,006	42.04	49.18	41.70	48.23	48.92	56.74
	Paid-up total	131	69,181	71.27	87.85	69.24	84.97	81.48	100.19
36–45	Extended	319	826,487	87.8	96.3	85.4	93.6	99.8	109.4
	Paid-up after 1879	148	100,473	70.98	80.46	62.45	70.53	71.85	81.20
	Paid-up total	478	300,761	93.15	101.54	81.34	88.39	93.19	101.40
46–55	Extended	297	818,195	97.5	95.4	86.4	84.1	97.5	94.9
	Paid-up after 1879	330	303,065	86.12	92.92	74.44	80.12	79.33	85.23
	Paid-up total	840	703,787	92.25	99.51	79.80	102.58	84.71	91.17
56–65	Extended	161	491,793	101.5	109.8	90.4	97.4	93.6	100.9
	Paid-up after 1879	424	545,230	96.00	101.39	85.81	90.52	85.70	90.16
	Paid-up total	857	935,334	99.53	101.36	89.06	90.54	88.90	90.24
66–75	Extended	60	164,433	140.1	128.8	129.7	119.1	125.8	115.4
	Paid-up after 1879	247	402,123	88.09	100.59	78.00	91.21	76.28	87.23
	Paid-up total	374	547,392	89.32	99.50	81.09	90.21	77.16	86.12

number 278 occurred within the first four years and 257 after twenty years. These figures show that there was a very different distribution of deaths in the paid-up experience from that which occurred under extended insurances. These figures are all the more significant when account is taken of the fact that the exposures in the extended insurance investigation were considerably greater than in the case of the paid-up experience. In the extended insurance investigation it was found that the percentage of actual to expected mortality in the first year of extension was heavy. The percentages steadily decreased during the first four years, after which the tendency appeared to be for the percentage to remain fairly constant. Remembering that the O[M] and the Compound Progressive are true select tables, it will be seen that under the paid-up policies the experience in the later years of duration was considerably higher than in the earlier years. In other words, as compared with the Compound Progressive Table, for example, it would appear that the experience under the paid-up policies showed a mortality with a greater effect of selection than that shown by the Compound Progressive Table. This tendency is shown in the following table, where the percentage of actual to expected deaths is shown for each five years under extended insurance or paid-up policies, as the case may be.

Year of Duration of Extension or Paid-up.	Experience.	Percentage of Actual to Expected Deaths by					
		Mod. Eng.		O[M]		Comp. Prog.	
		Pols.	Amt.	Pols.	Amt.	Pols.	Amt.
1–5	Extended.....	93.89	97.42	93.19	94.89	104.18	105.72
	Paid-up after 1879 ..	78.61	87.45	71.75	78.20	75.67	80.51
	Paid-up total.....	80.02	84.66	72.92	75.69	78.16	79.09
6–10	Extended.....	86.65	93.11	77.18	81.99	86.95	91.69
	Paid-up after 1879 ..	91.97	102.39	81.33	90.73	84.57	91.33
	Paid-up total.....	88.00	99.99	77.45	88.19	81.59	90.23
11–15	Extended.....	77.90	118.72	68.10	103.16	75.98	113.63
	Paid-up after 1879 ..	93.17	99.36	82.17	88.12	83.08	85.98
	Paid-up total.....	95.96	104.55	84.30	92.39	86.47	92.08
16–20	Extended.....	134.14	304.29	115.14	261.99	127.15	287.61
	Paid-up after 1879 ..	116.72	108.99	103.42	97.25	102.30	93.03
	Paid-up total.....	109.42	113.70	96.60	101.09	97.03	98.90
1-end	Extended.....	92.0	97.5	88.9	92.7	99.6	103.3
1-end	Paid-up after 1879 ..	92.28	97.85	82.38	87.08	84.70	86.95
1-end	Paid-up total.....	92.48	99.20	82.16	88.02	85.45	89.25

The above comparison shows that, while in the case of the extended insurances there appears to be a selection against the company with a heavy mortality in the first few years after taking advantage of the extended insurance option, the selection against the company under paid-up policies is of a different character. It will be seen that the mortality under paid-up policies is nearly as high as under extended insurances, but the effect of this higher mortality does not appear to be fully felt until some years after the issue of the paid-up insurances.

The above illustration shows clearly that a comparison as to the mortality experienced by a company with one of the standard tables must be viewed with caution. On the basis of policies, a comparison of the actual to expected deaths for the entire data according to the Modified English Table shows the percentage to be higher for the total paid-up business than for extended insurances, and the same is true for amounts. Where the comparison is made by the Compound Progressive Table it is seen that the percentage for extended insurance is very considerably higher than for the total paid-up business. In this connection it may be remembered that while the Modified English is not a true select table, an arbitrary allowance to give weight to the effect of selection was made.

TABLE B.

SUMMARY. PAID-UP POLICIES ISSUED SINCE 1879.

Year Paid-up.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by							
					Mod. Eng. No. 3.		O[M]		Comp. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.
2	1,507	$ 618,996	19	$13,825	124.50	201.82	121.51	193.89	136.74	215.74	115.25	186.15
3	7,797	2,259,828	117	37,465	93.32	95.91	86.49	87.92	93.77	95.09	86.46	88.66
4	7,250	2,706,315	107	41,843	79.09	78.28	71.61	70.76	76.87	75.59	72.97	72.18
5	5,231	2,692,412	98	51,944	99.35	104.94	89.56	94.61	95.91	101.75	91.58	97.06
6	5,019	2,938,943	99	58,434	97.36	92.94	86.66	82.60	92.65	87.85	89.74	85.45
7	3,590	2,550,388	75	54,831	90.86	95.80	80.76	85.25	84.76	89.86	82.83	87.66
8	2,851	2,051,380	56	36,330	78.80	67.89	69.48	59.81	72.90	62.50	72.11	62.03
9	2,384	1,823,844	55	47,582	90.34	92.91	79.12	81.44	82.75	83.58	82.26	83.27
10	2,363	1,871,568	58	56,427	80.84	83.53	71.27	73.94	72.42	73.52	72.12	73.48
11	2,593	2,487,040	77	72,744	107.98	102.77	94.98	90.87	98.49	93.51	98.10	92.90
12	2,438	2,117,564	67	64,116	85.48	97.60	75.23	85.79	76.12	87.86	76.19	87.99
13	2,389	2,457,910	61	75,011	74.28	74.82	65.67	66.40	66.62	66.21	66.69	66.40
14	1,835	1,987,462	83	103,014	105.56	110.43	93.46	97.98	90.48	93.20	90.82	93.66
15	1,874	2,092,165	71	88,508	89.69	88.96	79.55	79.14	79.32	78.00	79.82	78.51
16–20	6,662	8,128,316	282	387,878	94.34	101.19	83.73	89.92	82.17	87.65	82.61	88.15
21–25	2,915	5,069,404	156	283,584	102.02	105.92	91.08	94.49	88.98	92.10	89.55	92.70
26–47	1,856	3,054,560	121	221,047	97.23	100.95	87.14	90.51	82.50	84.53	82.95	84.93
Totals...	60,554	$46,908,095	1,602	$1,694,583	92.69	97.35	82.73	86.69	84.18	85.95	82.97	85.69

TABLE B.

Summary of Total Paid-up Policies.

Year Paid-up.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by							
					Mod. Eng. No. 3.		O[M]		Comp. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.
2	4,100	$1,580,510	81	$ 39,390	107.30	133.68	91.92	122.97	104.49	130.53	97.82	126.41
3	15,249	5,056,466	271	109,765	90.47	104.11	82.24	94.36	87.26	99.81	82.94	95.44
4	16,404	6,130,523	283	119,315	73.90	87.11	66.37	78.26	68.60	82.09	66.52	79.43
5	14,062	7,185,814	324	179,323	93.04	104.26	83.26	93.42	86.18	97.44	84.16	94.97
6	12,461	6,558,113	287	179,317	94.18	109.43	83.68	97.30	87.19	101.39	85.52	99.50
7	11,464	7,217,702	284	200,915	90.43	97.46	80.16	86.66	82.57	88.91	81.62	87.89
8	9,751	6,457,525	262	179,165	90.54	90.71	80.13	80.26	82.09	81.79	81.55	81.32
9	9,421	6,803,734	261	224,197	84.82	100.30	75.03	88.68	75.96	89.78	75.79	89.64
10	8,305	6,301,865	245	211,082	90.54	92.57	79.91	81.94	80.99	82.05	80.92	82.10
11	7,471	6,157,423	230	189,339	92.37	89.17	81.49	78.84	82.21	79.21	82.28	79.23
12	6,472	5,459,842	201	186,191	86.84	94.48	76.58	83.30	76.49	83.30	76.70	83.60
13	5,825	5,662,260	180	198,916	85.34	86.36	75.40	76.57	75.50	76.13	75.64	76.38
14	4,357	4,772,468	185	214,837	107.31	108.13	94.87	95.73	93.16	93.19	93.45	93.55
15	2,903	3,143,944	106	125,029	94.96	94.44	83.94	83.70	84.34	83.33	84.79	83.82
16–20	8,478	10,289,056	351	474,936	95.31	102.50	84.52	90.98	83.36	89.24	83.82	89.76
21–25	3,460	5,844,347	190	336,827	106.90	111.47	95.36	99.38	93.37	97.10	93.99	97.76
26–44	2,001	3,504,498	133	250,817	96.00	98.25	85.99	87.91	80.78	81.26	81.17	81.61
Totals ...	142,184	$98,126,090	3,874	$3,419,361	91.09	98.98	81.05	87.99	82.26	87.89	81.40	87.58

TABLE B[1].

Paid-up Policies Issued Since 1879.

Paid-up after being in Force over	Percentage of Actual to Expected by							
	Mod. Eng.		O[M]		Com. Prog.		American	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Am't.
1 year	92.69	97.35	82.73	86.69	84.18	85.95	82.98	85.69
2 years	92.66	97.23	82.70	86.59	84.14	85.84	82.94	85.58
3 years	92.41	96.93	82.41	86.30	83.79	85.53	82.70	85.31
4 years	92.34	96.96	82.11	86.26	83.09	85.33	82.41	85.23
5 years	93.58	97.57	83.06	86.76	83.62	85.62	83.26	85.64
6 years	93.15	97.34	82.60	86.52	82.80	85.17	82.67	85.30
7 years	92.81	97.52	82.27	86.68	82.06	85.07	82.12	85.30
8 years	92.95	97.58	82.37	86.73	81.88	84.89	82.07	85.21
9 years	93.86	98.70	83.21	87.76	82.43	85.69	82.69	86.04
10 years	94.07	98.92	83.45	88.00	82.41	85.77	82.72	86.14
11 years	95.05	99.72	84.37	88.73	83.14	86.39	83.49	86.80
12 years	94.02	99.55	83.52	88.61	81.97	86.00	82.36	86.46
13 years	94.84	99.66	84.32	88.77	82.52	85.90	82.94	86.37
14 years	97.14	102.00	86.42	90.89	84.24	87.71	84.70	88.21
15 years	96.12	101.19	85.57	90.21	83.48	87.17	83.96	87.67
16 years	97.01	102.59	86.40	91.47	84.03	88.20	84.52	88.70
21 years	99.89	103.69	89.30	92.70	86.02	88.62	86.54	89.13
26 years	97.27	100.95	87.11	90.51	82.48	84.53	82.93	84.93

TABLE B[1].

Total Paid-up Policies.

Paid-up after being in Force over	Percentage of Actual to Expected by							
	Mod. Eng.		O[M]		Com. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Am't.
1 year	91.09	98.98	81.05	87.99	82.26	87.89	81.40	87.58
2 years	91.03	98.88	80.98	87.90	82.19	87.80	81.34	87.49
3 years	90.80	98.68	80.74	87.70	81.89	87.56	81.11	87.29
4 years	90.83	98.51	80.62	87.49	81.50	87.20	80.97	87.09
5 years	92.68	99.00	82.17	87.88	82.86	87.40	82.54	87.35
6 years	92.65	98.70	82.05	87.57	82.51	86.86	82.36	86.93
7 years	92.48	98.08	81.87	87.01	82.03	86.07	82.03	86.23
8 years	92.74	98.13	82.09	87.04	81.96	85.86	82.08	86.11
9 years	93.02	98.73	82.34	87.59	81.95	86.18	82.15	86.49
10 years	94.33	98.57	83.51	87.48	82.89	85.82	83.15	86.17
11 years	94.95	99.26	84.10	88.12	83.19	86.25	83.51	86.63
12 years	95.40	100.47	84.56	89.23	83.36	87.07	83.72	87.50
13 years	97.08	101.21	86.14	89.97	84.70	87.53	85.09	87.98
14 years	99.64	103.74	88.49	92.26	86.66	89.42	87.12	89.91
15 years	97.98	102.98	87.10	91.66	85.26	88.78	85.74	89.28
16 years	98.47	104.09	87.62	92.70	85.40	89.46	85.89	89.97
21 years	102.15	105.42	91.27	94.14	87.75	89.64	88.28	90.15
26 years	96.03	98.25	85.97	87.91	80.80	81.26	81.20	81.61

TABLE C.

Summary of Paid-up Policies Issued Since 1879. First 20 Years of Duration Only.

Ages at Issue.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by							
					Mod. Eng. No. 3.		O[M]		Comp. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.
18–25	999	$ 344,483	1	$ 94	12.40	3.40	13.8	3.79	15.8	4.34	11.8	3.23
26–35	9,358	3,644,115	37	17,006	42.04	49.18	41.70	48.23	48.92	56.74	40.27	47.15
36–45	15,864	9,470,243	148	100,473	70.98	80.46	62.45	70.53	71.85	81.20	66.94	75.92
46–55	16,190	13,509,103	330	303,065	86.12	92.92	74.44	80.12	79.33	85.23	79.16	85.39
56–65	9,615	11,295,346	424	545,230	96.00	101.39	85.81	90.52	85.70	90.16	86.43	91.14
66–75	3,401	4,850,524	247	402,123	88.09	100.59	78.00	91.21	76.28	87.23	76.39	87.43
76–84	293	547,820	24	49,673	51.18	53.41	45.53	47.30	37.60	37.92	37.60	37.92
89–91	3	5,048	2	2,725	255.10	199.39	212.77	164.68	128.04	94.97	128.04	94.97
Totals...	55,723	$43,666,682	1,213	$1,420,389	83.22	93.43	74.29	83.14	76.39	83.02	74.98	82.67

Summary of Total Paid-up Policies. First 20 Years of Duration Only.

Ages at Issue.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by							
					Mod. Eng. No. 3.		O[M]		Comp. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.
18–25	1,834	$ 608,362	8	$ 3,909	52.93	78.70	57.63	86.05	66.58	99.14	50.49	74.75
26–35	19,032	8,143,198	131	69,181	71.27	87.85	69.24	84.97	81.48	100.19	68.31	84.38
36–45	37,423	21,588,881	478	300,761	93.15	101.54	81.34	88.39	93.19	101.40	87.67	95.62
46–55	37,238	28,745,214	840	703,787	92.25	99.51	79.80	102.58	84.71	91.17	84.75	91.42
56–65	18,487	19,407,275	857	935,334	99.53	101.36	89.06	90.54	88.90	90.24	89.57	91.11
66–75	4,999	6,638,208	374	547,392	89.32	99.50	81.09	90.21	77.16	86.12	77.27	86.31
76–85	326	615,297	43	81,498	82.49	78.06	73.35	69.09	60.63	55.57	60.63	55.57
89–91	3	5,048	2	2,725	25.51	19.94	21.28	16.47	12.80	9.50	12.80	9.50
Totals...	119,342	$85,751,483	2,733	$2,644,587	92.48	99.20	82.16	88.02	85.45	89.25	83.90	88.69

TABLE D.

Summary of Total Paid-up Policies.

Year of Duration.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by							
					Mod. Eng. No. 3.		O[M]		Comp. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.
1	9,501	$7,524,055	88	$108,140	55.30	67.49	51.65	60.97	55.48	64.31	50.81	61.67
2	9,082	7,105,854	146	150,885	91.06	94.26	83.95	84.69	90.09	88.62	84.01	85.84
3	8,642	6,661,867	132	140,104	83.08	89.58	75.65	79.91	81.20	83.76	76.81	81.76
4	8,219	6,233,709	138	137,245	87.13	88.55	78.38	78.83	83.94	81.99	80.33	80.59
5	7,820	5,882,948	132	129,166	83.53	83.64	74.44	74.01	79.49	76.70	76.85	75.77
6	7,433	5,529,768	141	175,942	89.18	114.95	78.93	101.48	83.89	104.53	81.80	103.70
7	7,095	5,175,704	142	137,185	90.50	92.49	79.72	81.54	84.46	83.86	82.87	83.42
8	6,750	4,842,906	123	130,786	78.64	90.26	69.10	79.57	72.82	81.56	71.83	81.31
9	6,409	4,538,525	140	140,508	90.05	99.06	79.09	87.35	82.93	89.05	82.06	88.95
10	6,081	4,259,105	142	144,084	91.64	102.59	80.45	90.50	83.87	91.56	83.25	91.60
11	5,736	3,967,773	145	124,686	94.70	91.71	83.10	80.88	86.10	81.19	85.67	81.34
12	5,361	3,664,819	133	127,739	88.61	96.51	77.78	85.19	80.24	84.90	80.03	85.15
13	5,006	3,382,490	141	156,821	95.59	123.12	83.97	108.87	86.26	109.06	86.18	109.48
14	4,694	3,117,926	133	106,216	90.75	86.61	79.75	76.61	81.31	76.18	81.37	76.52
15	4,347	2,877,911	158	151,244	110.78	126.68	97.47	112.17	98.80	110.59	99.02	111.13
16	3,994	2,611,118	148	115,103	108.68	101.69	95.70	90.14	96.69	88.26	97.03	88.71
17	3,692	2,397,854	127	113,882	96.93	105.34	85.41	93.46	85.96	91.18	86.34	91.67
18	3,441	2,192,926	154	138,428	118.86	131.59	105.00	117.14	105.41	114.36	105.96	114.98
19	3,135	1,980,650	128	107,490	103.50	109.53	91.55	97.70	91.68	95.82	92.25	96.39
20	2,904	1,803,575	142	108,933	119.93	122.58	106.12	109.13	106.07	107.05	106.79	107.72
21–end	22,842	12,374,607	1,141	774,774	87.95	98.24	78.51	87.89	75.51	83.55	75.97	83.99
Totals...	142,184	$98,126,090	3,874	$3,419,361	91.09	98.98	81.05	87.99	82.26	87.89	81.40	87.58

TABLE D.

Summary. Paid-up Policies Issued Since 1879.

Year of Duration.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by							
					Mod. Eng. No. 3.		O[M]		Comp. Prog.		American.	
	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.	Pols.	Amount.
1	4,963	$4,189,049	48	$ 76,941	52.39	75.60	48.81	68.08	51.43	70.52	47.76	68.54
2	4,709	3,920,966	75	88,789	81.48	87.63	75.13	78.62	79.26	80.98	74.69	79.26
3	4,454	3,665,026	70	83,379	77.32	84.56	70.53	75.58	74.62	78.14	71.11	76.89
4	4,197	3,438,486	85	102,092	94.30	103.05	85.16	91.80	89.77	94.28	86.44	93.25
5	3,938	3,194,834	78	84,531	88.10	86.68	78.86	77.03	82.94	78.64	80.56	78.06
6	3,701	2,974,421	76	100,597	87.44	105.89	77.70	93.92	81.38	95.43	79.61	95.00
7	3,502	2,760,528	75	86,838	88.18	95.13	78.03	84.24	81.50	85.15	80.16	84.91
8	3,289	2,538,884	70	78,818	84.25	90.43	74.39	80.09	77.38	80.76	76.40	80.64
9	3,077	2,349,461	82	91,726	101.54	109.28	89.58	96.81	92.80	97.08	91.90	97.08
10	2,875	2,165,649	78	91,235	99.46	112.19	87.74	99.42	90.40	98.79	89.73	98.89
11	2,655	1,978,405	76	72,969	100.57	94.94	88.62	84.04	90.67	82.67	90.22	82.84
12	2,410	1,781,994	62	62,019	85.95	84.36	75.76	74.73	77.03	72.66	76.80	72.89
13	2,176	1,596,614	70	96,121	102.24	140.34	90.17	124.59	91.42	122.63	91.30	123.13
14	1,965	1,414,409	51	42,077	77.38	66.29	68.25	58.84	68.51	57.34	68.53	57.59
15	1,750	1,275,431	62	67,975	99.29	111.41	87.69	98.98	87.25	95.18	87.40	95.61
16	1,534	1,120,691	76	63,351	133.69	112.43	118.12	99.93	117.18	95.06	117.49	95.49
17	1,354	992,678	45	48,877	86.27	94.25	76.24	83.80	75.22	79.38	75.46	79.74
18	1,212	880,559	65	66,243	129.44	133.52	114.82	119.37	113.37	113.79	113.87	114.34
19	1,036	759,427	54	48,150	118.77	107.87	105.58	96.85	104.42	93.66	105.00	94.14
20	926	669,173	47	34,863	114.12	93.07	101.34	83.14	100.41	80.64	101.03	81.10
21–31	4,831	3,241,410	257	206,992	94.89	93.87	84.65	84.00	81.57	79.42	82.04	79.79
Totals...	60,554	$46,908,095	1,602	$1,694,583	92.69	97.35	82.73	86.69	84.18	85.95	82.97	85.69

The Basis for Employers' Contributions Toward Service Pensions.

BY

A. H. MOWBRAY.

In 1907, the date as of which the investigation covered by the 23d Annual Report of the United States Commissioner of Labor closed, there were about fifty railroad relief funds established in the United States. The report presents data relative to fifty and says: "So far as known this report includes all but one large railroad fund and a few very small ones in existence in 1907." Of these but fifteen provided superannuation benefits and "14 are pension systems maintained entirely by the employing companies." These "are entirely maintained and controlled by the companies, the employees not contributing to them."

Under these circumstances it is natural to ask what was the motive for establishing them on their present noncontributory basis even though the pensions are not guaranteed. It has been generally assumed that the motive was the desire to secure stability of staff. It has also been charged that the fear of loss of prospective pensions was desired as a club for use in dealing with threatened strikes.

In a recent conference a prominent official of one road having a pension system, while admitting that there had been some gain in holding employees on the staff even at times in the face of labor troubles, took the position that the primary motive was economy through promotion of efficiency. He pointed out that if by pensioning a man of but 45 per cent. efficiency his place could be filled by one of 95 per cent. efficiency it would pay to do so. Mr. F. L. Hoffman also points out (Reports Seventh International Congress of Actuaries, Vol. 1, p. 255): "Obviously, the retention of old and worn-out employees for humane or other reasons on the pay roll of the Government" (or other service) "is equivalent to the grant of a pension, unless full service is rendered for the compensation paid. That this is not the case is a matter of everyday experience, and numerous investigations into the subject have conclusively

shown that the efficiency of the service as a whole is materially impaired by retaining on the pay roll of the Government superannuated and more or less physically and mentally disqualified employees."

Of course, inefficient employees could be replaced by simply dismissing them as is done when inefficiency is found among younger men. But with the faithful servant whose inefficiency is due to the strain of long service this may be, even from a dollars-and-cents point of view, bad policy, especially in view of the enlarged extent to which the relations of employer and employee are now in the public eye.

In his paper "On Staff Pension Funds" (45 J. I. A. 153) Mr. Manly refers to men put on pension before the usual age "generally at the request of Heads of Departments, who say that if their departments are to be kept efficient then the inefficients must be turned out." He goes on to say: "In some countries they would be scrapped: turned off without any compensation at all; but in this country (England) the Directors, backed by public opinion, take a large humanitarian view of the position and place their servants on the Pension Funds which they have established; often forgetting, however, that efficiency so secured, which has the effect of reducing the salary list, should be accompanied by larger contributions from the Company to the Funds." But Mr. Manly has not given any indication of his judgment as to how the amount of these "larger contributions" should be determined. Nor has any other discussion which has come to my attention dealt with this question. The explanation of this apparent oversight of so important a point may be found in the absence of standards of relative efficiency. The development of such standard is one work upon which the twentieth century has entered, and it therefore seems appropriate to investigate the measure of gain to an employer through the retirement of inefficients of advanced years and the amount and manner of outlay for pensions which can be profitably undertaken.

If, for example, it be the attitude of the public that a man who is reasonably well paid during his working years should so manage his affairs that on attaining the age of 75 years he would have the means to enable him to relinquish work without further assistance form his employer, and his employer's dismissing him at that age will therefore be accepted without criticism, but would not be so

accepted if dismissal came at a younger age; and if at age 65 his efficiency has so fallen that he can be replaced at the same compensation by a man whose efficiency rating is higher by fifty points, then there is a gain of the present value of a 10-year temporary annuity of 50 per cent. of his salary by and upon his retirement. This amount could be devoted to pensioning him and, if paid in instalments corresponding to the incidence of the gain, would cause no change in the employer's condition, except his incidental gain through the effect of the prospect of retirement on the attitude of the remaining staff and through a more favorable attitude of the public.

Expressing this condition in mathematical terms, we have as the gain

$$(e' - e_x) \cdot S_x \cdot a^{(12)}_{x\,:\,\overline{y-x}|}, \tag{1}$$

where y is the age of retirement without pension, x is the age of retirement on pension, e_x the probable efficiency rating for the next $(y - x)$ years of the man to be retired, S_x his average probable salary over the same period and e' the efficiency rating required of his successor.

The annual pension this would provide would be

$$\frac{(e' - e_x) \cdot S_x \cdot a^{(12)}_{x\,:\,\overline{y-x}|}}{a^{(12)}_x}. \tag{2}$$

If this system were adopted the annual instalments to be paid into the fund for $(y - x)$ years or until the prior death of the pensioner by the employer would be $(e' - e_x)S_x$, and the fund or reserve in respect of each pensioner at the end of n years after retirement (F_n) would be found by either the prospective or retrospective methods, as is the reserve on a life insurance policy, as follows:

$$F_n = \frac{(e' - e_x) \cdot S_x \cdot a^{(12)}_{x\,:\,\overline{y-x}|}}{a^{(12)}_x} \cdot a^{(12)}_{x+n} - (e' - e_x) \cdot S_x \cdot a^{(12)}_{x+n\,:\,\overline{y-x-n}|} \tag{3}$$

or

$$F_{(n)} = (e' - e_x) \cdot S_x \cdot {}_n u^{(12)}_x - \frac{(e' - e_x) \cdot S_x \cdot a^{(12)}_{x\,:\,\overline{y-x}|}}{a^{(12)}_x} \, {}_n u^{(12)}_x. \tag{4}$$

In the application of formula (4) it must, however, be remembered that after n becomes larger than $(y - x)$ the first term

increases by interest accumulations only. Hence for these values (4) becomes:

$$F_{(n)}=(e'-e_x)\cdot S_x\cdot {}_{y-x}u_x^{(12)}\cdot(1+i)^{n-y+x}-\frac{(e'-e_x)\cdot S_x\cdot a_{\overline{x:y-x}|}^{(12)}}{a_x^{(12)}}\,{}_nw_x^{(12)}. \quad (4a)$$

Using the ages and efficiency ratings noted in the illustration and Actuaries' 4 per cent. annuities, the gain from retirment at age 65 would be 3.2115 years' wages and the life pension purchasable, 38.72 per cent. of the average salary between 65 and 75.

Considerations of this kind would seem to furnish a good foundation for determining the attitude which should be taken by a corporation, from the standpoint of its own interests, both as to the policy of providing pensions, the amount of pension to be offered and annual outlay to be made in this way.

The careful observer of human nature and student of social psychology will at once object that this theory of mathematical determination of gains and pensions is built upon a false premise, viz., an age at which dismissal is expected. "For," he will observe, "however strong the community sentiment in favor of thrift and self-reliance, the dismissal, without some provision being made for his future of an aged faithful servant who has not made provision for himself, will not be viewed without something akin to indignation on the part of all to whose attention it comes, however ample may have been his compensation during his active service. The older the age at dismissal the stronger this feeling will be. Hence it will be impossible to fix any age for retirement without a pension and without criticism of the dismissal."

That there is much truth in this view cannot be denied. Yet the facts remain that life tenure, except in certain kinds of Government service, is practically unknown, and that, when there is no pension system, men are often retained a considerable time after they would have been compelled to retire had they been so provided for. We may then restate the proposition, substituting the age at which retirement is generally forced without regard to pension provisions, for the age at which dismissal on such terms will be approved by public sentiment.

If this theory be accepted and a certain minimum efficiency test, say 70 per cent. normal, be set up as the requirement for continuance on the staff, the age at which retirement on pension is to be made compulsory can be determined by a study of efficiency records

of the staff and consultation with department heads and efficiency experts. This age would probably be that at which the greatest number fall below the minimum test of efficiency. If the pension is to be based on the average earnings of the last ten years, such percentage of the ascertained gain to the service from such compulsory retirement as the company deems should be devoted to a pension fund, can be taken as a single premium for a life annuity, and the annual rent expressed as a percentage of such average salary. If the same mortality be assumed, and the same age of retirement were pension not provided, for those falling below the minimum efficiency requirement at an earlier age the company would gain more by their earlier retirement on pension and could afford to provide a slightly *more* liberal pension in such cases. For if y be constant $a^{(12)}_{x:\overline{y-x}|}$ increases more rapidly as x decreases than does $a^{(12)}_{x}$ The reason why it would not be advisable to make such an arrangement need not be dwelt upon. At least it is apparent that a uniform rule for determining the amount of pension, as a certain percentage of the average salary for the ten years preceding retirement based upon the term of service, fixed for the age of general compulsory retirement, will not cause loss if applied to retirements at an earlier age.

The basis of pension to be guaranteed having been fixed upon, the fund should be created by an annual payment of the annual gain (or percentage thereof devoted to pensions) from increased efficiency—$(e' - e_x)S_x$—for each pensioner until age y is reached or until prior death, which fund would be further increased by interest from its investment. Excess interest earned over that assumed in fixing pension rates would be a source of profit.

It is believed that this basis has at least two distinct advantages over the present common basis of an annual appropriation on an estimate of each year's need with a fixed maximum limit. It permits the pension to be guaranteed as to permanency and amount, making it much more attractive to the men. It requires the pension to be provided for out of the gain from increased efficiency as that gain is received.

It also seems that these considerations would well serve as the basis for determining the extent and character of the employer's participation in contributory funds. It would meet the objection of many employers to the usual form of contribution currently with employees, viz.: that it locks up too large an amount of

money for a long period of time, that it is planned for 75 to 80 years hence rather than to meet present problems. Since the employer's contribution in respect to any employee would not be paid into the fund until he has actually retired, there would be no expectation from the employee or his representatives of anything from this source on his leaving the service or death without pension.

The difficulty always encountered in establishing a contributory pension system on the usual basis in regard to the existing employees would also be lessened if the employer's contributions were made as here outlined. The employer makes the same gain in efficiency through retirement whether the employee has been contributing to a fund one or twenty years. Hence his contribution on behalf of a given employee should be independent of the age of the fund. If the employer will guarantee a minimum pension based upon the theory and methods here presented and require a supplemental pension to be provided by contributions from the employees on what is usually referred to as the "savings bank plan," the employees already at or above pension age will be retired upon the minimum pension, which will gradually increase for those retiring in later years through their own efforts, a discrimination of which there would be no reasonable ground for complaint. If the employees at or beyond pension age at the adoption of the system were below the efficiency rating adopted as the basis for the future, then the gain to the service being greater on their retirement than will be that on the retirement of those later retired, the employer's contribution might be increased proportionately. This will improve the condition for those retiring before their own contributions can have appreciably increased their pensions. The younger men cannot reasonably complain at this discrimination, since it is merely the exercise of sound business judgment on the part of the employer. The prospect of promotion will also tend to still such grumblings as might begin.

The methods above suggested doubtless can and will be much improved upon. The fact to which it is sought to draw attention is that, if we are to gain the confidence of men of affairs in the soundness and practicality of our professional labors in this field, we must correlate in extent and incidence the employer's contribution to a pension system with his probable gain from the increased efficiency thereby brought about.

ADDENDUM.

Since the feasibility of determining the efficiency percentages is the foundation of this paper, the suggestion has been made that some explanation should be given of the method of determining such percentages as otherwise the matter would seem to be left incomplete.

The unsatisfactory result of the attempt of the President's Commission on Economy and Efficiency to "ascertain the annual loss to the government through inefficiency of aged employees" by means of reports from heads of departments and bureau chiefs in which the estimate was largely a matter of personal judgment on the part of those reporting* is evidence that some scientific method must be used.

The Journal of Accountancy,† Vol. 12, contains a number of able papers dealing with efficiency methods. A study of these as well as the text books on the subject leaves no room for doubt that the foundation upon which all modern efficiency systems rest is standardization of working methods and product. While the text books and discussions as above primarily concern themselves with industrial processes, Mr. Hunter has shown in his paper, "Practical Application of the Piece Work System in Life Insurance Offices,"‡ that, appearances to the contrary notwithstanding, the development of such standards is possible in many lines of clerical work. Wherever work is standardized, particularly where the bonus system is in use, records may be and usually are kept of the number of standard units of work turned out by each worker in a given period of time. Study and experience with such records will permit the fixing of a standard allotment for a worker of 100 per cent. efficiency. Such allotment should not be so fixed as to require extraordinary effort on the part of a good man to accomplish it. The efficiency of any worker over a particular period would then be expressed as the ratio of his production to the standard. We are all more or less familiar with this system of allotments in life insurance field work.

* See "Retirement from the Classified Civil Service of Superannuated Employees," Government Printing Office, 1912, pages 37 et. seq. and Appendix E.

† Official Organ of the American Association of Public Accountants.

‡ T. A. S. A., IX, p. 285.

In some lines of work, for example teaching, it must be admitted considerable difficulty will be experienced in the determination of such standards and keeping of such records. Yet it is a self evident proposition that variations in efficiency exist and the writer makes bold to maintain that that profession has not reached its fullest development which cannot find standards of efficiency by which tests can be made so as to point out the inefficients and the degree of inefficiency.

Select and Ultimate Mortality Gain on Single Premium Policies.

BY

EDWARD W. MARSHALL.

The mortality gain on single premium policies is usually obtained by differencing the net American and select and ultimate reversions. This is a very simple and obvious mode of procedure but it necessitates the calculation of the select and ultimate net single premiums. These are tabulated on the life plan, in Mr. Dawson's "Comparative Reserve Tables," but so far as we know, the endowment reversions are not to be found in any publication.

As it is always better to have a uniform method of computing mortality gains in order that a clerk unacquainted with actuarial formulas might follow a similar process in all cases, the following formula and the accompanying table may be of some slight service.

The usual method, denoting assumed mortality gain by $G_{x\overline{n}|}$,

$$G_{x\overline{n}|} = A_{x\overline{n}|} - A_{[x]\overline{n}|}$$

can be simplified by adding to and subtracting from the righthand term,

$$A_{x\overline{n}|} \cdot \frac{D_x}{D_{[x]}}.$$

Then

$$\begin{aligned} G_{x\overline{n}|} &= A_{x\overline{n}|}\left(1 - \frac{D_x}{D_{[x]}}\right) + A_{x\overline{n}|} \cdot \frac{D_x}{D_{[x]}} - A_{[x]\overline{n}|} \\ &= A_{x\overline{n}|}\left(1 - \frac{D_x}{D_{[x]}}\right) + \frac{M_x - M_{x+n} + D_{x+n}}{D_{[x]}} - \frac{M_{[x]} - M_{x+n} + D_{x+n}}{D_{[x]}} \\ &= A_{x\overline{n}|}\left(1 - \frac{D_x}{D_{[x]}}\right) + \frac{M_x - M_{[x]}}{D_{[x]}} \text{ when } n \not< 5 \\ &= \frac{M_x - M_{[x]}}{D_{[x]}} - A_{x\overline{n}|}\left(\frac{l_x}{l_{[x]}} - 1\right). \end{aligned}$$

This result is similar in form to Mr. Henderson's formula for the

mortality gain on level premium policies, published in the "American Underwriter," viz.:

$$G_x = \frac{M_x - M_{[x]}}{D_{[x]}} - P_x \cdot \frac{N_x - N_{[x]}}{D_{[x]}},$$

and, in fact, could be derived therefrom by an obvious change in the last term.

In the case of single premium varying assurances, the above formula may be suitably modified by substituting for $\frac{M_x - M_{[x]}}{D_{[x]}}$ an expression corresponding to the benefit. For example, if A' is the single premium for a benefit starting at unity and increasing by unity each year for at least five years, then the mortality gain thereon is

$$(IG)_x = \frac{R_x - R_{[x]}}{D_{[x]}} - A'\left(\frac{l_x}{l_{[x]}} - 1\right).$$

The values of the function $\frac{R_x - R_{[x]}}{D_{[x]}}$ are given in a paper by Mr. Sheppard (T. A. S. A., IX, 295).

The factors $\left(\frac{l_x}{l_{[x]}} - 1\right)$ and $\frac{M_x - M_{[x]}}{D_{[x]}}$ at $3\frac{1}{2}$ per cent. interest are tabulated below for ages at entry 20 to 65, the l_x being based on the American Experience, the $l_{[x]}$ on Mr. Dawson's select and ultimate tables, and the $\frac{M_x - M_{[x]}}{D_{[x]}}$ being copied from the aforesaid article of Mr. Henderson's.

All of the above single premium formulas are applicable only to policies with a duration of at least five years, and for a shorter period the $\frac{M_x - M_{[x]}}{D_{[x]}}$ factor must be changed to $\frac{M_x - M_{[x]}}{D_{[x]}} - \frac{M_{x+t} - M_{[x]+t}}{D_{[x]}}$, t being the term of the policy.

The mortality gain having been obtained, the successive reserves may be found by means of Mr. Sheppard's select u and k* columns (T. A. S. A., X, 141) as follows:

$$A_{[x]+1\overline{n-1}|} = (A_{x\overline{n}|} - G_{x\overline{n}|})u_{[x]} - k_{[x]},$$

* Mr. Sheppard used U and K.

and thereafter

$$A_{[x]+t\overline{n-t}|} = A_{[x]+t-1\overline{n-t+1}|} \cdot u_{[x]+t-1} - k_{[x]+t-1}.$$

As this is a continued process, the agreement of the $A_{[x]+5\overline{n-5}|}$ with $A_{x+5\overline{n-5}|}$ proves all the preceding reserves for age at entry $[x]$. For this reason the formula is much superior to one that can be derived by a modification of the retrospective formula, as applied to single premiums.

In conclusion, it might be stated that, so far as we know, there has been no discussion of the subject of this paper before this Society. This fact constitutes the excuse for presenting such a simple and elementary topic for your consideration.

FORMULA FOR SINGLE PREMIUM MORTALITY GAIN.

$$1000 \cdot \frac{M_x - M_{[x]}}{D_{[x]}} = \text{(Single Premium per \$1000 Insurance) times } \left(\frac{l_x}{l_{[x]}} - 1\right)$$

Age.	$\frac{l_x}{l_{[x]}} - 1$	$\frac{M_x - M_{[x]}}{D_{[x]}} .1000$	Age.	$\frac{l_x}{l_{[x]}} - 1$	$\frac{M_x - M_{[x]}}{D_{[x]}} .1000$
20	.010328	9.600	43	.014390	13.348
21	.010399	9.666	44	.014902	13.834
22	.010482	9.743	45	.015443	14.332
23	.010533	9.791	46	.016056	14.900
24	.010618	9.869	47	.016760	15.550
25	.010705	9.950	48	.017574	16.305
26	.010805	10.040	49	.018518	17.179
27	.010917	10.145	50	.019602	18.182
28	.010997	10.220	51	.020789	19.279
29	.011125	10.337	52	.022102	20.479
30	.011244	10.448	53	.023599	21.880
31	.011377	10.572	54	.025247	23.409
32	.011549	10.718	55	.027076	25.101
33	.011662	10.835	56	.029104	26.982
34	.011840	11.001	57	.031405	29.114
35	.012022	11.168	58	.033955	31.479
36	.012220	11.352	59	.036804	34.120
37	.012462	11.575	60	.039952	37.037
38	.012708	11.804	61	.043482	40.312
39	.012973	12.048	62	.047392	43.938
40	.013298	12.350	63	.051765	47.997
41	.013630	12.657	64	.056637	52.517
42	.014012	13.009	65	.062056	57.553

Modern Surrender Values.

BY

JAMES F. LITTLE.

In the course of the discussion on Mr. Moir's paper on the "Liberality of Modern Policies," at the last meeting of the Actuarial Society, the subject of surrender values received a large share of attention. Differences of opinion upon more than one point developed, and it has occurred to me that a fuller discussion, limited to this one matter, will be of value.

I am the more tempted to bring forward this subject, because of the radical difference—in America at least—between the present-day point of view from which the question is regarded and that very generally accepted less than a quarter of a century ago; and because a clear statement of the existing point of view is not easily to be found, though many casual references thereto have been made from time to time.

Before proceeding further, I would like to state that, although in my opinion what I shall call the modern viewpoint is the proper one for America today, and ultimately for other countries, it does not follow that the earlier, and in many countries present-day, view is wrong. Circumstances may modify the conclusions that seem appropriate here and now, and without a full examination judgment should not be pronounced.

The chief source of information concerning the early history of life insurance is comprised in the records of British companies. From this source we may learn the interesting fact that surrender values were originally paid, not because it was deemed that the policyholders had a just claim thereto, but as a matter of policy—or profit. In due course a stage is reached in the history of a policy where the profit to the company will be greater if a certain surrender value is paid to terminate the contract than if the insured continues to pay premiums for the maximum term of the policy. Competition among companies soon made it advisable to promise the payment of an "equitable surrender value," and in some cases a minimum amount, generally stated as a percentage of premiums

paid, was guaranteed. Surrender values soon became an integral part of life insurance business, and the Act of 1870 called for a return showing the "minimum surrender values allowed," though, presumably, a company might have quoted these as nil.

Ere long a very generally accepted view was that the outgoing policyholder should put the company in as good a position as it would enjoy if he continued to pay premiums. This—the earlier viewpoint—arises naturally from the acceptance of the doctrine that the policy is a mutual contract for the remaining lifetime of the insured, or so much of it, in the case of endowment insurances, as is comprehended within a given term of years. According to English law, if one party to a contract desires to cancel it, he must put the other party in as good a position as if the original contract were to be carried out. Views even less favorable to the insured, though usually coupled with an admission that their logical result is neither practicable nor desirable, have been expressed from time to time. Thus the late Mr. C. Walford stated (J. I. A., xxi, 376): "I think the surrender value has grown out of a very mistaken idea. It has gone on growing until it has come to be regarded as a right. Now, there is no right about it. What other contract is there, which, entered into under seal between two parties, can be broken by one of them, and respecting which the person who has altered his mind can coerce the other into giving him something to his advantage?" In the course of the discussion of which the above forms a part, Mr. H. W. Manly remarked that "the policyholder is not equitably entitled to ask the office for any surrender value if he wishes to break the contract." It need hardly be added that Mr. Manly did not regard the paying of surrender values as either unwise or unjust.

As a consequence of this earlier viewpoint the method of calculating surrender values was, in principle, though not always in detail, to ascertain the present net accumulations of the policyholder's payments, usually taken as the valuation reserve, or such reserve diminished by any unliquidated portion of initial expenses, and to deduct therefrom (*a*) a sum sufficient to cover the increase in the mortality burden which was supposed to result, and (*b*) a further sum to cover the difference between the outgoing insured's contribution to the expenses, and the actual reduction in expenses consequent upon his surrender. The anticipated future profits from interest and sub-tabular mortality were not charged to the

insured, as it was assumed these would be returned to him as dividends in the case of a mutual corporation, while in the case of a stock company, although the shareholders' portion of the profits was considered a legitimate ground for a further deduction, it was agreed that it would be impolitic for proprietary offices to quote consistently lower values than their mutual competitors.

It may be noticed that this system of assessing surrender values is practically a valuation of policies by the *prospective* method, as distinguished from the *retrospective* method which we will find to be the equivalent of the modern plan of calculation. A net premium valuation by the prospective method gives results identical with those by the retrospective method, but in other than a net premium valuation the results may differ widely. In the present case the premium valued is not the net premium $P_{[x]}$ only, but the same plus an addition of ϕ, where $(\phi + \theta)$ represents the future annual share of expenses to be borne by the policy if continued, and θ the annual expenses saved by the discontinuance of the policy. The life being assumed to be subject to mortality lower than the average, equal, say, to a newly-selected life, introduces a further variation in the policy-value, which thus becomes:

$$A_{[x+n]} - (P_{[x]} + \phi)\,a_{[x+n]} = (P_{[x+n]} - P_{[x]} - \phi)\,a_{[x+n]}.$$

Very rough and ready methods were commonly used in practice to arrive at the actual values, the general view being that if anybody suffered it must not be the remaining policyholders, i. e., the company.

Although the above refers more particularly to British companies, similar views were held on this side of the Atlantic, as witness the following remarks of Insurance Commissioner Merrill, of Massachusetts, quoted and approved by Mr. Sheppard Homans in 1896 (T. A. S. A., IV, 384):

"There certainly is an equity somewhere in this surrender charge matter, but so far no one seems to have reached it to the general acceptation. (1) Everyone agrees that in going out of the company in advance of the normal termination of his contract a premium-paying member deprives those remaining of the advantage of his promised contributions to future expenses and contingencies, as expressed by the loading on the premiums. (2) And it is probable that the lives remaining may, after a time, deteriorate and fall

below the average, since it must be assumed that impaired lives will not, as a rule, retire from the company, and thus a loss be incurred for which the persistent members should be compensated. (3) Then it is also considered that a measurable disadvantage results from the disturbance of investments or from the necessity of keeping a supply of idle money or highly convertible low-interest bearing assets to meet the calls for cash surrender values."

The justness of the above principles depends on the validity of the view of the life insurance policy as a contract for its maximum term. This view, I submit, cannot be entertained in America today. Legislative enactments clearly regard the policy as an alternative contract, such, for instance, as a lease for a term of years with the right of renewal at a pre-determined rental for a further term. The exercise of this right, and the failure to do so are equally normal fulfilments of the original contract, involving no penalty. Whatever value the option may have is, presumably, included in the rental for the initial term. If this view be accepted, that the holder of a policy is entitled, as a part of his contract, to insurance for the maximum period prescribed, or any less period he may select, it is evident that no longer must he be required to put the company in as good a position as if he continued his payments, but that he shall be charged merely with the cost to date of his insurance. As he is not breaking his contract by surrendering, he may reasonably ask that as accurate a calculation be made as the circumstances admit of, instead of suffering a considerable diminution in the amount he receives in order to be sure the remaining members do not suffer. Where, however, reasonable doubt exists, the decision should naturally favor the continuing members, on the ground that the primary object of the insurance is the payment of the sum insured.

In addition to more accurate accounting, the outgoing policyholder may demand that he be not charged with anything on account of future expenses, his proper share of the current expenses during the term of his insurance alone being chargeable. This leads to a statement of what I believe to be the true principles upon which surrender values should be determined, viz., the premiums paid accumulated at interest should be credited to the policyholder, who should be debited with the following items, each also accumulated at interest:

i. Expenses incurred in connection with the policy.

ii. Dividends paid or allotted.

iii. The cost of covering the risk (of death and deterioration).

The premium payments are, of course, ascertainable prior to the issue of the policy, at which time it is necessary to know the future surrender values, but none of the three debit items is individually capable of very accurate estimate beforehand. The collective effect of these three items, however, may be estimated with sufficient accuracy, and the apparent difficulty may therefore be overcome by choosing a suitable method of attacking the problem.

Let it be supposed that the premium paid by the insured is divided into two portions, the valuation net premium and the loading. Further, suppose that interest in excess of the valuation rate and savings from light mortality are transferred to the loading fund, when the net premium will be found to provide for the payment of the actual claims and the building up of the valuation reserve, leaving no surplus other than that transferred to the loading fund. Also suppose that the loading fund, incremented as described, is first used to pay the expenses under the policy, and that the exact balance is paid as dividend to the insured. Deviations from this ideal condition will be dealt with later; meantime, assuming the conditions named, the question arises, is the policyholder entitled to the full valuation reserve upon surrender?

This question must, I hold, be answered in the negative, and I am convinced that a careful examination of the subject will prove the correctness of such answer. We have assumed that expenses and dividends are exactly balanced by loadings and profits from interest and light mortality, and we may therefore assume that these are nonexistent, and deal with a fund consisting of the accumulations of the net premiums; mortality and interest being exactly in accordance with the valuation assumptions. Now, if $l_{[x]}$ persons take out ordinary life policies of a unit each at age x, at age $x+n$, a premium being due and unpaid, the fund will amount to

$$l_{[x]+n} \times {}_nV_{[x]} = l_{[x]+n}(A_{[x]+n} - P_{[x]}a_{[x]+n}),$$

and this fund, with subsequent premiums, will exactly meet all requirements in the future. If, however, a policyholder whose life is better than the average surrender, the value of an annuity due on his life being greater than $a_{[x]+n}$, say $a_{[x]+n}+\phi$, the value of premium income lost would be $P_{[x]}(a_{[x]+n}+\phi)$, while the relief resulting from the cancellation of the sum insured would be

$1 - d(a_{[x]+n} + \phi) = A_{[x]+n} - d\phi$, so that the net liability will be reduced by

$$A_{[x]+n} - P_{[x]}a_{[x]+n} - \phi(P_{[x]} + d)$$

to

$$(l_{[x]+n} - 1)(A_{[x]+n} - P_{[x]}a_{[x]+n}) + \phi(P_{[x]} + d),$$

while, ${}_nV_{[x]}$ being paid as surrender value, the fund would be reduced to

$$(l_{[x]+n} - 1)(A_{[x]+n} - P_{[x]}a_{[x]+n}),$$

i. e., to $\phi(P_{[x]} + d)$ less than the liability, and would therefore be insolvent.

We thus arrive at the conclusion that the persistent policyholders, who have paid the full amount necessary to provide for their insurances, could not have their claims paid in full. In practice, of course, the claims of the continuing insured would be met in full, the excess payment to the outgoing insured being met either out of the profits of those who remain, which would not lessen the injustice, or out of some other credit due to the policyholder surrendering.

The proper amount to be paid upon surrender to a person whose health is superior to the average is evidently

$${}_nV_{[x]} - \phi(P_{[x]} + d).$$

If the life in question be equal to a select life at age $x + n$ the above becomes

$${}_nV_{[x]} - (a_{[x+n]} - a_{[x]+n})(P_{[x]} + d)$$

$$= \frac{(a_{[x]} - a_{[x]+n}) - (a_{[x+n]} - a_{[x]+n})}{a_{[x]}}$$

$$= (P_{[x+n]} - P_{[x]})a_{[x+n]},$$

which Dr. Sprague has called the value of a policy on a life still healthy.

The necessity for the proposed deduction may, to some, be clearer if it be remembered that a person who pays a premium for several years receives not merely insurance for those years but also has the option of continuing. This option may be very valuable, as, for

instance, if the insured is on his deathbed, and it is evident that in making up the reserve this has not been charged for, as the reserves consist of the accumulations of net premiums, less only actual emerged claims, and not less the cost of emerged deterioration. In fact, we may regard the reserve as consisting of two portions, first, the amount that would be necessary were all the lives now select, and a further sum, which we may call the deterioration reserve, required to cover the loss occasioned by the inferior health of a section of the insured. If the whole of this section retain their insurances for the maximum term, the whole of the deterioration reserve must also be retained.

From the foregoing it would appear that the full valuation reserve should not be handed over to the outgoing policyholder, unless

(*a*) There is some other fund to his credit, from which an amount equal to the deterioration reserve may be deducted, or

(*b*) There is reason to suppose that the average prospects of longevity of the persons surrendering are no better than the average of those continuing their insurances.

Dealing with the second point first, it has been said that we have no definite evidence that surrenderers are better average lives than the generality of the insured. This I am prepared to dispute. It is true we cannot show a mathematical proof one way or the other, and further, that we have no means of tracing the mortality of the outgoing policyholders after the termination of their policies, but there is, nevertheless, no inconsiderable body of circumstantial evidence available.

In the first place I would call attention to the fact that wherever the insuring public have an opportunity of exercising selection favorable to themselves, and we *can* definitely discover whether or not the selection is exercised, we never find the opportunity wholly neglected. Thus we find that upon taking out policies the superior lives tend more to endowment than ordinary life insurances while the reverse is the case with inferior lives. Thus the recent experience of British offices showed low mortality under endowment insurances and very high mortality under term and ascending premium insurances. The latter, too, showed a heavy rise in mortality after entering on the full premium period—five or seven years after entry—when a considerable percentage of the business lapses.

Next, we find annuitants equally alive to their own interests in the taking out of annuities, bad lives refraining from this step, as is evidenced by the low mortality rates in the early annuity years. A further instance of selection, and one coming closer home to our subject, is afforded by the published experience of some companies under term extension insurance, the high mortality shown, and its special incidence in the first year of the extended insurance, indicating a very marked choice by the policyholders.

Lastly, and still more closely related to the question at issue, we have available specimen mortality experiences of policyholders who surrendered reversionary bonuses for cash, compared with those who allowed the additions to remain in reversion. Not only was the mortality of the "Cash" section lower than that of the "Reversionary," but it was very materially lower, as witness the figures of the British Empire Mutual (J. I. A., XXIII, 1), and the Washington Life experience published in 1889.

Against this formidable array of confirmatory evidence I can recall only two instances where arguments on the other side have been offered as based on facts. Possibly other cases might be cited, but I cannot, at present, recall any. The first of the two instances referred to was a vague statement to the effect that policies had been purchased as a speculation and the investment had proved successful on account of high mortality. As to this it may be noted, first, that the experience, if accurate (figures were not supplied), is one of policies *kept in force* to the detriment of the issuing companies, and, second, that the circumstances connected with the transactions render it probable that the sale would sometimes be effected upon evidence of damaged health. In the last place, the statement was made many years ago, when the means of keeping a policy on foot were vastly less favorable to the insured than now.

The other instance to which I refer is contained in a prize essay by Mr. J. Chatham (J. I. A., XXIX, 170–171). Two tables are given. In the first are shown, for quinquennial entry ages, rates of mortality (unadjusted) for the first eleven calendar years from issue, and the corresponding rates of withdrawal of ten Scotch and twenty British (including the ten Scotch) offices. These Mr. Chatham deemed not clearly indicative of any general result, so he combined the figures for ages 25 to 40 and obtained the table reproduced below.

Year of Insurance.	Unadjusted Rate of Mortality.		Unadjusted Rate of Discontinuance.		
	Ten Scotch Offices.	Twenty British Offices.	Ten Scotch Offices.	Twenty British Offices.	Percentage of Scotch to British.
0	.0045	.0038	.007	.027	25.9
1	.0061	.0061	.056	.068	82.4
2	.0085	.0081	.039	.049	79.6
3	.0090	.0092	.033	.041	80.5
4	.0113	.0108	.027	.032	84.4
5	.0104	.0101	.023	.028	82.1
6	.0116	.0111	.018	.024	75.0
7	.0107	.0107	.020	.034	58.8
8	.0107	.0113	.017	.019	89.5
9	.0113	.0118	.012	.015	80.0
10	.0130	.0133	.012	.015	80.0

Mr. Chatham's deduction from the above figures is that, as the Scotch mortality is, on the whole, heavier than the British in the early years when withdrawals are heaviest, the lives withdrawing are certainly not better, and not improbably are worse than those remaining. A careful examination of the figures as they stand, however, leads to a diametrically opposite conclusion. Excluding the last three years shown, when the Scotch rates are lower than the British, the difference is never so much as five per cent. except for the year "0," i. e., the calendar year of issue, in which, of course, the mortality is unaffected by the withdrawals. In the initial year, the British rates are so much lower than the Scotch, that it is evident that the British lives, for some reason, were better at the commencement than the Scotch, and the fact that after the first year the British rates level up to the Scotch indicates that the heavier withdrawal rates of the British experience adversely affect the mortality—a point referred to in the discussion of Mr. Chatham's paper by Mr. G. F. Hardy.

Mr. Chatham endeavored to combat this view by pointing out that five of the Scotch offices, as compared with one only of the English, terminated their financial years in the first half of the calendar year, while the financial years of three only of the Scotch, as against five of the English offices, coincided with the calendar year. As new business piles up at the end of the financial year, Mr. Chatham argued that the average exposure of the Scottish policies in the first calendar year was much greater than that of the British, even venturing the extraordinary assertion that, notwithstanding their great apparent superiority, the English insurants

were really worse than the Scotch at commencement. It is, perhaps, worth while to examine this argument. Assuming new business is written to the extent of 15 per cent. of the total in the first, 25 per cent. in the second, 25 per cent. in the third, and 35 per cent. in the fourth quarter of the financial year, I find that the policies of a company closing its financial year on December 31 are exposed on an average for 5.1 months in the calendar year of issue, while if the books close either on March 31, June 30 or September 30, the corresponding exposure is 6.3 months. We need, therefore, divide the offices into two classes only, those who close their books on December 31 and those who do not. Now, as three out of ten Scotch offices closed their books on December 31, we have, assuming of necessity all offices of the same size, an average exposure in the year of issue for these offices of 5.94 months. Of the British offices eight out of twenty closed on December 31, whence we have an average exposure of 5.82 months. The difference is thus only 2 per cent., which would account for a proportionate difference in the rates of mortality. The actual difference in these latter is 20 per cent., and we may fairly conclude that nine tenths of this difference cannot arise from the causes offered by Mr. Chatham as explaining the divergence. The rates of mortality for the year "0" are affected by the inaccurate treatment of the withdrawals in that year as not having been at risk at all, and an adjustment on this account would *reduce* the British rates by one per cent at least, as compared with the Scotch.

As a matter of fact the whole comparison is of doubtful validity. Some of the English business was more than a hundred years old, and a number of short term insurances were included, the effect of which cannot easily be estimated. My own conclusions are as follows:

1. The comparison is of doubtful value at best, the two sections not being comprised of strictly homogeneous material.

2. If the foregoing objection be disregarded as not materially affecting the results, the mortality clearly increased more rapidly where the withdrawal rate was heavier.

To recapitulate, my reasons for believing in the existence on the part of surrendering policyholders of selection adverse to the companies are:

1. The *a priori* probability thereof.

2. The existence of evidence of similar selection in all available germane statistics.

3. The absence of any evidence to the contrary.

In concluding my remarks on this particular phase of the subject, I would point out that, although I have spoken of selection being exercised by policyholders as a whole, I believe actual deliberate choice is made by only a small percentage. In the case of surrenders the selection is probably not exercised to any marked extent by those who give up their policies, but rather by the small section of policyholders in decidedly bad health, who contribute to the claims an altogether disproportionate percentage, and none of whom, practically, ever surrenders.

The foregoing discussion appears to me to warrant the conclusion that the probability of surrendering policyholders being better than average lives is so high that justice requires the assumption of the condition stated as a fact. Payment of the full valuation reserve as a surrender value would appear, therefore, to be justified only if condition (*a*) noted above exist, viz., that the company holds an accumulation out of the policyholder's contributions, in addition to the reserve, at least equal to the sum which I have called the deterioration reserve, and from which the company may make the required deduction upon surrender. With the manner in which such a fund may come into existence I shall now proceed to deal.

In order to avoid complications it was premised above that the amount of loading plus profits from interest and light mortality (and other sources, if any) were exactly balanced by expenses and dividends. The conclusions arrived at will still be valid, though the assumptions be not in accordance with fact, if an adjustment be made for the deviation of assumptions from fact. It will ordinarily happen that the loading plus profits from interest and light mortality will be very much less, in the first year of insurance, than the expenses, consequently on this account, which we may call the loading account, there will be a deficiency. This will form a deduction from the surrender value otherwise available, arrived at from a consideration of the nature of the valuation reserve, and will generally reduce such surrender value to zero. After the first year the balance on the year's transactions on the loading account will be in the other direction, and if the dividend allotment is less than such balance, as will usually be the case, the initial deficiency will be reduced. In process of time this deficiency will be replaced by a small surplus. That this will be the case is evident upon consideration, as the full net level premium reserve being assumed to be held,

the deficiency in respect of recently issued policies must be counterbalanced by an equal amount of surplus derived from the older policies. Generally the tendency of the dividend system will be to keep these surpluses and deficiencies as low as possible, so that the difference between the valuation reserve and the accumulations under a particular policy will be near the practicable minimum.

As this surplus or deficiency enters into the surrender value it must be ascertained prior to the issue of the policy, and it is therefore important to notice that, although several of the elements entering into such surplus or deficiency are very variable, the latter itself will be but little affected even by considerable changes in the elements, under any reasonable dividend system. The important factors are the amount of loading, a fixed quantity; the rate of expenses, which may usually be estimated fairly closely; the profit from excess interest and light mortality, both subject to considerable variation; and the rate of dividend, which also may vary considerably. If an estimate be made of all these variable factors, the resulting surplus or deficiency at the end of successive years of duration may be ascertained. If, now, expenses, interest and mortality vary from year to year from the estimate, and so place a greater or less sum into the dividend fund, and the dividend be increased or diminished by an exactly corresponding amount, the calculated surplus or deficiency will be unaffected. In practice the variation in dividend may not exactly balance that in the margins for each individual policy, but the difference will usually be insignificant. The resulting error in the calculated surplus or deficiency will be only a small percentage of the *difference* between the actual and estimated dividends, and may safely be ignored.

The calculations of these surpluses and deficiencies will follow, in principle, the method used by Mr. Weeks in ascertaining assets-shares. The results show what is due to, or due from the policy-holder on account of payments and benefits received, other than the sums set aside for reserve. Adding the portion of the reserve available, we arrive at the total surrender value.

The final result, as was mentioned above, is a valuation of the policy by the retrospective method. Strictly considered, no regard at all is paid to the future. The past payments, properly adjusted for interest, are subject to deductions on account of (*a*) dividends paid, (*b*) expenses incurred on account of the policy, (*c*) mortality costs paid, and (*d*) cost of deterioration emerged. With reference

to the last item, although the cash has not been paid yet the event determining the payment has occurred.

The principles which, it appears to me, should underlie the calculation of surrender values being sufficiently set forth, it will be proper now to add that individual differences of opinion will inevitably occur as to the weight to be allowed certain factors. In particular, unanimity in the manner of calculating the deterioration reserve cannot be looked for in the present state of our knowledge. Varying dividend systems exist, as do various opinions as to the proper assessment of new business and renewal expenses, and all these factors modify surrender values. If, however, I have succeeded in setting forth correct principles, it is unlikely that serious injustice would result from the estimates put upon the various factors by a properly qualified individual.

Two further points require attention, if only to anticipate criticism. It will be observed that I have made no mention of a contingency reserve. This I have done intentionally, as it seems to me that such reserve should not be regarded as a safety margin over the ordinary valuation reserve. Into the latter enter estimates of interest and mortality, and in both cases a margin is allowed for unfavorable movements. The contingency reserve has a different function, viz., that of providing for a possible depreciation in assets. The latter are invested in various securities of which the cash value is not fixed and unalterable, and in order to be sure the aggregate assets may realize the value now put upon them, a safety margin is necessary. The surrendering policyholder being paid in immediate cash has no claim to any part of the fund which insures that the value put upon assets not immediately realizable shall be made good.

It may be objected that the total of the surrender values, calculated as above proposed, will be less than the total assets, exclusive of the contingency reserve. This is of importance only if we suppose that all the policyholders surrender, in which case it would be proper to give special surrender values to those in ill health. The practicableness of the latter step need not be considered, as the case would not arise in practice. Two practical possibilities may, however, arise. Those who have had occasion to consider the business of a company, where the insurance in force is steadily diminishing, will usually, if not invariably, have found heavy mortality prevailing. The usual margin contained in an ordinary valuation reserve has been reduced, and may in due course disappear, special reserves

being necessary on account of excessive mortality. The setting aside of the deterioration reserve set free by policies being surrendered would automatically make the necessary provision.

Another practical possibility is a sudden and heavy demand for surrender values. In such a case it is likely that some of the less healthy lives will go with the stream, but the gain therefrom would probably be more than balanced by the loss resulting from the necessity for finding large sums in cash at short notice. In such event the contingency reserve might prove an insufficient margin, and any profit from the surrenders would assist in restoring the equilibrium. It would probably be a fair assumption that an increase in the surrenders, sufficient to reduce sensibly the average vitality of those discontinuing, would be accompanied by an abnormal loss upon realization of securities that would justify retaining the rates of surrender value based upon the normal vitality of surrenderers.

A New Annuity Experience.

BY

JOHN S. THOMPSON.

An investigation into the mortality among the annuitants of The Mutual Life Insurance Company of New York has just been completed, and the results of it are herewith presented. Only immediate whole-life annuities paid for in cash, not supplementary contracts issued in settlement of other policies, entered into the observations. The numbers were so small as to preclude the possibility of exhibiting the results of the investigation in the form of select tables; accordingly, only aggregate tables have been derived, which may serve as an indication that the present bases of annuity valuation can be revised with advantage. Separate classes were made of annuities issued on residents of the United States and Canada and those issued elsewhere,—for the most part in Europe and practically all in England and France, making a sufficiently homogeneous class to justify independent study,—in order to discover what difference, if any, existed between the vitality of these two classes of applicants for annuities; these groups will hereinafter be designated as "domestic" and "foreign," respectively.

Cards were written embodying the usual necessary data, and included the entire annuity business of the company. If several annuity contracts existed on the same life, only the oldest was retained for observation, all subsequent policies being excluded. The age at issue was taken as the age nearest birthday at date of entry, and the entrants of the year 1909 were the last included, observations terminating upon the policy anniversaries in the year 1910. The "exposed to risk" formula used in operation was

$$E_x = E_{x-1} + n_x - e_x - d_{x-1},$$

with

$$E_x = \Sigma n_x - \Sigma e_x - \Sigma d_{x-1}$$

for checking. The extent of the data will be seen from the following table:

Class	Entrants.	Years of Risk.	Deaths.
Domestic male.....	976	8,025	341
Domestic female.....	1,467	13,024	446
Foreign male.....	2,993	25,119	1,137
Foreign female.....	1,868	18,408	561

As the tables must necessarily be more or less closely related to McClintock's annuity tables, it was determined to relate the constants to be employed in the graduation of the unadjusted data to those by which the graduation of Mr. McClintock's tables was effected, assuming that Makeham's law was likewise applicable to the present experiences. Throughout the adjustments $\log_{10} c$ was taken as .04 and the assumed value of μ_x was

$$(A + \delta A) + (B + \delta B)c^x.$$

If E_x be the exposed to risk at age x, d_x the actual deaths and d_x' the expected deaths according to Mr. McClintock's table, we have

$$\begin{aligned} d_x &= (A + \delta A)E_{x+\frac{1}{2}} + (B + \delta B)c^{x+\frac{1}{2}}E_{x+\frac{1}{2}} \\ &= (A + Bc^{x+\frac{1}{2}})E_{x+\frac{1}{2}} + (\delta A)E_{x+\frac{1}{2}} + (\delta B)c^{x+\frac{1}{2}}E_{x+\frac{1}{2}} \\ &= d_x' + (\delta A)E_{x+\frac{1}{2}} + (\delta B)c^{x+\frac{1}{2}}E_{x+\frac{1}{2}}. \end{aligned}$$

In the establishment of two equations for the determination of δA and δB, several arrangements of the whole data were tried, only two of which are worthy of notice. In relation to the domestic male data which extend from ages 15 to 102, inclusive, the first method of forming the equations is as follows:

$$\sum_{x=15}^{x=58}(d_x - d_x') = \delta A \sum_{x=15}^{x=58} E_{x+\frac{1}{2}} + \delta B \sum_{x=15}^{x=58} c^{x+\frac{1}{2}}E_{x+\frac{1}{2}},$$

$$\sum_{x=59}^{x=102}(d_x - d_x') = \delta A \sum_{x=59}^{x=102} E_{x+\frac{1}{2}} + \delta B \sum_{x=59}^{x=102} c^{x+\frac{1}{2}}E_{x+\frac{1}{2}},$$

and the second

$$\sum_{x=15}^{x=102}(d_x - d_x') = \delta A \sum_{x=15}^{x=102} E_{x+\frac{1}{2}} + \delta B \sum_{x=15}^{x=102} c^{x+\frac{1}{2}}E_{x+\frac{1}{2}},$$

$$\sum_{x=15}^{x=102}\sum_{x=x}^{x=102}(d_x - d_x') = \delta A \sum_{x=15}^{x=102}\sum_{x=x}^{x=102} E_{x+\frac{1}{2}} + \delta B \sum_{x=15}^{x=102}\sum_{x=x}^{x=102} c^{x+\frac{1}{2}}E_{x+\frac{1}{2}}.$$

The results are:

	First Graduation.	Second Graduation.
δA	—.0001301	+.0024154
δB	—.0000161	—.0000204

and the comparative value of the results is shown by the following table, the expected deaths by Mr. McClintock's mortality table being submitted for comparison:

Ages.	Actual Deaths.	Expected Deaths.		
		Graduation No. 1.	Graduation No. 2.	McClintock Male Table.
40–49	10	8.74	10.42	9.64
50–59	31	27.85	30.57	32.30
60–64	41	32.22	33.55	38.38
65–69	50	52.15	52.57	63.02
70–74	64	67.29	66.18	81.87
75–79	62	65.30	63.12	79.98
80–89	72	78.92	75.25	96.52
90–102	8	9.78	9.26	11.70
Total	338	342.25	340.92	413.41

Not only is the total number of expected deaths by graduation No. 2 somewhat closer to the actual than in case of graduation No. 1, but the deviations of expected from actual in case of the individual groups above presented are smaller in every case but one. Method No. 2 was accordingly used throughout, although strictly speaking its applicability to the other three tables should have been similarly investigated. The method of obtaining δA and δB necessarily makes the expected deaths agree with the actual; the slight discrepancy noticeable above is due partly to the fact that only the data are exhibited from age forty upwards, whereas the entire table was used in deriving δA and δB, but more particularly to the fact that the expected deaths by the "standard" table—Mr. McClintock's Table—were for the purposes of the equation assumed to be calculated by the application of $\mu_{x+\frac{1}{2}}$ to the "exposed to risk" at the middle of the year, but were, in fact, calculated by the application of q_x to the "exposed to risk" at the beginning of the year; it would have been more consistent to use the same method in both cases, in which case closer agreement between actual and expected deaths might have been looked for, but the expected deaths as used were already on record, and were accordingly utilized.

Below are the tables of log l_x, q_x and a_x which were derived from the new experiences, folowed by comparisons of the rates of mortality and annuities at 3 per cent. interest, by the McClintock, British Offices and French Annuity Tables.

The effect of applying the new tables to the valuation of a company's business was investigated by applying the new factors as

well as the McClintock annuity factors to the model annuity office suggested and used by Mr. Ryan in the Journal of the Institute of Actuaries, Volume XXX, page 193. For the purpose of comparison the factor a_x with interest at 3 per cent. was considered appropriate, and on that basis the total liabilities in respect of the model office, according to the various standards, assuming the lives all male or all female, are:

McClintock male	$2,203,274
McClintock female	2,528,166
New Domestic male	2,439,184
New Domestic female	2,763,763
New Foreign male	2,200,527
New Foreign female	2,581,296

The lives existing at close of observation, at all ages in the present experience, are classified as follows:

	Number.	Percentage.
Domestic male	635	13.177
Domestic female	1,021	21.187
Foreign male	1,856	38.514
Foreign female	1,307	27.122
Total	4,819	100.000

The total amounts of annuities for these four classes not being immediately available, these proportions were applied to the reserves above derived, with the following results:

Approximate reserve by McClintock's tables on present sex-distribution of business:

Male annuities, domestic and foreign	$1,138,899
Female annuities, domestic and foreign	1,221,326
	$2,360,225

Approximate reserve by new basis, on present sex-distribution of business:

Male annuities, domestic	$ 321,411	$1,168,927
Male annuities, foreign	847,516	
Female annuities, domestic	585,558	1,285,652
Female annuities, foreign	700,094	
	$2,454,579	

The difference $94,354 between the two estimates of liability indicates that, for an average sex-distribution of business, the use of the above annuity tables would increase a company's liability in respect to its annuity business by about 4 per cent. of the reserve computed on the basis of McClintock's annuity tables.

log l_x.

Age.	Domestic Male.	Domestic Female.	Foreign Male.	Foreign Female
10	5.0000000	5.0000000	5.0000000	5.0000000
11	4.9956852	4.9991752	4.9968358	4.9992745
12	4.9913640	4.9983451	4.9936629	4.9985426
13	4.9870358	4.9975091	4.9904805	4.9978037
14	4.9827000	4.9966667	4.9872877	4.9970571
15	4.9783558	4.9958173	4.9840835	4.9963020
16	4.9740024	4.9949602	4.9808668	4.9955376
17	4.9696389	4.9940946	4.9776364	4.9947630
18	4.9652644	4.9932197	4.9743909	4.9939773
19	4.9608778	4.9923346	4.9711289	4.9931794
20	4.9564779	4.9914383	4.9678488	4.9923681
21	4.9520634	4.9905298	4.9645489	4.9915421
22	4.9476329	4.9896079	4.9612272	4.9907000
23	4.9431849	4.9886713	4.9578816	4.9898402
24	4.9387177	4.9877186	4.9545098	4.9889610
25	4.9342295	4.9867482	4.9511093	4.9880606
26	4.9297182	4.9857584	4.9476773	4.9871369
27	4.9251816	4.9847473	4.9442108	4.9861877
28	4.9206173	4.9837129	4.9407065	4.9852105
29	4.9160226	4.9826529	4.9371607	4.9842026
30	4.9113946	4.9815649	4.9335694	4.9831610
31	4.9067300	4.9804462	4.9299282	4.9820825
32	4.9020253	4.9792938	4.9262323	4.9809635
33	4.8972767	4.9781045	4.9224765	4.9798001
34	4.8924800	4.9768747	4.9186550	4.9785880
35	4.8876305	4.9756005	4.9147614	4.9773225
36	4.8827231	4.9742776	4.9107888	4.9759985
37	4.8777522	4.9729013	4.9067296	4.9746104
38	4.8727116	4.9714665	4.9025754	4.9731520
39	4.8675946	4.9699675	4.8983170	4.9716165
40	4.8623939	4.9683981	4.8939444	4.9699964
41	4.8571014	4.9667515	4.8894466	4.9682836
42	4.8517082	4.9650203	4.8848115	4.9664691
43	4.8462046	4.9631963	4.8800258	4.9645431
44	4.8405800	4.9612706	4.8750750	4.9624949
45	4.8348227	4.9592334	4.8699432	4.9603127
46	4.8289199	4.9570739	4.8646129	4.9579836
47	4.8228575	4.9547803	4.8590650	4.9554934
48	4.8166202	4.9523397	4.8532785	4.9528265
49	4.8101911	4.9497379	4.8472303	4.9499659
50	4.8035517	4.9469593	4.8408952	4.9468929
51	4.7966817	4.9439869	4.8342455	4.9435870
52	4.7895588	4.9408020	4.8272509	4.9400258
53	4.7821586	4.9373841	4.8198781	4.9361846
54	4.7744544	4.9337107	4.8120906	4.9320364
55	4.7664169	4.9297571	4.8038484	4.9275516
56	4.7580139	4.9254963	4.7951076	4.9226977
57	4.7492101	4.9208986	4.7858201	4.9174391
58	4.7399669	4.9159315	4.7759332	4.9117367
59	4.7302430	4.9105594	4.7653890	4.9055477
60	4.7199908	4.9047432	4.7541241	4.8988252
61	4.7091593	4.8984401	4.7420690	4.8915177
62	4.6976926	4.8916031	4.7291475	4.8835688
63	4.6855294	4.8841807	4.7152760	4.8749166
64	4.6726026	4.8761164	4.7003628	4.8654933
65	4.6588385	4.8673483	4.6843074	4.8552254
66	4.6441563	4.8578085	4.6669996	4.8440286
67	4.6284674	4.8474226	4.6483186	4.8318161

log l_x (CONTINUED).

Age.	Domestic Male.	Domestic Female.	Foreign Male.	Foreign Female.
68	4.6116747	4.8361089	4.6281319	4.8184890
69	4.5936717	4.8237779	4.6062942	4.8039397
70	4.5743416	4.8103315	4.5826463	4.7880503
71	4.5535564	4.7956621	4.5570135	4.7706915
72	4.5311757	4.7796517	4.5292043	4.7517216
73	4.5070456	4.7621709	4.4990087	4.7309851
74	4.4809973	4.7430778	4.4661965	4.7083116
75	4.4528458	4.7222169	4.4305153	4.6835142
76	4.4223881	4.6994176	4.3916883	4.6563880
77	4.3894017	4.6744929	4.3494120	4.6267083
78	4.3536427	4.6472378	4.3033536	4.5942287
79	4.3148436	4.6174274	4.2531482	4.5586791
80	4.2727111	4.5848152	4.1983957	4.5197633
81	4.2269236	4.5491309	4.1386574	4.4771566
82	4.1771285	4.5100781	4.0734523	4.4305029
83	4.1229391	4.4673318	4.0022529	4.3794117
84	4.0639425	4.4205357	3.9244809	4.3234549
85	3.9996628	4.3692990	3.8395022	4.2621631
86	3.9295903	4.3131933	3.7466215	4.1950216
87	3.8531661	4.2517489	3.6450765	4.1214660
88	3.7697774	4.1844507	3.5340312	4.0408775
89	3.6787523	4.1107339	3.4125691	3.9525776
90	3.5793541	4.0299793	3.2796852	3.8558223
91	3.4707749	3.9415079	3.1342775	3.7497958
92	3.3521290	3.8445752	2.9751378	3.6336037
93	3.2224451	3.7383649	2.8009412	3.5062652
94	3.0806583	3.6219819	2.6102350	3.3667049
95	2.9256010	3.4944447	2.4014264	3.2137437
96	2.7559928	3.3546772	2.1727690	3.0460887
97	2.5704299	3.2014994	1.9223478	2.8623223
98	2.3673730	3.0336175	1.6480630	2.6608901
99	2.1451343	2.8496129	1.3476123	2.4400877
100	1.9018632	2.6479301	1.0184713	2.1980463
101	1.6355305	2.4268636	0.6578720	1.9327168
102	1.3439113	2.1845433	0.2627794	1.6418524
103	1.0245660	1.9189186	$\bar{1}$.8298656	1.3229895
104	0.6748197	1.6277412		0.9734269
105	0.2917393	1.3085458		0.5902027
106	$\bar{1}$.8721088	0.9586292		0.1700693
107		0.5750275		$\bar{1}$.7094658
108		0.1544908		

q_x.

Age.	Domestic Male.	Domestic Female.	Foreign Male.	Foreign Female.
40	.01211	.00379	.01031	.00393
41	.01233	.00398	.01063	.00416
42	.01258	.00418	.01097	.00443
43	.01286	.00443	.01133	.00471
44	.01318	.00469	.01174	.00501
45	.01349	.00496	.01220	.00535
46	.01386	.00526	.01270	.00572
47	.01427	.00560	.01324	.00613
48	.01470	.00597	.01383	.00656
49	.01517	.00638	.01449	.00704
50	.01569	.00682	.01520	.00759
51	.01626	.00730	.01597	.00816
52	.01689	.00784	.01683	.00880
53	.01757	.00841	.01778	.00951
54	.01834	.00905	.01879	.01026
55	.01916	.00976	.01992	.01111
56	.02006	.01054	.02116	.01204
57	.02105	.01138	.02252	.01304
58	.02213	.01229	.02398	.01415
59	.02333	.01331	.02559	.01535
60	.02467	.01440	.02739	.01669
61	.02606	.01563	.02931	.01814
62	.02761	.01694	.03143	.01972
63	.02933	.01839	.03375	.02146
64	.03119	.01999	.03630	.02337
65	.03324	.02173	.03907	.02546
66	.03548	.02364	.04210	.02772
67	.03792	.02571	.04543	.03023
68	.04060	.02799	.04904	.03295
69	.04353	.03050	.05300	.03593
70	.04674	.03321	.05733	.03918
71	.05023	.03619	.06203	.04274
72	.05405	.03945	.06718	.04663
73	.05822	.04300	.07276	.05086
74	.06276	.04690	.07887	.05550
75	.06773	.05114	.08553	.06056
76	.07315	.05577	.09276	.06606
77	.07904	.06084	.10063	.07206
78	.08547	.06634	.10918	.07860
79	.09245	.07234	.11844	.08572
80	.10007	.07887	.12851	.09345
81	.10834	.08599	.13942	.10185
82	.11731	.09375	.15121	.11098
83	.12703	.10216	.16395	.12090
84	.13758	.11129	.17772	.13162
85	.14902	.12120	.19254	.14324
86	.16135	.13192	.20850	.15581
87	.17470	.14355	.22563	.16937
88	.18909	.15612	.24397	.18398
89	.20457	.16967	.26359	.19972
90	.22121	.18430	.28453	.21662
91	.23906	.20004	.30680	.23474
92	.25814	.21695	.33042	.25414
93	.27854	.23507	.35540	.27483
94	.29988	.25448	.38171	.29686
95	.32384	.27518	.40872	.32026
96	.34737	.29722	.43625	.34530
97	.37366	.32061	.47619	.37089

q_x (CONTINUED).

Age.	Domestic Male.	Domestic Female.	Foreign Male.	Foreign Female.
98	.39914	.34567	.50000	.39956
99	.42857	.37058	.54545	.42546
100	.46250	.40000	.50000	.45570

a_x, INTEREST AT 3 PER CENT.

Age.	Domestic Male.	Domestic Female.	Foreign Male.	Foreign Female.
40	17.676	20.492	17.456	19.955
41	17.430	20.187	17.167	19.635
42	17.177	19.875	16.872	19.309
43	16.918	19.558	16.571	18.977
44	16.652	19.234	16.263	18.638
45	16.381	18.904	15.950	18.294
46	16.103	18.569	15.632	17.944
47	15.819	18.227	15.308	17.589
48	15.530	17.879	14.978	17.228
49	15.234	17.526	14.644	16.862
50	14.933	17.168	14.305	16.492
51	14.626	16.805	13.962	16.116
52	14.314	16.436	13.614	15.736
53	13.997	16.063	13.262	15.352
54	13.675	15.685	12.907	14.965
55	13.348	15.304	12.549	14.574
56	13.017	14.918	12.189	14.180
57	12.682	14.529	11.826	13.783
58	12.344	14.137	11.461	13.384
59	12.002	13.743	11.095	12.983
60	11.657	13.346	10.728	12.582
61	11.310	12.947	10.361	12.179
62	10.961	12.547	9.994	11.776
63	10.611	12.146	9.628	11.373
64	10.259	11.745	9.263	10.972
65	9.907	11.344	8.900	10.571
66	9.555	10.944	8.540	10.173
67	9.204	10.545	8.183	9.777
68	8.854	10.148	7.829	9.384
69	8.506	9.754	7.480	8.994
70	8.159	9.362	7.136	8.609
71	7.816	8.975	6.797	8.229
72	7.477	8.591	6.463	7.855
73	7.141	8.212	6.137	7.486
74	6.810	7.839	5.817	7.124
75	6.484	7.471	5.504	6.769
76	6.163	7.110	5.200	6.421
77	5.849	6.756	4.903	6.081
78	5.542	6.409	4.615	5.750
79	5.241	6.071	4.336	5.428
80	4.949	5.740	4.067	5.115

RATES OF MORTALITY.

Age.	Mutual Life.		McClintock.	$O^{[am]}$	$O^{am(5)}$	RF*
	Domestic.	Foreign.				
Male Lives.						
40	.01211	.01031	.01056	.00471	.00992	.00834
50	.01569	.01520	.01542	.00821	.01544	.01275
60	.02467	.02739	.02750	.01658	.02853	.02411
70	.04674	.05733	.05721	.03637	.05926	.05298
80	.10007	.12851	.12790	.08220	.12904	.12403
Female Lives.						
40	.00379	.00393	.00590	.00402	.01044	.00834
50	.00682	.00759	.00960	.00611	.01533	.01275
60	.01440	.01669	.01884	.00935	.02179	.02411
70	.03321	.03918	.04166	.02351	.04358	.05298
80	.07887	.09345	.09668	.06574	.10793	.12403

ANNUITIES AT 3 PER CENT. INTEREST.

Age.	Mutual Life.		McClintock.	$O^{[am]}$	$O^{am(5)}$	RF*
	Domestic.	Foreign.				
Male Lives.						
40	17.676	17.456	17.410	17.604	17.379	18.114
50	14.933	14.305	14.286	14.403	14.156	14.844
60	11.657	10.728	10.731	10.882	10.585	11.116
70	8.159	7.136	7.151	7.441	7.069	7.355
80	4.949	4.067	4.084	4.537	4.094	4.140
Female Lives.						
40	20.492	19.955	19.321	18.257	17.987	18.114
50	17.168	16.492	16.031	15.514	15.197	14.844
60	13.346	12.582	12.275	12.231	11.902	11.116
70	9.362	8.609	8.423	8.406	8.042	7.355
80	5.740	5.115	5.013	5.054	4.610	4.140

* Sexes not distinguished.

A Theory of Sub-standard Lives.

BY

ALBERT W. WHITNEY.

The fundamental nature of the problem of rating is the same for every kind of insurance; the problem can be stated in mathematical language,—it is to express the hazard as a function of those certain elements that identify the risk. For instance here is a four-story brick building with wooden floor-joists and wooden cornice, enclosed elevator shafts, equipped with fire-pails, used for mercantile purposes, etc., etc., etc. Find a mathematical function that will describe its hazard in terms of these details, a function sufficiently general to serve to give the rate not only for this particular building but for all buildings or at any rate for buildings that are all of the same general character.

Here is an applicant for life insurance; he is 39 years of age, 20 pounds overweight; he has a good family record with regard to longevity but he has a slight heart murmur; he is a railroad brakeman in an unhealthful district of Louisiana and he uses intoxicating liquors. Find a mathematical function that will give the premium for an ordinary life insurance in terms of these details, and let it be sufficiently general to apply not merely to this risk but to any other risk.

The practical method of attacking the problem of rating will differ with each different kind of insurance. Schedule rating in fire insurance has grown up on an empirical basis as a method of assessing the variations from a standard risk. The method actually pursued may be said to be roughly equivalent to using the first term and the first derivative terms in a Taylor's expansion, and hence conditions of convergence limit the schedule in its application to risks that closely resemble the standard. This necessitates the employment of a large number of different schedules for different classes. Whether it is possible to analyze the nature of the function in fire insurance closely enough to allow of building it up *ab initio* and so using a single schedule is a question that has not been answered yet in practice, although in theory it seems possible.

The first step in the case of any kind of insurance is an analysis of the structure of the hazard; in fire insurance, for instance, this step consists in recognizing the elements of ignition, combustion, damage-production, exposure, protection, etc., and their logical relations to each other. In life insurance it is necessary in the same way to inquire into the nature of the tendency to die. In neither fire insurance nor life insurance can this inquiry be carried out upon a wholly empirical basis; it is necessary to use *a priori* reasoning. New truth must be focused by a theory.

It happens in the case of life insurance that a satisfactory theory of this kind already exists; I refer of course to Makeham's law of mortality. This formula in its application to life insurance has been regarded too exclusively merely as an empirical statement of facts; but Gompertz and Makeham did not so conceive of it.

I propose, instead of trying to formulate a new theory, to describe the causes of death, to make use of Makeham's hypothesis, backed up as it is by such abundant proof of its substantial agreement with fact.

Makeham assumes that the tendency to die, that is, the force of mortality, consists of two parts; one of these is a constant, that is, independent of the age; the other obeys the natural law of growth, as though the tendency to die were the effect of an organism planted in the system and multiplying according to the ordinary law governing increase of population. Gompertz and Makeham lived before the days of bacteriological research but their theory can hardly fail at least to suggest modern theories of disease.

Let us, following Makeham, put this theory into mathematical language.

$$\mu_z = \mu'_z + \mu''_z.$$

Let μ'_z be a constant, independent of z, say α; let μ''_z obey the law:

$$\frac{1}{\mu''_z}\frac{d\mu''_z}{dz} = \gamma.$$

γ, the rate of growth per unit of μ''_z, is a constant independent of the age z.

Integrating and determining the constant of integration by means of the initial condition: when $z = x$, $\mu''_z = \mu''_x =$ say β_x, we have:

$$\mu''_z = \beta_x e^{\gamma(z-x}$$

and therefore,

$$\mu_z = \alpha + \beta_x e^{\gamma(z-x)}. \tag{1}$$

To recapitulate: α is that part of the force of mortality that is independent of age; it may describe, for instance, the effects of environment, occupation, accident, infection, or, at any rate, some parts thereof. β_x is the value at age x of the growing part of the force of mortality; it will consist of tendencies, partly antenatal, that is, inherited, and partly the embodiment of the past life of the individual. γ is the rate per unit at which the tendency to death is growing; it may depend partly upon the effects of habits and environment and partly upon heredity and past life.

Since $\beta_x = \beta_0 e^{\gamma x}$, where β_0 is a constant, it is possible to use instead of β_x a canonical form β_0,—not the actual β of birth* but a β of quasi-birth, as though a man could be reborn into a condition which would automatically reproduce his present condition. In that case equation (1) would take the form:

$$\mu_z = \alpha + \beta_0 e^{\gamma z}.$$

Whether β_x or β_0 would be the more serviceable is a practical question which could be answered only when the detailed attempt was made to translate the findings of the medical examiner into parametric form. In any case the transition from one to the other would be easy.

Since

$$\mu_z = -\frac{1}{l_z}\frac{dl_z}{dz},$$

we have:

$$-\frac{1}{l_z}\frac{dl_z}{dz} = \alpha + \beta_x e^{\gamma(z-x)}.$$

Integrating and determining the constant of integration by means of the initial conditions: when $z = x$, $l_z = l_x$, we have:

$$-\log_e \frac{l_z}{l_x} = \alpha(z-x) + \frac{\beta_x}{\gamma}(e^{\gamma(z-x)} - 1).$$

Letting $z = x + n$, and observing that $\frac{l_{x+n}}{l_x}$ is ${}_np_x$, we have:

$$\log_e {}_np_x = -\alpha n - \frac{\beta_x}{\gamma}(e^{\gamma n} - 1). \tag{2}$$

* Makeham's law, as a matter of fact, does not purport correctly to express the facts regarding infant mortality.

α, β_0 and γ are connected with the ordinary constants, s, g and c, of the Makeham formula, by the relations:

$$\begin{aligned} \alpha &= -\log_e s \\ \beta_0 &= -\log_e g \log_e c \\ \gamma &= \log_e c \end{aligned} \tag{3}$$

The probability of living is now completely described in terms of α, β_0, γ and x, and hence, for actuarial purposes, a life may be symbolized by $(\alpha, \beta_0, \gamma, x)$. We will suppose, furthermore, that $(\alpha, \beta_0, \gamma, x)$ is a standard type of life for which tables have been prepared and, on the other hand, that $(\alpha', \beta_0', \gamma', x)$ is some other type of life—the non-standard life that we are to study—for which no tables exist.

Let us proceed for this non-standard life, $(\alpha', \beta_0', \gamma', x)$, to find, for instance, the value of an annuity, a_x. We may now formulate our problem along the lines that have just been suggested. The elements of the hazard are evidently the three parameters that describe the life, α', β_0', γ', the age, x, and the rate of interest, i. We must introduce these elements into a function that will be determined partly by the actuarial nature of the annuity and partly by the nature of Makeham's law.

We shall see into the nature of this function more clearly by considering the instantaneous annuity, $\bar{a}_x$, instead of a_x; for thus we shall have the advantage of dealing with an integral.
We have:

$$\bar{a}_x = \int_0^\infty e^{-\delta n}\, {}_np_x\, dn. \tag{4}$$

Here δ is the force of interest; that is,

$$e^{-\delta} = v = \frac{1}{1+i},$$

where i is the rate of interest.

From formula (2) we get:

$${}_np_x = e^{-\alpha n} e^{-\frac{\beta_x}{\gamma}(e^{\gamma n}-1)}. \tag{5}$$

Substituting this in formula (4) we have:

$$\bar{a}_x = e^{\frac{\beta_x}{\gamma}} \int_0^\infty e^{-\left(\frac{\alpha+\delta}{\gamma}\right)\gamma n} e^{-\frac{\beta_x}{\gamma}e^{\gamma n}}\, dn. \tag{6}$$

Let

$$\frac{\alpha+\delta}{\gamma}=h_i, \quad \frac{\beta_x}{\gamma}=b_x;$$

then:

$$\bar{a}_x = e^{b_x}\int_0^\infty e^{-h_i\gamma n}e^{-b_x e^{\gamma n}}dn.$$

Let

$$b_x e^{\gamma n}=z,$$

then

$$\gamma z dn = dz, \quad \text{and} \quad e^{-h_i\gamma n}=\left(\frac{z}{b_x}\right)^{-h_i}:$$

$$\therefore\ \bar{a}_x = \frac{e^{b_x}b_x^{h_i}}{\gamma}\int_{b_x}^\infty z^{-(h_i+1)}e^{-z}dz. \tag{7}$$

Formula (7) applies to the life, $(\alpha, \beta_0, \gamma, x)$, or, since we wish also to indicate the rate of interest used, we may say the life, $(\alpha, \beta_0, \gamma, x, i)$. Formula (7) is therefore:

$$\bar{a}_{(\alpha,\beta_0,\gamma,x,i)} = \frac{e^{b_x}b_x^{h_i}}{\gamma}\int_{b_x}^\infty z^{-(h_i+1)}e^{-z}dz,$$

and for the non-standard life we have:

$$\bar{a}_{(\alpha',\beta_0',\gamma',x,i)} = \frac{e^{b_x'}b_x'^{h_i'}}{\gamma}\int_{b_x'}^\infty z^{-(h_i'+1)}e^{-z}dz, \tag{8}$$

where the primes on h_i' and b_x' indicate that they are formed from α', β_0' and γ', instead of from α, β_0 and γ.

It will be noticed that

$$e^{b_x'}b_x'^{h_i'}\int_{b_x}^\infty z^{-(h_i'+1)}e^{-z}dz$$

is a function solely of h_i' and b_x'; represent it by $f(h_i', b_x')$; then:

$$\gamma'\bar{a}_{(\alpha',\beta_0',\gamma',x,i)} = f(h_i', b_x'). \tag{9}$$

A remarkably interesting and important principle now comes to light. Consider the standard life, $(\alpha, \beta_0, \gamma, x', i')$, differing however from the life, $(\alpha, \beta_0, \gamma, x, i)$, in age and rate of interest. Let us determine x' and i' so that the h_i' and b_x' for the life, $(\alpha', \beta_0', \gamma', x, i)$

shall be the same as the $h_{i'}$ and $b_{x'}$ for the life, $(a, \beta_0, \gamma, x', i')$, namely by the equations:

$$\begin{aligned} h'_i &= \frac{\alpha' + \delta}{\gamma'} = \frac{\alpha + \delta'}{\gamma} = h_{i'}, \\ b'_x &= \frac{\beta'_x}{\gamma'} = \frac{\beta'_0 e^{\gamma' x}}{\gamma'} = \frac{\beta_0 e^{\gamma x'}}{\gamma} = \frac{\beta_{x'}}{\gamma} = b_{x'}. \end{aligned} \tag{10}$$

Solving for δ' and x' we have:

$$\delta' = \frac{\gamma}{\gamma'}(\alpha' + \delta) - \alpha; \tag{11}$$

$$x' = \frac{\gamma'}{\gamma} x + \frac{\log \frac{\beta'_0}{\beta_0} + \log \frac{\gamma}{\gamma'}}{\gamma \log e}. \tag{12}$$

When δ' and x' are so determined the life, $(\alpha', \beta'_0, \gamma', x, i) \equiv$ the life, $(a, \beta_0, \gamma, x', i')$, in the sense of producing the same values of h and b and hence of $f(h, b)$.

Hence from formula (9) we shall have the relation:

$$\gamma' \bar{a}_{(\alpha', \beta'_0, \gamma', x, i)} = \gamma \bar{a}_{(a, \beta_0, \gamma, x', i')}. \tag{13}$$

This means in words that, for any sub-standard life, it is always possible so to choose the age and rate of interest for a *standard* life that the values of the instantaneous annuity for the two shall be the same except for a simple factor.

The instantaneous annuity value $\bar{a}_x$ may be reduced to the annuity value a_x by means of the relation:

$$\bar{a}_x = a_x + \tfrac{1}{2} - \tfrac{1}{12}(\mu_x + \delta).$$

This is an approximate formula derived from the theory of finite differences, but the series is very rapidly convergent so that the value as given above is an exceedingly close approximation.

Since $\mu_x = \alpha + \beta_x$, $\mu_x + \delta$ can be written, $\gamma(h_i + b_x)$, and we have:

$$\bar{a}_x = a_x + \tfrac{1}{2} - \frac{\gamma}{12}(h_i + b_x). \tag{14}$$

Substituting this value in formula (13) we have:

$$\gamma' \left\{ a_{(\alpha', \beta_0', \gamma', x, i)} + \tfrac{1}{2} - \frac{\gamma'}{12}(h_i' + b_x') \right\} = \gamma \left\{ a_{(\alpha, \beta_0, \gamma, x', i')} + \tfrac{1}{2} - \frac{\gamma}{12}(h_{i'} + b_{x'}) \right\}.$$

But since by hypothesis $h_i' = h_{i'}$ and $b_x' = b_{x'}$, we obtain the following:

$$a_{(\alpha', \beta_0', \gamma', x, i)} + \tfrac{1}{2} = \frac{\gamma}{\gamma'} \{ a_{(\alpha, \beta_0, \gamma, x', i')} + \tfrac{1}{2} \} + \frac{\gamma'^2 - \gamma^2}{12\gamma'}(h_{i'} + b_{x'}). \quad (15)$$

γ, it will be remembered, is the rate of growth per unit of the tendency to die and must undoubtedly be closely connected with habits; for example a man living a dissolute life is burning his life up faster than the ordinary man and this doubtless should be expressed by a larger γ. But dissolute, intemperate lives are exactly the lives that should be rejected by the insurance companies. It is not for the public good that a man with dissolute habits should be allowed to obtain insurance. On the other hand there are very strong sociological reasons why men in adverse environmental conditions ($\alpha' > \alpha$) and those who find themselves with impaired constitutions ($\beta_x' > \beta_x$) should be given insurance. There is reason to believe that the habits of such lives would in general be such that γ' would not be larger than the standard γ. We shall thus undoubtedly include nearly all the insurable cases if we limit ourselves to the lives for which $\gamma' = \gamma$.

In this case equations (11), (12) and (15) are very greatly simplified. Equation (15) becomes,

$$a_{(\alpha', \beta_0', \gamma', x, i)} = a_{(\alpha, \beta_0, \gamma, x', i')}, \quad (16)$$

and equations (11) and (12) become,

$$\delta' - \delta = \alpha' - \alpha, \quad (17)$$

$$x' - x = \frac{\log \frac{\beta_0'}{\beta_0}}{\gamma \log e}. \quad (18)$$

That is, in words: for a non-standard life of good habits ($\gamma' = \gamma$) it is possible by changing the age and the rate of interest to obtain

a *standard* life having the same annuity value. Furthermore, the increase in the age and the increase in the rate of interest are both independent of the age; not only that, but the increase in the environmental part of the tendency to die is entirely taken care of by the increase in the rate of interest, while, on the other hand, the increased value of the constitutional impairment is entirely taken care of by a change in the age. Here we have a theoretical justification for the not uncommon practice of *marking up* an impaired life; we have shown, however, not merely the structural correctness of this principle, but our formula gives the quantitative relation between degree of impairment and increase of age. It also shows that certain environmental conditions such as the hazard of occupation can be taken account of by *marking up the rate of interest.*

For the sake of an example let us suppose the following sub-standard conditions:

(1) $a' = 2a$, that is, the environmental force of mortality is double the standard.

(2) $\beta_0' = 2\beta_0$, that is, the constitutional force of mortality is double the standard.

(3) $\gamma' = \frac{5}{4}\gamma$, that is, the constitutional force of mortality is growing at a rate per unit 25 per cent. greater than the standard.

Let us study eight types of life, namely those having none, any one, any two, or all three, of these sub-standard conditions. They will be namely the following:

Case I	(a, β_0, γ), a standard life.
Case II	$(2a, \beta_0, \gamma)$.
Case III	$(a, 2\beta_0, \gamma)$.
Case IV	$(2a, 2\beta_0, \gamma)$.
Case V	$(a, \beta_0, \frac{5}{4}\gamma)$.
Case VI	$(2a, \beta_0, \frac{5}{4}\gamma)$.
Case VII	$(a, 2\beta_0, \frac{5}{4}\gamma)$.
Case VIII	$(2a, 2\beta_0, \frac{5}{4}\gamma)$.

A table is given herewith in which are given the increase in the rate of interest, $i' - i$, the increase in the age, $x' - x$, the annuity value, a_x, the single premium for an insurance, A_x, and the premium for an ordinary life insurance, P_x. The rate of interest used is 3 per cent. and the ages taken are 35 and 45. The two annuity values admit of the calculation of ${}_{10}V_{35}$ and this is given and also the ratio ${}_{10}V_{35}/P_{35}$.

For purposes of calculation formula (11) can be thrown, by means of equations (3), into a form involving i' and i instead of δ' and δ, as follows:

$$1 + i' = (1 + i)^{\frac{\gamma}{\gamma'}} s^{1-\frac{\gamma}{\gamma'}\frac{\alpha'}{\alpha}}. \tag{19}$$

For cases I to IV this reduces to:

$$1 + i' = (1 + i) s^{1-\frac{\alpha'}{\alpha}}.$$

For cases V to VIII this reduces to:

$$1 + i' = (1 + i)^{\frac{4}{5}} s^{1-\frac{4}{5}\frac{\alpha'}{\alpha}}.$$

The values of a_x for the example were got by simple interpolation from tables of a_x which, for several rates of interest, are given herewith. These tables were computed from the values of the constants, g, c, s, upon which Mr. Hunter's Makehamized American Experience Table are based, viz.,

$$\log c = .04579609,$$
$$\log s = -.003296862,$$
$$\log g = -.00013205,$$

p_x was calculated from the relation:

$$\log_e p_x = -\alpha - \frac{\beta_0}{\gamma}(e^{\gamma} - 1)e^{\gamma x},$$

which can be reduced to the following working formulas:

$$\begin{aligned} G_0 &= (-\log g)(c-1), \\ G_{x+1} &= cG_x, \\ \log p_x &= -(G_x - \log s). \end{aligned} \tag{20}$$

a_x was calculated from p_x by means of the recursion formula:

$$a_x = vp_x(1 + a_{x+1}).$$

It will be observed that it is necessary to go out to age 120 to obtain an initial value of a_x that has no significant figure in the last

place. The value of a_0 was checked (and this was of course a check upon all other values of a_x) by means of the following formulas:

$$a_x = \bar{a}_x - \tfrac{1}{2} + \frac{\gamma}{12}(h_i + b_x),$$

$$\bar{a}_x = \frac{e^{b_x} b_x^{h_i}}{\gamma} \int_{b_x}^{\infty} z^{-(h_i+1)} e^{-z} dz = \frac{1}{h_i \gamma} \left\{ 1 - e^{b_x} b_x^{h_i} \int_{b_x}^{\infty} z^{-h_i} e^{-z} dz \right\}. \quad (21)$$

This transformation is necessary in order to obtain an expansion which will represent the function for the value of h_i in question.

$$\int_{b_x}^{\infty} z^{-h_i} e^{-z} dz = \Gamma(1 - h_i) - \left\{ \frac{b_x^{1-h_i}}{1 - h_i} - \frac{b_x^{2-h_i}}{2 - h_i} + \frac{b_x^{3-h_i}}{2(3-h_i)} - \right\} \quad (22)$$

This holds for $h_i < 1$.

$$\Gamma(1 - h_i) = \frac{\Gamma(2 - h_i)}{1 - h_i}.$$

h_i is in general between 0 and 1 and hence $2 - h_i$ is between 1 and 2 and hence $\Gamma(2 - h_i)$ can be found from tables of $\log \Gamma$.

h_i and b_x are expressed in terms of g, c, s as follows:

$$h_i = \frac{\log(1 + i) - \log s}{\log c}, \quad b_x = \frac{c^x(-\log g)}{\log e}. \quad (23)$$

For computing a_0 we must use the value b_0, that is $-\log g / \log e$.

It will be noticed that the agreement between the table herewith given for 3 per cent. and Mr. Hunter's table of a_x for 3 per cent. is very close. They are both right but they are calculated on a different basis. Mr. Hunter's table is based on a radix; the table given herewith is calculated directly from the constants. This method was adopted, partly because it was somewhat easier and more direct for the present purpose, and partly because, in the case of a theory which is founded upon these constants, it seemed the more natural and desirable. Values of other functions can be obtained from the table of a_x by simple computation or by means of conversion tables.

To return to the example, let us suppose that the new age and the new rate of interest have been calculated for cases I–VIII by means of formulas (12) and (19), and, furthermore, that by interpolation the values of $a_{(\alpha, \beta_0, \gamma, x', i')}$ have also been found. For cases I–IV for which $\gamma' = \gamma$, $a_{(\alpha', \beta_0', \gamma', x, i)} = a_{(\alpha, \beta_0, \gamma, x', i')}$ and there-

fore the work is complete. For cases V–VIII, however, we must still apply formula (15). It will be found that in general the term

$$\frac{\gamma'^2 - \gamma^2}{12\gamma'}(h_{i'} + b_{x'})$$

will be small. For example, in cases V–VIII the largest value it has is about .002. If this term is neglected the formula is much simplified.

It must be remembered that in getting A_x and P_x from the values of a_x thus obtained, we must use the standard rate of interest i instead of the derived rate i'; in other words, the increased rate of interest must not be applied to A_x and P_x directly but only through the medium of a_x.

If the values of ${}_{10}V_{35}/P_{35}$ are examined carefully, it will be noticed that the influence of $(a' > a)$ is in the direction of lowering the proportion of P_x that goes to the reserve, while the influence of $(\gamma' > \gamma)$ is in the direction of increasing the proportion of P_x that goes to the reserve. This will be readily understood if one remembers that an increase of a operates immediately upon the death-rate while the effect of an increase of γ will be cumulative with the age.

The question may legitimately be raised as to whether Makeham's law will adequately describe the conditions in the case of an impaired life. In the first place it must be remembered that the law is not the law for an individual but for a mass. It is perfectly evident that for the individual, a, β_0 and γ do not remain constant. Impairment of a constitution by disease means an increase of β_x, and this could be produced under this theory only by an increase of γ; that is, when the upset comes, γ no longer remains constant but increases in an erratic way, and, at the moment of death from disease, γ would have to be thought of as infinite. After the upset, provided death does not occur, a new β_0 and a new γ emerge. Whether for this new—so to speak static—condition, taken not for the individual but for the mass, Makeham's law holds, is, of course, a matter for statistical inquiry.

We can answer, however, *a priori*, that it will certainly be found that for conditions not too abnormal it will be a close approximation.

Doubtless some conditions will be more successfully described by it than others. But it must be remembered that a method of this kind to be practical must not be too complicated. Doubtless some

pathological conditions would be better described by, for instance, a law that made γ a function of the age, but doubtless we should at the same time break up the simple property that Makeham's law possesses for the treatment of sub-standard lives as well as its property of handling multiple lives.

We now come to a quite different part of the subject—the determination of the quantitative values of the parameters, α, β, γ, for any particular life. For this purpose the actuary, the medical examiner and the statistician must meet on a common ground; the findings of the medical examiner and the statistician must be transcribed into parametric form.

While this part of the process will call for a high degree of a certain mixture of experience and intuition called *underwriting judgment* we are by no means ready to abandon our attempt at actuarial analysis; there are still several things to be said as to the form which these judgments will assume.

In the first place we observe that the force of mortality, μ, is a frequency, that is, a time-rate of probability, in structure, therefore, a fraction whose numerator is a probability and whose denominator is a time. The probability of dying during time dx is therefore $\mu_x dx$. This probability is got by adding the probabilities of death from many causes. These, by Makeham's theory, fall into two classes, those, in value αdx, which are independent of the age and those, in value $\beta_x dx$, which depend upon the age.

In order to examine more carefully into the structure of α and β, let us suppose that a life is characterized by the qualities, A, B, C, etc. For example, A means living in an unhealthful part of Louisiana, B means being a brakeman, etc. We wish to know the probability of death for a life so conditioned. Properly the values of α and β should be separately determined by statistical analysis for every complex of qualities, A, B, C, ... Practically this would be out of the question and so we are forced to make some simplifying assumption.

Let us suppose the probability of dying because of quality A is independent of the coexistence of quality B. For example, the probability of dying because of the supposed unhealthfulness of some part of Louisiana is largely independent of whether one is or is not a brakeman, and similarly, the probability of death because of being a brakeman is largely independent of whether one does or

does not live in Louisiana. But the probability of death because of being a brakeman would be very different in a country in which the cars and tracks were continually covered with slippery ice. Similarly, if the unhealthfulness of a certain locality were due to malaria, the likelihood of contracting the disease might be largely conditioned by the nature of the occupation.

In many cases, however, we shall be justified in assuming the independence in question, particularly since the complexity of the problem absolutely demands some simple assumption, and since the assumption that we have made is unquestionably the best for a first approximation.

Under this assumption the frequencies of death because of different causes combine additively. The probability of not dying during time dx from any of the causes, A, B, C, . . ., is the probability of not dying from cause A, multiplied by the probability of not dying from cause B, etc. Let us suppose for the sake of simplicity that causes, A, B, etc., contribute only to α, then the relation just given can be written:

$$1 - \alpha dx = (1 - \alpha_{(A)}dx)(1 - \alpha_{(B)}dx) \cdots . \tag{24}$$

where $\alpha_{(A)}dx$ is the probability of death from the environmental cause, A, etc. From this equation follows:

$$\alpha dx = \alpha_{(A)}dx + \alpha_{(B)}dx + \cdots - \alpha_{(A)}\alpha_{(B)}(dx)^2 - \cdots ,$$

or

$$\alpha = \alpha_{(A)} + \alpha_{(B)} + \cdots - \alpha_{(A)}\alpha_{(B)}dx - \cdots ,$$

and since dx is a differential, we have:

$$\alpha = \alpha_{(A)} + \alpha_{(B)} + \cdots . \tag{25}$$

That is, frequencies of death due to different causes unite additively.

We may therefore, except in cases where the independence is flagrantly lacking, form α and β by the addition of the components that arise from the different causes. It is easy enough, however, to give examples of cases in which this independence, and hence this additive property, does not obtain. A notable example is the conjunction of pregnancy and tuberculosis. The frequency of death in such a case because of this complex of hazards would be more than the probability of death because of pregnancy plus the probability

of death because of tuberculosis. In such cases as this it would be necessary to treat the complex of such interdependent qualities as a statistical unit, or else to make some assumption regarding the nature of the dependence.

The determination of γ is of less importance, since, as we have said, most insurable lives have the standard γ. γ is not a probability nor a frequency. The γ of a sub-standard life could, however, doubtless be obtained additively with a sufficient degree of accuracy.

γ, we observe, is a function of the qualities, A, B, C, . . ., and, if we think of these as defined in some quantitative way, we may expand this function and we shall get as an approximation: $\gamma' = \gamma$ plus the increase due to the increased value of A over the standard plus the increase due to the increased value of B over the standard, etc. The additive method of approximation would doubtless be sufficiently accurate, since in most practical cases $\gamma' - \gamma$ would be small.

It is very easy to pick out difficulties. One we may notice is that the causes of death which are commonly reported are the proximate causes and not the ulterior causes which characterize the life. In fact the transition from theory to practice is beset with difficulties; but that there should be difficulties is not peculiar—the knitting together of the ideal and the real is always difficult. The difficulty is, however, no greater in this case than in most other cases, certainly not as great as, for instance, in the case of the theory of the strength of materials. After statistics have been gathered and the medical examiner and the actuary have become familiar with the problem the parametric valuation of an impaired life should not show any great difficulty.

A TABLE FOR VARIOUS TYPES OF SUB-STANDARD LIVES.

		Rate of Interest 3 %.									
		Age 35.						Age 45.			
Case	$\alpha'\ \beta_0'\ \gamma'$	$i'-i$	$x'-x$	a_x	A_x	P_x	${}_{10}V_x$	${}_{10}V_x/P_x$	$x'-x$	a_x	A_x	P_x
I	$\alpha\ \beta_0\ \gamma$	.0	0	18.92	.4198	.02108	.1454	6.9	0	16.02	.5042	.02962
II	$2\alpha\ \beta_0\ \gamma$	.00785	0	16.98	.4763	.02649	.1305	4.9	0	14.63	.5446	.03483
III	$\alpha\ 2\beta_0\ \gamma$	.0	6.57	17.10	.4730	.02614	.1852	7.1	6.57	13.74	.5706	.03870
IV	$2\alpha\ 2\beta_0\ \gamma$	.00785	6.57	15.52	.5189	.03141	.1699	5.4	6.57	12.71	.6007	.04381
V	$\alpha\ \beta_0\ \frac{5}{4}\gamma$	−.00762	6.63	15.07	.5320	.03310	.2560	7.7	9.13	10.96	.6518	.05452
VI	$2\alpha\ \beta_0\ \frac{5}{4}\gamma$	−.00140	6.63	13.84	.5678	.03826	.2395	6.3	9.13	10.29	.6713	.05948
VII	$\alpha\ 2\beta_0\ \frac{5}{4}\gamma$	−.00762	13.21	12.99	.5927	.04238	.3116	7.4	15.71	8.63	.7196	.07474
VIII	$2\alpha\ 2\beta_0\ \frac{5}{4}\gamma$	−.00140	13.21	12.06	.6195	.04742	.2962	6.2	15.71	8.19	.7322	.07964

A TABLE OF a_x FOR VARIOUS RATES OF INTEREST.

x	2 Per Cent.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.	7 Per Cent.
0	30.5040	24.2765	19.9392	16.8080	14.4728	12.6803
1	30.3522	24.1962	19.8955	16.7835	14.4586	12.6718
2	30.1964	24.1129	19.8498	16.7576	14.4435	12.6627
3	30.0363	24.0266	19.8020	16.7303	14.4274	12.6529
4	29.8719	23.9371	19.7519	16.7015	14.4103	12.6424
5	29.7031	23.8444	19.6996	16.6711	14.3921	12.6311
6	29.5298	23.7483	19.6449	16.6391	14.3728	12.6191
7	29.3518	23.6487	19.5877	16.6053	14.3522	12.6062
8	29.1692	23.5455	19.5279	16.5697	14.3303	12.5923
9	28.9816	23.4386	19.4654	16.5321	14.3071	12.5775
10	28.7891	23.3279	19.4000	16.4925	14.2824	12.5617
11	28.5915	23.2131	19.3317	16.4508	14.2562	12.5448
12	28.3888	23.0943	19.2604	16.4068	14.2284	12.5266
13	28.1807	22.9713	19.1859	16.3605	14.1989	12.5073
14	27.9672	22.8439	19.1080	16.3118	14.1675	12.4865
15	27.7482	22.7120	19.0268	16.2604	14.1342	12.4644
16	27.5235	22.5755	18.9419	16.2064	14.0989	12.4407
17	27.2931	22.4343	18.8533	16.1495	14.0615	12.4155
18	27.0568	22.2881	18.7609	16.0896	14.0218	12.3885
19	26.8145	22.1369	18.6644	16.0267	13.9798	12.3597
20	26.5661	21.9805	18.5638	15.9605	13.9352	12.3290
21	26.3115	21.8188	18.4589	15.8910	13.8881	12.2963
22	26.0506	21.6517	18.3495	15.8179	13.8381	12.2614
23	25.7833	21.4789	18.2356	15.7411	13.7853	12.2242
24	25.5095	21.3003	18.1168	15.6604	13.7294	12.1846
25	25.2291	21.1159	17.9931	15.5758	13.6702	12.1425
26	24.9421	20.9255	17.8643	15.4870	13.6078	12.0977
27	24.6482	20.7289	17.7303	15.3939	13.5418	12.0500
28	24.3475	20.5260	17.5909	15.2962	13.4721	11.9993
29	24.0399	20.3167	17.4459	15.1939	13.3986	11.9455
30	23.7254	20.1008	17.2951	15.0867	13.3210	11.8883
31	23.4038	19.8783	17.1385	14.9746	13.2393	11.8277
32	23.0752	19.6490	16.9758	14.8572	13.1531	11.7634
33	22.7395	19.4129	16.8070	14.7345	13.0624	11.6953
34	22.3967	19.1697	16.6318	14.6062	12.9670	11.6232
35	22.0468	18.9196	16.4501	14.4722	12.8667	11.5469
36	21.6898	18.6623	16.2619	14.3324	12.7612	11.4662
37	21.3258	18.3978	16.0669	14.1865	12.6505	11.3809
38	20.9547	18.1261	15.8651	14.0345	12.5344	11.2909
39	20.5768	17.8472	15.6564	13.8762	12.4126	11.1960
40	20.1920	17.5610	15.4406	13.7114	12.2850	11.0960
41	19.8005	17.2675	15.2178	13.5400	12.1515	10.9907
42	19.4024	16.9669	14.9879	13.3619	12.0119	10.8799
43	18.9978	16.6590	14.7507	13.1771	11.8661	10.7635
44	18.5871	16.3441	14.5064	12.9853	11.7139	10.6413
45	18.1703	16.0222	14.2550	12.7867	11.5552	10.5132
46	17.7477	15.6934	13.9964	12.5810	11.3900	10.3791
47	17.3196	15.3580	13.7307	12.3684	11.2181	10.2387
48	16.8863	15.0160	13.4580	12.1488	11.0395	10.0920
49	16.4482	14.6677	13.1784	11.9222	10.8541	9.9390
50	16.0055	14.3134	12.8922	11.6887	10.6620	9.7795
51	15.5588	13.9534	12.5993	11.4485	10.4631	9.6135
52	15.1084	13.5879	12.3002	11.2015	10.2575	9.4410
53	14.6549	13.2175	11.9950	10.9481	10.0454	9.2620
54	14.1987	12.8423	11.6841	10.6883	9.8267	9.0766
55	13.7404	12.4630	11.3677	10.4224	9.6017	8.8848
56	13.2806	12.0799	11.0462	10.1507	9.3705	8.6868

A TABLE OF a_x FOR VARIOUS RATES OF INTEREST (continued).

x	2 Per Cent.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.	7 Per Cent.
57	12.8199	11.6937	10.7201	9.8736	9.1333	8.4826
58	12.3588	11.3048	10.3898	9.5913	8.8905	8.2725
59	11.8982	10.9138	10.0559	9.3042	8.6424	8.0567
60	11.4386	10.5213	9.7187	9.0129	8.3892	7.8355
61	10.9808	10.1281	9.3790	8.7178	8.1314	7.6092
62	10.5255	9.7347	9.0373	8.4194	7.8695	7.3782
63	10.0735	9.3419	8.6942	8.1182	7.6039	7.1428
64	9.6255	8.9505	8.3505	7.8149	7.3351	6.9036
65	9.1823	8.5611	8.0067	7.5102	7.0637	6.6609
66	8.7447	8.1745	7.6637	7.2046	6.7904	6.4154
67	8.3134	7.7915	7.3222	6.8988	6.5156	6.1676
68	7.8892	7.4128	6.9829	6.5936	6.2401	5.9180
69	7.4729	7.0393	6.6465	6.2897	5.9645	5.6674
70	7.0651	6.6716	6.3139	5.9878	5.6896	5.4164
71	6.6665	6.3105	5.9857	5.6886	5.4161	5.1656
72	6.2778	5.9567	5.6627	5.3929	5.1446	4.9157
73	5.8996	5.6109	5.3456	5.1014	4.8759	4.6674
74	5.5325	5.2737	5.0352	4.8148	4.6107	4.4215
75	5.1770	4.9458	4.7319	4.5338	4.3497	4.1785
76	4.8336	4.6276	4.4366	4.2590	4.0935	3.9392
77	4.5026	4.3198	4.1497	3.9910	3.8429	3.7042
78	4.1843	4.0227	3.8717	3.7305	3.5983	3.4742
79	3.8792	3.7367	3.6032	3.4780	3.3604	3.2497
80	3.5873	3.4621	3.3445	3.2338	3.1296	3.0314
81	3.3089	3.1993	3.0960	2.9986	2.9066	2.8196
82	3.0440	2.9483	2.8579	2.7725	2.6915	2.6148
83	2.7925	2.7093	2.6305	2.5558	2.4849	2.4176
84	2.5545	2.4824	2.4140	2.3489	2.2870	2.2281
85	2.3298	2.2676	2.2084	2.1519	2.0981	2.0467
86	2.1183	2.0648	2.0137	1.9649	1.9182	1.8736
87	1.9197	1.8738	1.8299	1.7879	1.7476	1.7090
88	1.7337	1.6945	1.6569	1.6209	1.5863	1.5531
89	1.5601	1.5267	1.4946	1.4638	1.4342	1.4057
90	1.3984	1.3701	1.3429	1.3166	1.2914	1.2670
91	1.2483	1.2244	1.2013	1.1791	1.1576	1.1369
92	1.1094	1.0892	1.0698	1.0510	1.0329	1.0153
93	.9812	.9643	.9480	.9322	.9169	.9021
94	.8633	.8492	.8355	.8223	.8095	.7971
95	.7552	.7435	.7322	.7211	.7104	.7000
96	.6566	.6469	.6375	.6284	.6195	.6108
97	.5669	.5589	.5512	.5436	.5363	.5291
98	.4857	.4792	.4728	.4666	.4606	.4547
99	.4126	.4073	.4021	.3971	.3922	.3874
100	.3471	.3429	.3387	.3346	.3306	.3267
101	.2890	.2855	.2822	.2789	.2757	.2725
102	.2376	.2349	.2322	.2296	.2270	.2245
103	.1927	.1905	.1884	.1864	.1844	.1824
104	.1538	.1522	.1505	.1489	.1474	.1459
105	.1207	.1194	.1181	.1169	.1157	.1145
106	.0927	.0918	.0908	.0899	.0890	.0881
107	.0696	.0689	.0682	.0675	.0669	.0662
108	.0509	.0504	.0499	.0494	.0489	.0484
109	.0361	.0358	.0354	.0351	.0347	.0344
110	.0248	.0245	.0243	.0241	.0238	.0236
111	.0163	.0162	.0160	.0159	.0157	.0156
112	.0103	.0102	.0101	.0100	.0099	.0098
113	.0062	.0061	.0061	.0060	.0060	.0059

A TABLE OF a_x FOR VARIOUS RATES OF INTEREST (continued).

x	2 Per Cent.	3 Per Cent.	4 Per Cent.	5 Per Cent.	6 Per Cent.	7 Per Cent.
114	.0035	.0035	.0035	.0034	.0034	.0034
115	.0019	.0019	.0019	.0018	.0018	.0018
116	.0009	.0009	.0009	.0009	.0009	.0009
117	.0004	.0004	.0004	.0004	.0004	.0004
118	.0002	.0002	.0002	.0002	.0002	.0002
119	.0001	.0001	.0001	.0001	.0001	.0001
120	.0000	.0000	.0000	.0000	.0000	.0000

Legal Notes.

BY

WENDELL M. STRONG.

Corporation Tax:—(Mutual Benefit Life Ins. Co. *vs.* H. C. Herold, Collector, U. S. District Ct. for the District of N. J., 198 Fed. Rep. 199.) The situation under the Federal Corporation Tax with regard to dividends is at best an anomalous one. The premium rates on non-participating business are lower than on participating, because in the latter the policy receives back, in some form, whatever excess is charged over what proves to be required. It would seem then that the amount to be reckoned as income should be the amount paid to the company less what is returned as dividend. Congress, however, did not allow dividends "paid" as a deduction.

Dividends, however, may not be paid, but may be used in reduction of premium, or taken in the form of additions to the policy. Under the ruling made in the administration of the Corporation Tax no account has been taken of this, and the tax has been levied on the premiums which might have been received rather than what actually were received. In other words where, for instance, dividends have been applied in reduction of premium the company has been taxed on the full premium which by the contract it might have demanded if there had been no dividend credit, but which it did not demand and did not receive. How extreme this is may be seen when we consider that premiums in a participating company may be so reduced by dividends that less actual cash is received for a given insurance than on non-participating policies. Nevertheless, the ruling followed levied a much higher tax on the participating insurance by using the basis of the full contract premium. The Mutual Benefit resisted this ruling on the ground that it was not authorized by the statute.

The claim of the Mutual Benefit was that in determining the tax the dividends available to reduce premiums should not be treated as cash income and, consequently, should not be taxed. Whether this claim was justifiable depended upon the statute which

provided for the tax. The deciding clauses in the statute are the clauses providing for the income to be taxed; they are two: (*a*) "upon the entire net income over and above $5,000 received by it," and (*b*) (describing deductions allowable) "and, in the case of insurance companies, the sums other than dividends paid within the year on policy and annuity contracts and the net addition, if any, required by law to be made within the year to reserve funds."

The court in its opinion noted the rule of interpretation that statutes providing for the imposition of taxes are to be construed strictly, and that their provisions are not to be extended by implication beyond the clear import of the language used. In the clauses quoted above the word "received" is used in the first with reference to income, and in the second clause, which describes allowable reductions, the word "paid" is used. It would seem from such wording that the dividends which could not be deducted were dividends paid, and dividends which are simply deducted from premiums are not "paid." Again, if we turn to the first clause and the word "received," we could not, even if the interpretation were not very strict, consider that a hundred dollars was received by the company for a second year's premium when only $90 was demanded from the policy holder and he was informed that the remaining $10 of the premium which the company could, under the contract, demand from him, was covered by the dividend. In regard to such points the court spoke in part as follows:

"Hence, when it refers to dividends 'paid' it means dividends paid, and not an application of excess premium payments in abatement or reduction of subsequent premiums. The word 'paid,' as used by Congress, is highly significant. It clearly shows that it had cash payments in mind. Not only does this appear from its inherent meaning, but by its use in other clauses of the section providing for deductions from gross incomes. The expression 'gross income,' as used in the act, means gross cash receipts and the deductions which were directed to be made therefrom, in order to ascertain net 'income' 'received,' were deductions of cash expenditures. The principle of cash receipts and cash expenditures underlies the structure of the entire section. To hold that 'paid' has a different meaning when applied to dividends from that given it in several clauses of the immediate context would be unwarranted. It should be held to mean dividends which have been paid in cash during the year, and repaid to the company as premiums. Counsel for the complainant speaking of such dividends well say, 'unless so received and paid back to the company, they do not

constitute "income" "received," the question of income is to be determined not by what the parties might do, but by what they do do.' If, therefore, a policy holder by the express provision of his policy elects to have a previous over payment of premium applied in reduction of a succeeding stipulated premium, what he pays, and all that he pays, or can be required to pay, is the reduced premium, and that is all that the company received by way of income and all that it is liable to be taxed for. Such a construction of the act in no wise contravenes its purpose, which was to subject to taxation cash dividends, which, as statistics show, form a very large item in insurance business."

Two other questions of not as great but still of considerable importance were also decided in this case. Of these the first was, whether the increase in reserve on supplementary contracts could be used as a deduction in determining the net income taxable. Such reserves are required by law, and it would seem that they should, without question, be included with other reserves in determining the "net addition to reserve funds," and the court so held. The second point is whether, "for the purpose of taxation, the corporation's statement should be made on a cash or a revenue basis, or, stated in another way, whether uncollected and deferred premiums and interest accrued and due but not actually received, are proper subjects of taxation." The court took notice of the fact that insurance departments of the states require the returns to be made on a cash basis, and while they require a report of uncollected and deferred premiums do not allow them to be entered upon the books as assets. It also took notice that the deductions allowed from gross income are amounts "paid" or amounts "received." In consequence of these and certain other considerations it held that a cash basis was intended and that, in consequence, uncollected and deferred premiums could not be included to determine the income taxable.

Dividend Estimates:—(Grange *vs.* Penn Mut. Life Ins. Co., Supreme Ct. of Pa. 84 At. Rep. 392.) The case was that of a bill in equity praying for specific performance of the contract in accordance with its terms, and for an accounting, and for a discovery in aid of plaintiff's proof. The grievance of the plaintiff was that the surplus at the end of the distribution period had not come up to the amount estimated. Besides verbal estimate the insured received from the assistant secretary a letter inclosing an

estimate of $7,800 and saying that the company did not furnish inflated estimates. The court said in regard to this, in part:

"It will not do to construe the contract in this case as an agreement by which the company was bound to guarantee to appellant a certain definite amount of surplus. That was something which, from the circumstances, the future alone could determine. It depended, for one thing, largely upon the number of lapsed policies, which could not be foretold. The company is a mutual one, and in its accumulations all its policy holders had the right to share in the proportions fixed by the terms of their contracts. Whatever representation may have been made to appellant, he is and can be entitled to nothing more than his proportionate share of the surplus which actually accrued. It is obvious that a mutual insurance company cannot discriminate among its policy holders, and any agreement which would result in the payment of larger proportionate dividends to one of its policy holders than to others in the same class would be illegal and void. . . . That the amount which appellant was to derive from surplus and accumulations was merely an estimate appears plainly from the language of the option, as set forth in the policy."

In the latter part of the first extract above given a point is made which is too often overlooked in considering the question of the right of an individual policy holder, that is whether to grant his demand would be discrimination against other policy holders. The second of the above extracts also brings out an important point; that is, that the policy showed clearly by its language that any statement of amount of surplus was merely an estimate. Practically all participating policies have shown this clearly, and yet, in certain cases, like that of Timlin *vs.* the Equitable in Wisconsin, this clear statement has been brushed aside as of no importance.

As regards the question of an accounting the court spoke as follows:

"If there was anything in the evidence to indicate bad faith or serious error in the distribution of the surplus, or abuse of their discretion by the trustees, it would, of course, be the duty of the courts to give relief to an aggrieved policy holder. But, in the absence of any such showing, we see no reason to justify any interference with the exercise of the discretion conferred by the charter of the defendant company upon its board of trustees. We do not understand that defendant refuses to set forth the basis upon which the sum tendered to appellant is made up. No averment of mistake or miscalculation in making up the amount is made. It is

the methods pursued in creating the surplus fund, which was to be accumulated and divided among the policy holders to which plaintiff belongs, to which exception is taken. If those methods were right, and if the company was justified in their adoption and application, the details of the calculation are not questioned by plaintiff."

This derives more importance because of the later decision in White *vs.* Provident Life & Trust Co. by the same court, in which an accounting was granted.

ACCOUNTING:—(White *vs.* Provident Life and Trust Co., Supreme Ct. of Pa. Not yet reported.) In contrast to the Grange case an accounting was granted in the present case. It will be seen, however, that the circumstances are entirely different and the decisions are consistent with each other.

The Act of Incorporation of the company provided "That all net profits to be derived from the business of life insurance, after deducting the expenses of the company, shall be divided *pro rata* among the holders of the policies of such life insurance, equitably and ratably, as the directors of such company shall and may, from time to time, ascertain, determine, and report the same for division." According to the testimony of the President, endowment policies received, in addition to the annual dividends, a dividend at maturity amounting to one-tenth of one per cent. a year of the face of the policy. This dividend at maturity was admitted by him to be arbitrary and not the result of calculation.

Thus we have in this case two peculiar features. First, the provision quoted from the Act of Incorporation regarding the division of the surplus, and, second, the fact, explicitly admitted in the testimony, that the division as made was arbitrary and not the result of calculation. Under such circumstances there was no reason to believe that the dividend declared was the amount to which the policy holder was equitably entitled. The determination of what his share should have been could be obtained only by an accounting, and the court held that he was entitled to this.

It would seem that this case can have little bearing in general upon the results in other cases where bills for accounting are filed. In the first place, the act quoted above governing dividends placed this company in a different position from most companies, since it is generally specifically provided, in charter or otherwise, that

the determination of dividends lies within the discretion of the directors. The courts have pretty uniformly held that where this discretion is properly exercised with a view to making an equitable distribution, the court should not go back of the result. In the second place, there is probably no other case on record in the courts of this country in which the method of ascertaining the dividends was admittedly arbitrary and without reference to any attempt to ascertain the contribution of the policy to surplus. Not only is this true of reported cases but, so far as is known, practically all companies do make careful calculations to determine what the proper dividends should be.

The attitude of the Supreme Court of Pennsylvania can be judged by comparing the present case with the preceding one of Grange *vs.* Penn Mutual in which an accounting was denied. In that case there was nothing to show "bad faith or serious error in the distribution of surplus or abuse of their discretion by the Trustees."

Date of Policy:—(Nicoud *vs.* N. Y. Life Ins. Co., Supreme Ct., Appellate Division, N. Y., 134 App. Div. 937.) The question of the date of the policy, and, depending thereon, the due date of the second annual premium determined whether the policy was in force at the time of the death of the insured. The application stated "unless otherwise agreed in writing the policy shall then relate back to and take effect as of the date of this application." The policy was dated February 26, 1904, which was the date of the application, and stated clearly that it took effect "as of the twenty-sixth day of February, nineteen hundred and four," and that the sum paid constitutes "payment for the period terminating on the twenty-sixth day of February, nineteen hundred and five." The policy was not delivered and paid for until May 20, 1904. The agent who had been only temporarily in New York had left the policy with a clerk in the medical department for delivery, and it was claimed that this clerk agreed with the plaintiff, insured's wife, that the date should be really May 20, instead of February 26, and the second premium not payable until May 20. Due notice was received from the company stating that the next premium would be due February 26, 1905, and the clerk was again consulted, and, it is claimed, told plaintiff that she did not have to pay until May. Thus the statements of this clerk were relied on to

change the conditions of the written contract and this, notwithstanding the fact that the application had contained a distinct statement that the company would not be bound by any statements by the soliciting agent unless they were reduced to writing and presented to the company in the application. The policy further stated "Only the President, a Vice-President, a Secretary or the Treasurer has power on behalf of the Company to make or modify this or any contract of Insurance or to extend the time for paying any premium, and the Company shall not be bound by any promise or representation heretofore or hereafter made, unless made in writing by one of said officers." In the face of all this the agreements made by the medical clerk were relied on. The court held that the contract expressed clearly what its terms were and that there was no mutual mistake or modified contract or a new contract, and expressed strongly the opinion that the clerk whose advice was relied on had no authority nor semblance of authority to modify the contract.

Wife's Policy: Vested Right of Beneficiary:—(Bradshaw *vs.* Mutual Life Ins. Co., N. Y. Ct. of Appeals, 205 N. Y. 467.) The policy involved was a "wife's" policy "for her sole use if living in conformity with the statute and if not living to their children or their guardian for their use." The insured's wife died and, there being no children, insured wrote the company that he wished the policy made payable to his estate. He received a request for a certain affidavit setting forth the facts of death, etc., which was furnished. A letter was written to him in the name of a "General Agent" by Pratt, "cashier," which included the statement "The records of the company will show the fact that the policy has been made payable to your estate." No change was ever made in the company's records. The executors of the insured brought suit for the amount of the policy. The court held that the policy was vested in the wife and, consequently, passed under her will, and that neither insured nor the company could change this; furthermore, that the mistake of the agent in his letter did not change this or create a new contract. The decision was by a divided court. An extract from the opinion is:

"If we might assume, which I gravely doubt, that an agent could commit the defendant to a new and different liability, the letter which is relied upon could not alter the existing contract,

and it did not effect a new one. If the assured supposed that the policy could be made payable to his estate his ignorance of the law would not excuse him. Equally, the mistake of an agent of the defendant in construing the contract and the rights of the assured under it would not estop the defendant from thereafter taking that position which the correct legal interpretation required."

ANSWERS IN APPLICATION:—(Forwood *vs.* Prudential Life Ins. Co., Ct. of Appeals of Md., 83 At. Rep. 169.) Two questions interest us in this case. The first is that of materiality. The insured in answer to questions in the applications stated that he used no liquor and that he had never used liquors to excess. The court held that these answers were material as a matter of law.

The second question came under the claim that the answers of the insured were incorrectly recorded. The court quoted as part of its opinion from the case of The New York Life *vs.* Fletcher decided by the Supreme Court of the U. S. (117, U. S. 519), as follows:

"There is another view of this case equally fatal to a recovery. Assuming that the answers of the assured were falsified, as alleged, the fact would be at once disclosed by the copy of the application, annexed to the policy, to which his attention was called. He would have discovered by inspection that a fraud had been perpetrated not only upon himself, but upon the company, and it would have been his duty to make the fact known to the company. He could not hold the policy without approving the action of the agents, and thus becoming a participant in the fraud committed. The retention of the policy was an approval of the application and of its statements. The consequences of that approval cannot, after his death, be avoided."

It would seem as if this doctrine should be followed in the several states, since it should be held that, when there is added to the insured's signing the application, his receiving and holding a copy of that application as part of his policy without protest, he has thereby incontrovertibly certified that the answers recorded are his.

DIVIDEND ILLUSTRATIONS:—(State ex. rel. Mutual Benefit Life Insurance Co. *vs.* McMaster, Insurance Commissioner, Supreme Court of South Carolina, 75 S. E. Rep. 547.) This case arose from the ruling of the Insurance Commissioner against a pamphlet on the Accelerative Endowment, under the provision of the law that no illustration should be issued misrepresenting the

terms of a policy, or the benefits, or the dividends to be received therefrom. The illustration was made upon the basis of the dividend scale of 1912 and showed ages at which various policies would mature as endowments for the face, on the basis of such scale. It was stated specifically that this was not an estimate of future dividends and that "Such dividends are necessarily contingent upon existing business conditions and their amount cannot be predicated or ascertained in advance."

The court held against the company, basing this chiefly upon two points: that the results of the circular depended upon the assumption that there would be no decrease in dividends for a long period of years in the future, and that the scale of dividends used was the scale for a single year, in which the dividends were considerably larger than in previous years. An extract from the opinion is:

"The proposition for which the petitioner contends is not tenable, for the reason that the statement in the circular is based, not only upon the scale of dividends for the year 1912, but upon the assumption that there will not be a decrease in the scale of dividends during the two periods mentioned in the circular, to wit, 27 and 37 years. This assumption is unreasonable, and tends to mislead the public, for the reason that the scale of dividends is based upon a single year, and upon the further fact that the scale of that year resulted in a considerable increase in the dividends over any previous year."

Right to Exercise Option After Death:—(New York Life Insurance Co. *vs.* Noble, Supreme Court of Oklahoma, 124 Pac. Rep. 612.) Policies which on lapse give several options, one of which must be elected within a certain time after the lapse, lead to an interesting situation when the insured dies between the date of lapse and the end of the period within which election must be made.

In the present case there was the option of surrendering the policy within six months after lapse, either for paid up insurance or for extended insurance. The insured died within six months of the default in payment of premium without surrender of the policy under either option. The beneficiary, after the death of the insured, made demand for the extended insurance for the full amount of the policy and this was refused by the company. The court held that such demand was sufficient, and that the full

amount of the policy was payable under the extended insurance option.

The further question whether, had such demand by the beneficiary not been made until after the end of the six months period, it would still have been sufficient on the ground that the rights of the parties were fixed by the death of the insured and that there could be no question of election between the larger sum payable under extended insurance and the smaller sum under paid up insurance, did not enter into consideration.

Effect of New York Statute Requiring Premium Notice: —(Adam *vs.* Manhattan Life Insurance Co., Ct. of Appeals of N. Y., 97 N. E. Rep. 740.) A policy issued in 1885 lapsed by nonpayment of premium due in 1904 and the insured died in 1907. The statute of 1876, as amended in 1877, provided that a policy thereafter issued could not be declared lapsed or forfeited unless a premium notice had been duly addressed and mailed. The plaintiff claimed that such notice had not been given. In behalf of the company the statute of 1897 was invoked. This statute differed from the earlier statute by limiting the time within which the company was prohibited from declaring a policy lapsed or forfeited, to one year from default. The statute of 1897 defined its applicability as to "any policy hereafter issued or renewed." The court held that the expression above quoted in the statute of 1897 expressly limited the application of that statute, thus excluding policies issued prior to its enactment. The court apparently gave no effect to the word "renewed." It would seem from the wording of the statute that "renewed" meant the renewal by the payment of a premium, thus making the statute apply to every policy on which a premium was paid subsequent to its enactment.

The court further held that in the present case the provisions of the statute of 1877 were a part of the contract in the case of policies issued while it was in force and that these provisions could not in any case be annulled or varied by a later statute. On this point it was contended by the company that because of the injustice which could result from the earlier statute the later statute was intended to apply to policies in force and that the one year period was a statute of limitation. The court held that such a statute as a statute of limitation was not within the power of the legislature since it would annul the provisions of a contract

before its maturity, which was something entirely different from a statute of limitation.

False Statements in Application-Effect of:—(Ætna Life Ins. Co. *vs.* Outlaw, U. S. Circuit Ct. of Appeals, 4th Circuit (So. Car.), 194 Fed. Rep. 862.) This case turned upon the interpretation of a provision in the policy required by the laws of a number of states for several years past in policies issued in those states. The provision in its essential parts followed the exact language of some of the statutes, reading as follows:

"All statements made by the insured shall, in the absence of fraud, be deemed representations, and not warranties, and no such statement shall avoid the policy or be used in defence to a claim under it, unless it is contained in the written application for this policy and copied hereon."

The company claimed that the trial judge "should have charged the jury explicitly that a misstatement or misrepresentation in a material matter would avoid the policy, although such misstatement may have been honestly made in a sincere belief that it was correct, and without any fraudulent intent."

The court held against this reasoning as follows. (1) Under the company's claim the only effect of the word "fraud" in this clause would be to avoid the policy in case of fraud for immaterial misstatements. (2) That fraud under the general rule of law could be relied on as a defense, consequently the construction in (1) would deprive the clause of its effect. (3) A representation under the terms of the policy must have been knowingly false and therefore fraudulent to serve as a defense.

Apparently the court, considering the words "in the absence of fraud" to be surplusage as regards immaterial misstatements, instead of letting them go as surplusage, applied them to material misstatements, thus interpreting the clause to mean that material misstatements should not avoid the policy unless there was fraud. Such application does not seem to be justified even as a possible interpretation of the clause above quoted.

The general rule of law is that a material misrepresentation, whether fraudulently or honestly made, avoids the policy. This is as it should be. The questions asked and the answers are about

matters peculiarly within the applicant's knowledge and generally not at all within that of the company. If the applicant makes a misstatement in a material matter, the company is likely to be deceived to its detriment. The intent of the applicant in making such misstatement or his knowledge of its falsity are extremely difficult matters to prove, and the requirement of such proof opens the door to fraud against the companies.

RIGHTS OF POLICY HOLDERS UNDER ABSORPTION OF ONE COMPANY BY ANOTHER:—(Washington Life Insurance Co. et al. *vs.* Lovejoy et al., Court of Civil Appeal of Texas, 149 S. W. Rep. 398.) The Washington Life was absorbed by the Pittsburgh Life and Trust, transfering practically all its assets to the latter company, in return for which that company assumed all its obligations. Lovejoy refused to assent to the transfer of his policy from the one company to the other. According to the findings of the court it was, nevertheless, transferred without his consent. Lovejoy allowed his policy to lapse and brought suit for damages. The court upheld his right to damages, and, on the conclusive evidence that he could not procure other insurance, fixed the amount of damages at the premiums paid, together with interest thereon.

The bearing of such a decision as this upon the reinsurance of one company by another, where the reinsured company, either actually or practically, goes out of existence, is evident. A few policy holders who refused to accept the transfer of their policies to the reinsuring company, could, under this ruling, make the transaction an expensive one.

Abstract of the Discussion of Papers Read at the Previous Meeting.

Address of the President, Archibald A. Welch:—Extended Insurance.

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Written Discussion.

Mr. Rhodes:

Mr. Welch is entitled to the thanks of the Society for placing before it the experience of the Phoenix Mutual Life Insurance Company under extended insurance, corroborating as it does the similar experience of the Mutual Benefit, which was presented to the Society in October, 1908. Mr. Macdonald has kindly shown me the similar experience of the Confederation Life Association, which, so far as it goes, agrees with the experiences of the Phoenix and the Mutual Benefit.

In view of these several experiences it may be accepted that there is an adverse selection on the part of policyholders who avail themselves of the contract provisions for extended insurance. The full effect of this selection, however, has not been, and probably cannot be, measured. The fact that this adverse selection exists is not of itself sufficient to condemn the practice of granting extended insurance. Admitting that the practice involves some additional cost to policyholders generally, the question is whether the privilege is worth what it costs. The admission is made only for the purpose of argument, because the fact of such additional cost has not been demonstrated; in fact, I am inclined to the opinion that the interest earned upon the reserves on automatic extended insurance in excess of the rate required to maintain the reserves will generally suffice to meet any excess mortality cost and the expense of handling the extended insurances.

We should not forget that the experiences which have been published relate to extended insurances which have been granted without any action by the insured. If extended insurance is granted only at the request of the policyholders, we may reasonably expect a more serious selection against the companies. It would seem reasonable to expect that any defence of the practice of granting participating extended insurance would come from those who may fairly be presumed to have had the most favorable results, but such

is not the case. During the discussion of Mr. Moir's paper at the Society's meeting last spring it was argued that, as each individual policyholder must be accepted as equal in every particular to the average man of the group in which he is originally placed, so, as far as the company is concerned, he always must remain as a perfect illustration of this average individual. The conclusion drawn from this argument was that a policy which was originally a participating policy must always remain a participating policy. I think the conclusion is unwarranted. If default occurs in premium payments, and the form of insurance carried by the individual is changed from that originally issued to extended insurance, it is the policyholder who, voluntarily and by his own act, takes himself out of his original class and, by so doing, requires the company, if mutual principles are being observed, to see that the policyholder so acting does not unduly profit at the expense of other policyholders. In my remarks upon Mr. Moir's paper last spring I showed that this rule had not been followed by one company which granted participating extended insurance. If a policyholder removes himself from a dividend-earning class to one which does not earn dividends, there would appear to be no good reason why his policy should continue to be credited with dividends. Allowing some surplus when none should be allowed, violates the principle of mutuality quite as much as the allowance of none when some should be allowed.

I think we can all agree that the practice of continuing a policy in force for its full amount in case of default in the payment of premiums meets a need of the insuring public which may be said to be almost vital. The only other plan which meets this need is one which is looked upon with approval and preference by some actuaries. I refer, of course, to the plan of continuing the original policy in force by charging unpaid premiums as a lien, so long as such a lien is secured by the policy reserve. The relative desirability of these two plans, so far as individual policyholders are concerned, cannot be settled by any general statement, but we can determine, in a measure, which plan is more desirable for the average policyholder, and this is really all that can be done in any question of like character.

Under the policies of the Mutual Benefit the extended insurance is granted automatically, but the insured may avail himself of an automatic premium loan provision. A comparison of the results under these two provisions, as applied to continuous premium life and to twenty premium life policies issued at ages 30 and 50, shows that for policies of short duration, under which, of course, the majority of lapses occur, the insured would receive more insurance, both in amount and time, under the extended insurance provision than he would receive under the automatic loan provision. For example, under a whole life policy, issued at age 30, after payment of five premiums he would have extended insurance in the

full amount of the policy for six years and nine days. Under the automatic loan provision he would have insurance (in case of a thousand dollar policy) ranging in amount from $981.02 to $888.15 for 5 years and 338 days, assuming that the company's present dividend scale is continued without change. If the calculation be made according to the rule adopted by the company which may be looked upon as the leading advocate of the automatic premium loan plan, which involves charging 4 per cent. yearly of the indebtedness as a revival and expense charge in addition to 6 per cent. interest, the policyholder would receive only four years' additional insurance under the automatic premium loan plan as against six years under the extended insurance plan.

There is, of course, a certain advantage to the policyholder under the automatic premium loan plan in that, while the insurance is continued, a medical examination is not required to restore the policy to its original condition. This advantage, to my mind, is not as important as it appears at first sight, for the reason that if a policy is entitled to extended insurance for, say, three or four years, the company need not be very particular in its inquiry regarding the insured's condition of health; in fact, it will generally be to the interest of the company to reinstate the policy without a medical examination; and I am of the opinion that if a medical examination is required the company is justified in looking upon it more leniently than it would look upon an application for original insurance. Moreover, there is an advantage to the policyholder in having his insurance continued for a definite amount and fixed time instead of for a constantly decreasing amount and for an indefinite time. The experience of the Mutual Benefit shows that 53 policies per thousand lapse or surrender during the second year; 7 per thousand during the tenth year, and 3 per thousand during the twentieth year. It is clear that under the extended insurance provision more policyholders would benefit, both in the amount and term of extended insurance, than under the automatic loan provision. It has been shown that in extended insurance the effect of selection against the company is heavier immediately after lapse than it is during the later period of the extended insurance. It has also been shown that for policies having a longer duration between the time of issue and the time of extension the deaths are proportionately greater. These facts have an important bearing in determining the relative advantage of the two plans, so far as the company is concerned. Although for policies with short durations the period of extended insurance is greater than the period during which the insurance would be continued under the loan plan, the company still retains in some measure the benefit of the medical selection. Under policies with long durations, where the heaviest mortality under extended insurance is experienced, the term of extended insurance may be less than the period during which the insurance is continued under the loan plan. The com-

pany, of course, realizes, under the loan plan, the benefit of any premiums charged as a lien; but this, of itself, is not sufficient to make the loan plan preferable to the extended insurance plan. The definite time and fixed amount for which the insurance is continued under the latter plan, as well as the longer term available to the greater number of policies, in my opinion make the extended insurance plan preferable. I have appended to these remarks a table showing the results of the application of the two plans to Mutual Benefit policies to which I have referred. In my calculations of the results of the automatic loan provision I have followed the plan of applying the value, when it was not sufficient to meet the full premium, to the purchase of extended insurance. This resulted, in a number of instances, in giving the insured more than a year's additional insurance to which he would not be entitled under the loan plan as practiced by some of its advocates.

COMPARISON OF EXTENDED TERM INSURANCE AND THE AUTOMATIC PREMIUM LOAN PROVISION.

Based upon the Premium Rates and Surrender Values of the Mutual Benefit and assuming that Dividends correspond to the 1912 Dividend Scale of the Mutual Benefit.

Plan of Insurance.	Age at Issue.	Lapse at End of Year.	Extended Insurance.			Automatic Premium Loan.			
			Amount of Insurance.	Years	Days.	Maximum Insurance.	Minimum Insurance.	Years	Days.
Ordinary Life..	30	2	$1,000.00	1	290	$983.14	$979.51	1	245
		5	"	6	9	981.02	888.15	5	338
		10	"	13	98	981.75	692.94	13	47
		20	"	18	45	984.28	306.17	26	120
	50	2	"	2	331	960.33	913.77	2	191
		5	"	6	103	961.45	774.19	6	151
		10	"	9	33	963.57	520.81	11	12
		20	"	9	178	968.98	187.65	17	249
Life 20 Payments	30	2	"	4	82	970.85	936.54	3	273
		5	"	12	313	971.70	624.31	11	276
		10	"	24	80	973.41	179.39	37	191
	50	2	"	3	298	953.06	898.01	3	164
		5	"	8	47	954.53	609.07	8	256
		10	"	12	28	957.37	127.66	24	177

ORAL DISCUSSION.

MR. MACAULAY: It strikes me that a comparison such as Mr. Rhodes has made between the extended term insurance and the automatic non-forfeiture system, by which premiums are advanced as loans, is hardly to the point. He is comparing things which are not alike.

The extended term system concerns itself with the form which the surrender value shall take; the automatic non-forfeiture system, on the other hand, rests on an entirely different basis. It assumes that when a policyholder, because of financial difficulties, or carelessness, or other reason, misses the payment of a premium, he should have a further chance. It is a system of trying to prevent the policy from lapsing at all.

It is not to the point to compare the length of time that the insurance will be continued under the two systems. One is a surrender value, and is to be compared with surrender values; the other is a method of enabling the policyholder, who is unfortunate, to keep his policy in force. In one case the policyholder gets out of the company; in the other case he is kept in the company.

I looked over the statistics of my own company a few days ago, and noted particularly the result that by means of the system of automatic non-forfeiture a very large proportion of the premiums in arrears are within a few months or years paid up and the policy continued. It is surprising how large this proportion is.

I have some figures showing the effect, as regards the continuing of policies, of automatically advancing premiums. This system was introduced in the company with which I am connected in September, 1894, and the statement up to December, 1911, is as follows:

Number of policies extended by automatic premium loans	38,707
Indebtedness paid in cash	19.314
Automatic loan repaid by obtaining regular policy loan	920
Automatic loan repaid by deduction from death claim	488
Automatic loan repaid by deduction from matured endowment	128

Only 1,366, or about 3½ per cent., took surrender values or paid-up policies. The total number forfeited by the expiring of the term was 8,478, or less than 22 per cent. The number still in force on the books is 8,013, or nearly 21 per cent.

We thus have the remarkable result that about 54 per cent. of the total advances have actually been repaid, and less than 22 per cent. of the policies on which such advances were made were forfeited by the expiry of the term. The figures are large, and I think they bear out the statement, that this system has a very marked effect in reducing lapses.

Another point is that of the value to the policyholder of the option under the automatic non-forfeiture provision of keeping the policy in force. Suppose a man has consumption and lapses a policy under which the extended insurance is two years and six months. Suppose he lives out the two years and six months, but dies within two years and seven months. Under the extended insurance he would get nothing; under the automatic loan he could, before the policy went out of force, pay up the premiums in arrears and leave his insurance to his widow. Extended insurance where

it gives a longer extension than the automatic premium loan is the best for anyone who can tell within how many years and months after default in payment of premiums he is going to die, or for anyone else who knows that he won't become permanently uninsurable within the time of extension, but not for the vast majority who do not know either of these things.

I have been an advocate of making the penalty of allowing premiums to fall into arrears under the automatic non-forfeiture provision heavy; *i. e.*, to 6 per cent. I should add an additional 4 per cent. as penalty for carelessness. This penalty can be avoided by going through the process of taking a policy loan. We do not want, by the automatic non-forfeiture, to encourage policyholders to be lax in their dealings with the company, consequently we make it against his interests to be so by means of this additional 4 per cent.

Thus our whole system is that of holding the policy good and protecting the policyholder against a little carelessness, a little forgetfulness, a little financial embarrassment, and in that way helping him to keep his policy in force.

The automatic system of advancing premiums has, however, a weakness, particularly if the company charges a low rate of interest, in the risk of encouraging its policyholders to become careless in their payments. Take any body of men who, like the Civil Service employees and other classes of people, are sometimes hard up. A notice comes saying the premium is due. Insured says, they will only charge me interest for delay, and the money is worth interest to me; and he does not pay and lets his policy run behind. This is true if the rate of interest is low; if it is 6 per cent. for instance. If, however, a penalty is charged, such as the 4 per cent. mentioned above, the insured thinks twice before he lets the premium run far in arrears and allows 10 per cent. to be charged up against him. With a low rate of interest, however, there is a very real weakness, illustrated in the Barbados Mutual Life Society. They have had the system for a long time, and charged only 6 per cent. Some years ago they were in the position, if I remember aright, of having over 40 per cent. of their total assets in the form of automatic premium loans, because the condition of trade in Barbados had not been good, and the low rate of interest encouraged people to let their premiums go into arrears. I am afraid that other companies who do not charge a fine may find the automatic system will not work out as favorably for them as for companies that remove the inducement to the policyholder to impose on them.

The point has been raised that, under the system of charging a penalty, the policyholder would only have to sign a loan agreement to avoid this penalty, and that the company would not be any better off in regard to the rate of mortality than under the extension of the insurance by means of the automatic loan. This

is true. It is not a question of mortality at all, but a question of an inducement to keep the policies in force.

Mr. A. B. Wood: In listening to Mr. Rhodes' comparison I wondered whether he based his figures upon the cash surrender value or upon the full reserve. I am informed it was the cash surrender value. In the company with which I am connected we advance on automatic premium loans not merely the cash surrender value but the full reserve, taking into account the premium paid by the loan. This would make the comparison more favorable to the automatic premium loan than the figures Mr. Rhodes has given.

Another point which should be emphasized is that in the case of a policy in arrears where the value is not sufficient to carry it farther, the policy holder in order to continue it does not need to pay up all the arrears; he has only to pay the five or ten dollars, or whatever it may be, which, together with the balance of the reserve, if any, is sufficient to carry him along.

Mr. Little: I did not intend to enter upon the discussion of Mr. Welch's address at all, but it has wandered a little off the main point to a subject with which I have had a considerable degree of familiarity—the keeping of policies in force by automatic loans.

Of the two companies that claim to have originated this system, one is in a somewhat exceptional position, as both its surrender values and its dividends are very large. As a consequence it sometimes happens that insurance is kept in force for a longer period of time than under the extended term provision. In practice, however, the conclusion of Mr. Rhodes, that the period of insurance is longer under the extended term than under the automatic loan provision, is usually correct.

I might mention with regard to insurance in Australia, where this system is in general use, that the total rate of interest charged is not 6 per cent. plus 4 per cent., 10 per cent. in all, but 8 per cent., which doubtless affects the number of those who make use of the automatic loan privilege.

A point to be remembered, also, is that there is no temptation for the policyholder to take advantage of the automatic loan provision if he expects to die, since there is no saving to him from it, the amount of loan being deductible from the amount payable under his policy. Consequently this privilege can hardly affect the mortality, and in that differs from the extended term insurance.

Mr. Rhodes: I do not question at all the fact that a great many of the automatic premium loans are paid. It does not appear to me that that determines the question at issue, or shows a comparative advantage of such system over extended term insurance. If the two systems are to be considered according to the degree in

which business is retained on the company's books, I shall be very glad to pit the automatic extension of the Mutual Benefit against the automatic premium loan plan of the Sun.*

MR. FERGUSON: Mr. Rhodes stated that in his company the policyholder received a greater benefit by accepting extended insurance instead of taking the automatic premium loan. This does not seem to me to show any inherent advantage that extended insurance possesses over the automatic loan, but it suggests to me that possibly the extended insurance offered by Mr. Rhodes' company is perhaps unduly liberal. There should be some equation between the two which would work out the results equitably, allowing on the side of the automatic loan the advantage that the policyholder has in being able to continue his policy without examination, and charging as a disadvantage of that system the reducing insurance. If such an equation is formed, and if the extended insurance is proved to be a better plan for the policyholder, then we have simply a proof that either the extended insurance is too liberal or the automatic loan is not liberal enough.

* The figures included in Mr. Macaulay's discussion were given at the end of the meeting and after Mr. Rhodes's discussion.

SURVIVORSHIP AND DEFERRED SURVIVORSHIP ANNUITIES. SOME POINTS RAISED BY RECENT RULINGS AND LEGISLATION— HERBERT N. SHEPPARD.

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WRITTEN DISCUSSION.

MR. A. B. WOOD:

Mr. Sheppard has directed our attention to several interesting points in connection with survivorship annuities. The question whether legally they are contracts of life assurance or annuities is also of importance in connection with Section 96 of the New York law limiting the volume of new business. I am inclined to agree with Mr. Sheppard that the position taken by the Insurance Commissioner of Massachusetts declaring them to be contracts of life assurance is the logical one from this standpoint. By such a contract the company undoubtedly assures the life of one person "A" in favor of another "B," the object being to protect "B" against the contingency of "A's" death. The fact that the company's liability is discharged by the payment of a life annuity to "B" instead of by a single cash payment, would not seem to alter the nature of the contract, so far as the question of its being an assurance on the life of "A" is concerned. On the other hand, it may be argued that the contract is one to pay an annuity to "B" upon the happening of a particular event. In reality it combines the features of both a life assurance and an annuity contract, and is so regarded actuarially. Viewing it as a contract of life assurance; when a claim arises by the death of the assured the value of the annuity would, I presume, be charged as a claim against the assurance branch and credited to the annuity branch. From the actuarial standpoint an objection could be urged against this practice on the ground that the annuity branch might thus profit at the expense of the life branch in the event of an unfavorable mortality occurring amongst the assured lives. The point is of importance in the case of any company in which the policyholders do not share in the profits of the annuity branch. The most equitable course would seem to be to include these policies from their inception until the death of the beneficiary in either the one branch or the other, and from this point of view I consider that the most natural course would be to regard them as annuities.

As regards the basis of valuation of ordinary survivorship annui-

ties under the New York law, it seems to me that, if they are considered contracts of life assurance, a reasonable interpretation of Section 84 is that the American Table must be used for the assured life and the McClintock Table for the beneficiary. The latter is the standard prescribed for the valuation of annuities,—presumably all annuities whether immediate or deferred,—with the one exception expressly provided for in the law, namely that annuities deferred ten or more years and written in connection with life or term assurance *shall* be valued by the same mortality table from which the premiums are computed. The annuity payable under the survivorship annuity contract can hardly be construed as coming under this heading, for it may become payable at any time, and when it falls in I should judge that Section 84 would require it to be valued by the McClintock Table. If so, the mortality by this table should be assumed for the beneficiary from the inception of the contract.

The proper actuarial basis for the calculation of the net premiums and reserves undoubtedly is the combination of an assurance table for the assured life with an annuity table for the beneficiary. We shall, naturally, look for a twofold selection against the company, but, as regards the beneficiary, we should hardly expect as low a rate of mortality as under immediate annuities. This view is supported by the comparisons given in Mr. Jensen's paper (*T. A. S. A.*, Vol. X, 265) where it appears that the mortality of Danish female annuitants under survivorship annuity policies, although lower than that of corresponding female assured lives, is nevertheless higher than the mortality of ordinary female annuitants. The net premiums based upon the American Table for the assured life and the McClintock Table for the annuitant probably err on the side of safety. Mr. Sheppard presents a table showing the level net premium for certain combinations of ages of the assured and beneficiary calculated on this basis, 3 per cent. interest being assumed during the currency of the policy and 3½ per cent. during the currency of the annuity. Comparing these with the gross premiums charged by one company, he expresses the opinion that the rates are probably too low even if the contracts are issued on the nonparticipating plan. In Canada, the $O^{M(5)}$ Table would be used for the assured life and the $O^{[af]}$ for the beneficiary. The following are the corresponding net premiums per $100 annuity on this basis, 3½ per cent. interest being assumed throughout.

Age of Assured.	Age of Beneficiary.				
	20	30	40	50	60
25	$20.28	$16.45	$12.96	$ 9.74	$ 6.87
35	30.23	24.56	18.98	13.91	9.47
45	48.37	40.21	32.27	23.57	15.64
55	81.42	70.75	58.18	44.14	29.81

These are naturally lower than those given by Mr. Sheppard except in a few instances at the advanced ages. The gross premiums referred to are not quoted, but apparently they provide for a loading of 15 per cent. or more on the average over the above net rates. I should therefore consider them to be unquestionably sufficient for the non-participating plan, but, as regards participating contracts, Mr. Sheppard's criticism is probably well founded if dividends of any substantial amount are expected. It may be remarked that under the survivorship annuity policy profits would be looked for chiefly from loading and favorable mortality, for, owing to the nature of the reserves, interest profit would be comparatively small.

As regards the comparison of the level net premium with the net premium for the first year's risk, it will be noted that only in extreme cases where the beneficiary is considerably older than the assured does the level net premium fall below that for the first year's risk. As such cases will form but a very small proportion of the total business, this condition does not present any real practical difficulty.

As regards deferred survivorship annuities issued in connection with ordinary instalment polices, it may reasonably be assumed that there is little, if any, selection against the company on the part of the beneficiary. It therefore seems justifiable to calculate the net premiums and reserves on a less stringent basis than that adopted for immediate survivorship annuities. Mr. Sheppard gives the net premiums for a 20-year deferred survivorship annuity of $100, using Mr. McClintock's Tables "Male" and "Female" for the assured and beneficiary respectively. The following are the corresponding premiums by the combination of the $O^{M(5)}$ and $O^{[af]}$ tables at $3\frac{1}{2}$ per cent. interest.

Age of Assured.	Age of Beneficiary.				
	20	30	40	50	60
25	$ 5.51	$ 3.88	$ 2.50	$1.37	$0.47
35	8.06	5.35	3.45	1.80	0.59
45	13.76	9.43	5.69	2.82	0.90
55	24.86	17.69	10.86	5.40	1.66

The use of the $O^{[af]}$ Table in this connection probably gives unnecessarily high premiums, but they are nevertheless somewhat lower than those given by Mr. Sheppard for most of the combinations of ages at which these policies are chiefly taken out.

Referring to the question of reserves for the continuous instalment benefit, I agree with Mr. Sheppard that when the deferred survivorship annuity produces a negative reserve, the reserve on the instalment policy should not be reduced. Should the beneficiary predecease the assured, the policy both as regards premium and reserve is automatically changed to an ordinary instalment

policy. The full reserve to provide for the guaranteed instalments should, therefore, be maintained at all times.

Mr. Sheppard also discusses the question of the sufficiency of the reserves in the aggregate under continuous instalment life policies, according to the present practice of reserving on the basis of $780 or $800, according as the valuation basis is 3½ per cent. or 3 per cent. From an investigation of the actual policies issued by one large company, he finds that the average net premium for the deferred survivorship benefit is equivalent to the net annual premium for an ordinary life policy of $147, or of $115, according as the calculations are made by the McClintock, or the American Table. From this he draws the inference that the present approximate basis of valuation indicates an understatement of the liability under these policies. No definite conclusions can, however, be drawn from a mere comparison of the average net premium with the corresponding premium for a straight life policy, owing to the essential difference in the nature of the reserves. A comparison of the net level premiums for the continuous instalment with the net premiums for the first year's risk, given on page 16, shows how frequently negative reserves will occur. Assuming the business to be distributed in the proportion indicated by the table on page 17, the net level premium will be less than the net premium for the first year's risk under probably 70 per cent. of the policies. Taking the case of an assured life aged 35 and beneficiary 30, the net premium is $5.58 as compared with $6.30 for the first year's risk, and not until about the twentieth year of the contract, that is until the assured and the beneficiary have attained the ages of 55 and 50, is the net premium for the year's risk ($5.46) less than the net level premium. The preponderance of negative reserves, even if these are eliminated in the valuation and positive reserves maintained wherever they occur, would indicate that a comparatively small extra reserve carried under every continuous instalment policy will produce sufficiently high reserves in the aggregate. Nevertheless, as Mr. Sheppard has stated, the matter should be tested by a comparison with exact values.

MR. LAIRD:

This paper presents good reasons for considering survivorship and deferred survivorship annuities during the lifetime of the insured as insurance contracts for a continually decreasing amount. Mr. Sheppard then calculates the mortality gains under a survivorship annuity, and, on page 15, shows that a very close approximation to the exact gains may be obtained by considering the contract as ordinary life insurance for an amount equal to the present value of an annuity on the life of the beneficiary at the end of the second year.

If it is correct to take mortality gains on a survivorship annuity,

it follows that it is equally proper in the case of a deferred survivorship annuity, whether issued as a separate contract or incorporated in an instalment policy to continue payments until the death of the beneficiary. Owing to the popularity of continuous instalment policies, this question is of importance to a company whose expenses are but little less than the limit allowed by Section 97 of the New York law.

In the case of a continuous instalment policy providing $50 a year for 20 years certain and as long thereafter as the beneficiary, now aged 20, shall survive, we have, on the basis of the American Experience 3½ per cent. table, $735 as the commuted value of the instalments certain and $357 as the value of the deferred annuity at the end of the second year. Applying Mr. Sheppard's approximation, we find that the mortality gains on the deferred part (commuted value $357) are 48 per cent. of the gain on the instalments certain (commuted value $735). For beneficiary aged 40 at issue, the ratio is 25 per cent., but at age 60 it drops to 2 per cent.

If the number of instalments certain is less than 20, the mortality gains for the deferred part are, of course, relatively greater. Recently a company issued a continuous instalment policy with instalments certain for only 5 years, and in this somewhat exceptional case the mortality gains on the deferred part are actually 3½ times those on the instalments certain.

If we admit the theoretical propriety and practical importance of taking mortality gains on survivorship and deferred survivorship annuities, we must still consider whether such action would be in accordance with the New York law as interpreted by the Superintendent.

On page 269 of Part 5 of the 52d Annual Report, we notice the following extract from his letter to a New York company: "After referring to the limitations of the section as to expenses other than investment expenses, you write as follows:

"'We are uncertain whether the loadings on the various forms of annuity contracts, namely, immediate, temporary, survivorship or deferred, are to be included in the calculation of the amount of expense to which we may go either on first year's business or on the total business of the Company. A ruling on this point will be very greatly appreciated.'

"In my opinion the loadings on the various forms of annuity contracts should be included in the calculations referred to."

No distinction is here made between immediate and survivorship annuities, or between annuities and insurance contracts, in regard to limitation of expenses. If then we are to include loadings on annuity premiums in margins on first year's business, it might be argued that, on those annuities which give mortality gains, namely, survivorship annuities, it would be quite proper to take advantage of those savings, but on the same page we have the following rul-

ing: "I would advise you that, in my opinion, annuity contracts, whether they are survivorship annuities or not, do not come within the provisions of sections 96 and 97 of the Insurance Law, except as to the provisions of the latter section relating to expenses other than investment expenses."

As the part of Section 97 relating to first year margins does not refer to "expenses other than investment expenses" it is apparently the Superintendent's intention to exclude all annuities from first year margins but to include them in total business. Under this ruling, it seems hard to justify taking mortality gains on survivorship annuities. Furthermore on page 343 we read: "In my opinion the term 'annuities' as used in section 102 of the Insurance Law does include survivorship annuities."

If New York adheres to this view, which is contrary to that of Massachusetts and Connecticut, the loadings and commissions on the premiums for the deferred survivorship annuity part of a continuous instalment policy should, strictly speaking, be excluded from the calculations of first year margins, but included in the figures for total business. The rulings quoted, however, are somewhat conflicting and I should welcome an expression of opinion on the propriety of taking mortality gains on survivorship and deferred survivorship annuities.

MR. DAWSON:

Mr. Sheppard's paper is especially valuable by reason of the adaptation of his previously published retrospective formula for select and ultimate valuation to the particular case of a survivorship or deferred survivorship annuity policy. This formula will unquestionably be useful to American actuaries, if called upon to make such computations in respect of survivorship annuities.

The fact may be worth noting here, that when Mr. McClintock, Mr. Moir, and I were at work, developing formulas for select and ultimate values, before this method of valuation was made known at the May meeting of the Society, in 1903, we were all investigating retrospective expressions, when the now familiar and always self-explanatory prospective form:

$$ {}_nV'_{[x]} = A_{[x]+n} - P_x a_{[x]+n} $$

was brought forward by me and at once accepted by my collaborators as the most suitable for the initial presentation of the method.

Mr. Sheppard's discussion of the whole subject is also of much value, not because of the novelty of any portion, but because the whole had not been comprehensively set forth in one paper hitherto.

There is, however, at least one important thing wanting, the omission of which is the more surprising, in that, if the ruling he omits to mention, be adhered to, precisely the most practical por-

tion of the paper becomes mere theory. I refer to this: the Insurance Superintendent of the State of New York, in which state alone select and ultimate valuation and the computation of select and ultimate margins are required, ruled that survivorship annuities are annuities and not life insurances and that, in consequence, such annuities are not to be counted in making up the total of new insurances; from which it follows, also, as a matter of course, that neither the limitation of cost of new business, involving select and ultimate margins, nor the requirement of select and ultimate valuation applies to such annuities. On principle, such a ruling would apply equally to deferred survivorship annuities when separately issued; but, in my opinion, certainly not to the deferred survivorship annuity portion of a continuous instalment life insurance policy, which would be dealt with as a whole, as in the past.

Mr. Sheppard, without mentioning this ruling, quotes the opinion of the Massachusetts Insurance Commissioner to the contrary, and says: "the present writer feels that it is the logical one"; but in Massachusetts, where the courts have also spoken in a manner tending to sustain that view, there is no select and ultimate valuation and no requirement to report the select and ultimate margin, while in New York, where both are matters of practical interest to actuaries, the contrary rule has been established. So far as New York is concerned, pending a reversal of this departmental ruling, it can scarcely be true that: "the legal status of both survivorship and deferred survivorship annuities does not at present seem to be well-defined."

It seems highly probable, also, that the ruling would be upheld by the courts, were it contested, which is most improbable. If not upheld, the whole contract, it would seem, would become life insurance and the table by which to value it, as respects both lives, would be the American Experience and, for the minimum valuation, the American Experience select and ultimate, perhaps, though that would be inconsistent with the plain legislative intent. It is true that arguments can be adduced on both sides, as to whether or not a survivorship annuity should be classed as an insurance or an annuity, for actuaries often speak of the nominator as "the insured life" and of the nominee as "the beneficiary"; but the survivorship or reversionary annuity has always been known as an annuity, the formulas for its premiums and values are annuity formulas in form, and the rules of statutory construction, which apply to criminal laws, such as the limitations of expenses and of new business, are strict and, in effect, would give the benefit of the doubt, if any, to the company.

Mr. Sheppard seems to think (p. 10) that, if survivorship annuities are annuities, within the meaning of Section 84 of the New York law, this calls for using the American Experience Table "for the insured life" and of McClintock's Table "for the beneficiary"; because the former is prescribed for life insurances and the latter

for annuities. But that would be inconsistent, would it not? If these are *life annuities,* would not McClintock's Table apply throughout?

The solution of this quandary is not, I am confident, that which Mr. Sheppard suggests; but is to be found in the special power given the Superintendent to use other tables in special cases.

No designation of tables has been made by the Superintendent, who is, therefore, under the ruling that survivorship annuities are annuities and, of necessity, special cases of such and not mere "life annuities," free to use, or accept valuation by, other tables at discretion. There is no doubt that he would, and indeed does, insist upon the American Experience Table for the life of the nominator, and equally no doubt that he will accept any demonstrably safe table for the life of the nominee.

McClintock's Table would be a *safe* table, unquestionably. Mr. Sheppard finds (bottom p. 11) that "one of the principal companies publishing a complete set of rates" brings out gross premiums but 3 per cent. to 4 per cent. higher than his net rates, so computed, "for the combination of ages under which most of the contracts would probably be written"; but the difference is chiefly due, I think, to his employing 3 per cent. interest for the joint lives and $3\frac{1}{2}$ per cent., only, for the survivorship while the company in question probably employed 3 per cent. throughout.

In addition to being a safe table a table should also be suitable and convenient. Mr. McClintock's Table constructed for life annuitants is not a suitable table for survivorship annuitants, because among survivorship annuitants there is feeble initial selection and none when the annuity is entered upon. That it is not a convenient table is shown by Mr. Sheppard's resort to an approximation formula to obtain a few values by it.

Two tables graduated by Makeham's formula and having the same value of c can be conveniently used together, for the equal age formulas for the value of a_{xy} can then be employed. The Danish Female Survivorship Annuity Table used the same value of c as the Makehamized American Table for this reason (*T. A. S. A.*, x, 253 and 503). This table is also *safe,* being from the largest group of such annuitants ever investigated, all females and residing in a country which, next to Sweden, has the lowest mortality of any nation which has investigated its mortality.

Mr. Sheppard, referring to the Danish Tables, objected to the inclusion of negative terminal reserves. He overlooked the fact that the points he raises are all covered by the explanation in that book of the significance of negative reserves. Mr. Sheppard also makes the statement that it is commonly the practice to take the mean reserve as one half the initial reserve when the initial reserve is insufficient to cover the risk of a policy year. This is in error, as competent actuaries do not so take the mean reserve.

MR. SHEPPARD:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

In his criticism Mr. Dawson refers to an omission on my part which, he says, might reduce the most practical part of the paper to mere theory. It was precisely on account of what seemed to me the inconsistent rulings of the New York Insurance Department that the ruling of the Insurance Commissioner of Massachusetts was alone referred to at length. Mr. Laird in his discussion refers to the rulings which appear to me to be inconsistent (N. Y. Ins. Report, Part v, Vol. i, 1911, p. 269). According to the second ruling, survivorship annuities are not life insurance for the purpose of limitation of new business under Section 96, but they do come within the provisions of Section 97 for the purpose of limitation of expense other than investment expenses. Mr. Laird concludes from this that it is apparently the Superintendent's intention to exclude all annuities from first year margins but to include them in total business. It is hard to reconcile this with the ruling immediately above on the same page, that the loadings on the various forms of annuity contracts should be included in the calculations referred to, viz.: first year's business and the total business of the company. Mr. Dawson says it follows as a matter of course, that, if survivorship annuities are not to be counted on making up the total of new insurance, they are not subject to the limitation of the cost of new business. Here I must disagree with him, for it might be urged that, as the amount of insurance represented by a survivorship annuity depends upon the time when it becomes a claim and the method of valuation of the annuity afterwards followed, it is not a clearly defined quantity for the purpose of Section 96, and it would therefore be better to exclude it altogether from consideration.

In passing, I might point out that the wording of the law is "the present values of the assumed mortality gains for the first five years of insurance on policies in force, etc.," not "on policies of insurance in force," so, with due respect for what Mr. Dawson says with regard to the rules of statutory construction, the whole question, in my judgment, depends upon whether the expression "year of insurance" is applicable to a survivorship annuity policy or not. I think the majority of actuaries will claim that it is applicable.

Mr. Dawson says that my use of the "Nugget Formula" is an argument that Mr. McClintock's Table is not a convenient one for use in such cases. The question, however, is not primarily one of convenience, but whether the nominator and the beneficiary are both subject to mortality which follows Makeham's first law with the same value of "c" in the two tables without doing undue violence to the original statistics on which the tables are based.

If they are subject to such mortality it is certainly a convenience, but even in this case, a formula of approximate summation is advantageous where one rate of interest is used during the joint lifetime of the nominator and the beneficiary, and another rate thereafter.

I cannot agree with Mr. Dawson's statement that in his book "Survivorship Annuity Tables" the significance of negative reserves is fully explained and I still think their publication inadvisable. Mr. Dawson is right in criticising the words "as is commonly the practice" on page 12, and these words should have been omitted and attention drawn to the importance of avoiding a possible error in such a case.

I am glad to read that Mr. Wood is of the same opinion as myself with regard to some of the points covered by the paper and the discussion on it. I feel inclined to agree with him that (as a result of the fact that most continuous instalment policies are written with a period of not less than twenty yearly payments certain) in the aggregate a comparatively small extra reserve is needed.

ON THE METHODS USED IN THE CALCULATION OF THE l_x^{aa} COLUMN, WITH A NEW METHOD OF CALCULATING D_x^{aa} VALUES
SIDNEY H. PIPE.

VOL. XIII, PAGE 20.

WRITTEN DISCUSSION.

FRANKLIN B. MEAD:

Mr. Pipe states that the main object of his paper is to promote a discussion as to whether theoretically the employment of a "single analyzed table" beginning at the youngest age or whether "separate analyzed tables" constructed for each age at entry are the more correct for use in calculating premiums and reserves for disability benefits. As Mr. Pipe points out, Mr. Hunter has shown that the financial results are practically the same, as the premiums by the two methods do not differ by as much as one tenth of one per cent. That this difference is not greater than it really is, is due to the fact that by the employment of a "single analyzed table" or "single combined table" (it seems to me that the word "combined" is to be preferred to "analyzed" for the reason that the latter term is custamarily used in this connection as synonymous actuarially with the term "select") the value of one portion of the expression for the value of the annuity entered upon at the end of the year when disability occurs, is increased, whereas the value of the other portion is decreased, as compared with the resulting values when separate combined tables are employed. The question then is of little practical importance, aside from the fact that the use of a single combined table eliminates the necessity for separate combined tables for each age at entry, although by the use of the separate tables the formulæ for the computation of premiums are greatly simplified.

I have already given the reasons why I think a single combined table is the more theoretically correct in previous discussions upon this subject (*T. A. S. A.*, XI, p. 566 and XII, p. 327) to which I would refer those who might be interested in this phase of the subject. I might say in passing, however, that by the use of single analyzed tables in connection with the American Experience Table allowance is made for the fact that the rate of mortality by the American Table, being an ultimate aggregate table, embraces the mortality of some lives which are totally and permanently disabled, whereas by the use of the separate combined tables it is assumed

that the rate of mortality by the American Table is not influenced by its embracing a proportion of lives a portion of which became totally and permanently disabled at previous ages. I am informed, upon reliable authority, that in Germany, where this question has been given consideration for a number of years, the single combined table is considered theoretically the more correct.

Mr. Pipe calls attention to the fact that when separate combined tables are employed p_x^{aa} will vary with age at entry. It seems to me that p_x^{aa} should not vary with age at entry as we are dealing with an ultimate basic table of mortality. As ${}_np_x$ is the correct probability of mixed lives, some of which are disabled, surviving n years, it seems to me that it is incorrect to assume that that is the correct probability of active lives living n years, especially since we are interested in determining the true theoretical extra cost of waiving premiums in case of total and premanent disability.

Toward the bottom of page 21 of his paper Mr. Pipe makes the statement that I, though disagreeing with separate combined tables as being theoretically unsound, agree with Mr. Hunter's reserve formula which is based upon that theory. On page 330 of Volume XII of the *Transactions* I express the opinion that Mr. Hunter's formula for reserves appeared to me to be proper for use in practice, although it is subject to some theoretical objections; that it is probably best on account of our various valuation and non-forfeiture laws. However, I disagree with Mr. Pipe's contention that Mr. Hunter's formula is based upon the theory of separate anlyzed tables. The expression for Mr. Hunter's formula is as follows:

$$l_{x+n}A_{x+n} - l^{aa}_{x+n}(\pi_x + P^{ai}_x)\mathrm{a}^{aa}_{x+n}.$$

This formula is for the total reserves upon both active and disabled lives and it may be readily demonstrated that the reserves upon an active life corresponding to the foregoing formula may be reduced to the same form as that which I gave for the reserve in my paper on page 324, Volume XI of the *Transactions*.

$l_{x+n}A_{x+n}$ is the present value of the insurance on both active and disabled lives according to the basic mortality table without reference to whether the lives may be separated into active and disabled lives by the use of single or separate analyzed tables. The functions l^{aa}_{x+n}, P^{ai}_x and a^{aa}_{x+n} may either be derived from single or separate analyzed tables. π_x is the net premium according to the basic mortality table, and that is the premium properly assumed to be lost in case of disability, whether we use single or separate combined tables, for that is the premium payable during the period of disability if there is no waiver of premium benefit. For these reasons I do not see the basis of Mr. Pipe's contention that Mr. Hunter's formula applies to separate analyzed tables only.

In the second portion of his paper Mr. Pipe gives an ingenious method of calculating the D columns direct from the D_x of the

basic mortality table. Mr. Pipe's method may be followed whether we desire a single combined table or separate combined tables, although in some of the formulas arising from the use of the single combined table the values of l_x^{aa} and l_x^{ii} are required, and these values are not obtained by Mr. Pipe's method which eliminates the calculation of the l_x^{aa} and l_x^{ii} columns.

Mr. Pipe's method is a rearrangement of the fundamental method of constructing combined tables, as may be shown as follows:

As Mr. Pipe indicates, L_x, the radix of the analyzed table, may be made equal to l_x of the basic mortality table, in which case the ratio of $L_x \div l_x$ becomes one and his formula becomes perfectly general.

The usual formula for the construction of a combined table is

$$l^{aa}_{x+n+1} = l^{aa}_{x+n} - d_{x+n} + q^{i}_{x+n} l^{ii}_{x+n} - r_{x+n} l^{aa}_{x+n}.$$

This readily reduces to the form which Mr. Pipe gives on page 22 as follows:

$$\begin{aligned} l^{aa}_{x+n+1} &= l^{aa}_{x+n} - d_{x+n} + (l_{x+n} - l^{aa}_{x+n}) q^{i}_{x+n} - l^{aa}_{x+n} r_{x+n} \\ &= l^{aa}_{x+n}(1 - q^{i}_{x+n}) - l^{aa}_{x+n} r_{x+n} + l_{x+n}(1 - q_{x+n}) - l_{x+n}(1 - q^{i}_{x+n}) \\ &= l^{aa}_{x+n} p^{i}_{x+n} - l^{aa}_{x+n} r_{x+n} + l_{x+n} p_{x+n} - l_{x+n} p^{i}_{x+n} \\ &= l^{aa}_{x+n}(p^{i}_{x+n} - r_{x+n}) + {}_{n}p_{x} l_{x}(p_{x+n} - p^{i}_{x+n}) \\ &= l^{aa}_{x+n}(p^{i}_{x+n} - r_{x+n}) + l_{x+n}(p_{x+n} - p^{i}_{x+n}). \end{aligned}$$

MR. PIPE:

(AUTHOR'S REVIEW OF DISCUSSION.)

The object of the paper was to promote a general discussion on the theory of this subject. Mr. Mead's views are very familiar to me and, no doubt to all the members, and if I had anticipated that he would have been the only one to discuss the paper, I would not have ventured to present one which contained very little that is new.

Mr. Mead says that he disagrees with my contention that the formula

$$l_{x+n} A_{x+n} - l^{aa}_{x+n}(\pi_x + P^{ai}_x) \mathrm{a}^{aa}_{x+n}$$

is based upon the theory of separate analyzed tables. This formula holds good for all values of n. Hence, if we put $n = 0$,

$$l_x A_x - l^{aa}_x(\pi_x + P^{ai}_x) \mathrm{a}^{aa}_x = 0$$

or

$$P^{ai}_x = \frac{A_x}{\mathrm{a}^{aa}_x} - \pi_x = \pi_x \left[\frac{\mathrm{a}_x - \mathrm{a}^{aa}_x}{\mathrm{a}^{aa}_x} \right].$$

This is the formula obtained by using separate analyzed tables.

Mr. Mead also states that the above formula "is for the total reserves upon both active and disabled lives and it may be readily demonstrated that the reserves upon an active life corresponding to the foregoing formula may be reduced to the same form as that which I gave for the reserve in my paper on page 324, Vol. XI of the *Transactions.*"

He also attempts to prove this on page 329, Vol XII (formula 13 and following), the inference being that if his methods produce the same results as Mr. Hunter's, the single analyzed table must be theoretically correct. I have already shown above, by deducing the net premium for the "waiver of premium" risk directly from the reserve formula which Mr. Mead claims to support his views, that his view is incorrect.

By an analysis of the annuity a_x^{ai} it can also be shown that Mr. Hunter's corrected formula for the reserve on an active life is based upon separate analyzed tables.

Mr. Mead's formula for the reserve on an active life is

$$\pi_x a_x^{ai} - P_x^{ai} \mathrm{a}_{x+n}^{aa}.$$

The annuity

$$a_x^{ai} = \mathrm{a}_x - \mathrm{a}_x^{aa} + \frac{l_x^{ii}}{l_x^{aa}}(\mathrm{a}_x - \mathrm{a}_x^{i}).$$

Therefore the reserve may be stated in the form

$$\pi_x \mathrm{a}_{x+n} - (\pi_x + P_x^{ai})\mathrm{a}_{x+n}^{aa} + \frac{l_{x+u}^{ii}}{l_{x+n}^{aa}} \pi_x (\mathrm{a}_{x+n} - \mathrm{a}_{x+n}^{i}),$$

which is Mr. Hunter's formula. It is not possible to say from this formula alone whether it depends upon a single or separate analyzed tables. This question can be decided by the annuity a_x^{ai}.

In considering this annuity it must be remembered that it represents, when used in the above reserve formula, the value of an annuity on a life which is still active n years after entry at age x, and which is payable for the rest of life. In other words, it is computed from an analyzed table, the radix age of which is x. This can be demonstrated by the method given on pages 20 and 21 of my paper. If we assume x as the age at entry, n as the duration, then the value of $a_{[x]+n}^{a}$ may be obtained from

$${}_t p_{x+n}^{a} l_{[x]+n}^{aa} = l_{x+n+t} - {}_t p_{x+n}^{i} l_{[x]+n}^{ii}$$

the brackets indicating the age at entry. This is distinctly an annuity based upon separate anlyzed tables. Referring to this annuity, I said on page 21, "If the entrant at age x survives as an

active life for n years, the above formula applied to age $x+n$ will give the value of an annuity payable till death, when separate analyzed tables are used."

Mr. Mead inserted this annuity in his reserve formula and produced Mr. Hunter's reserve formula for an active life. This latter formula is merely part of the general reserve formula

$$A_{x+n} - (\pi_x + P_x^{ai})\mathrm{a}_{x+n}^{aa},$$

which has been shown to depend upon separate analyzed tables, and I think that it can be fairly claimed that a_x^{ai} and a_x^a depend upon the same basis.

It appears to me, also, that the use of a single table assumes the same entrant to be subject to two different rates of mortality. The net premium lost by the company is assumed to be the net premium calculated from the basic mortality table, and therefore it is assumed the assured is subject to that mortality. On the other hand, the annuity on his life is assumed to be a_x^a which depends upon a different rate of mortality from the basic table.

I think is is very fortunate that the results calculated from the single table give such close approximations to those based upon separate tables. But I cannot see that the former is theoretically correct.

ON THE DETERMINATION OF THE "EXPECTED MORTALITY ON NET AMOUNT OF RISK" AND "INTEREST REQUIRED TO MAINTAIN RESERVE." [NOTE ON GAIN AND LOSS EXHIBIT.]—MERVYN DAVIS.

VOL. XIII, PAGE 26.

WRITTEN DISCUSSION.

MR. THOMPSON:

One method of calculating the interest required to maintain the reserve may be given symbolically as

$$\frac{\frac{i}{2}(A+B)}{1+\frac{i}{2}}$$

where A is the assurance (or annuity) fund at the commencement of the year, B the amount at the end of the year and i the rate of interest at which the reserve maintained is computed. This is so simple in application that it should be utilized as a check on the formula given by Mr. Davis; when the expected mortality on the net amount at risk has been derived as the balancing item after the calculation of interest required in the manner above suggested, it in turn could be compared with that obtained from following the process outlined by Mr. Davis in his paper. In a large company, however, on account of the magnitude of its operations the chance of error is so great, and an error, if made, might be so large in amount, absolutely, if not relatively, that it is almost imperative that the expected mortality be computed independently from the valuation books.

The suggestion as to obtaining the interest required in a single sum (instead of as the total of two or more sums) by means of an average rate of interest is a good one. At the same time it may be observed that the error introduced is a persistent one because the amount found in this way is always in excess of the true amount and, in case of a large company, over a period of years might ultimately have some effect on the company's surplus. If reserves be calculated at, say, two rates of interest, i and i' the "mean" rate of interest computed in the manner suggested by Mr. Davis would be

$$\frac{\frac{i}{2}(A+B)+\frac{i'}{2}(A'+B')}{\frac{1}{2}(A+B)+\frac{1}{2}(A'+B')}$$

where A', B' are the assurance funds at the beginning and end, respectively, of the fiscal year in respect of the business on which reserve is carried at the rate i', or $\frac{iS + i'S'}{S + S'}$, say, I, where

$$S = \tfrac{1}{2}(A + B) \text{ and } S' = \tfrac{1}{2}(A' + B'),$$

and the interest required is $\frac{iS + i'S'}{1 + \frac{1}{2}I}$, which exceeds

$$\frac{iS}{1 + \frac{i}{2}} + \frac{i'S'}{1 + \frac{i'}{2}},$$

the true amount, by

$$\frac{(i - i')^2 SS'}{2(S + S')\left(1 + \frac{I}{2}\right)\left(1 + \frac{i}{2}\right)\left(1 + \frac{i'}{2}\right)},$$

which is seen to be always positive, as stated above.

MR. SHEPPARD:

Mr. Davis' "Note on the Gain and Loss Exhibit" seems to me to decrease the value of the exhibit as an indication of the sources of profit and loss to the company during the year under observation. Mr. Davis characterizes the suggestion that the cost of insurance be determined as the item required to balance the exhibit as evidently unsatisfactory. To the writer the use of the difference between the cost of insurance and the interest required to maintain the reserve as a balancing item is equally unsatisfactory as it presupposes absolute accuracy in all the other figures of the exhibit.

One advantage of the Gain and Loss Exhibit lies in its being a check on the calculating of reserves held and net premiums received, and the correct entry of reserves released on policies cancelled. In my discussion of Mr. Kaufman's paper I pointed out how the accurate balancing of the Gain and Loss Exhibit is complicated by a number of incidents of the business, such as the issue of policies with premiums payable at intervals of less than one year, the issue of dated-back policies, the case of policies unsettled at the beginning but in force at the end of the year, changes in policies, etc. If the difference between the cost of insurance and the interest required to maintain the reserve is made a balancing item, we cannot test the correctness of our treatment of these cases.

It seems to me advisable for every company to make an investigation of its mortality classified according to the judgment of its actuary by years of issue, classes of insurance, ages at issue or ages

attained, or such combinations of these factors as he needs for his investigations into the effect of medical selection, choice of plan of insurance, etc. If this be done the figures thus obtained can be used for the Gain and Loss Exhibit after necessary determinable modifications have been made. The interest required to maintain reserve is then calculated and an approximate balance should be obtained. If there be a large balance unaccounted for, an error is to be expected in some item and an effort should be made to discover it. Mr. Davis' method precludes this, and for this reason as well as those given above, I cannot approve its use.

MR. HENDERSON:

This paper throws an interesting light on the construction of the Gain and Loss Exhibit. I do not think, however, that the author has gotten away entirely from the use of the cost of insurance as a balancing item. In view of the balance which must be maintained among the actuarial items of the Gain and Loss Exhibit, it follows that the balance required, which he denotes by A and expresses as the difference between the expected mortality on net amount at risk and interest required, may also be expressed as the difference of items already known. We have already on one side of the account the mean reserve at the beginning of the year and the net premiums earned during the year. On the other side of the account, we have the mean reserve at the end of the year and the reserves released on terminated policies, including in this the payments incurred for maturing endowments and annuities.

Referring now to equation (2) on page 27 and noting that the interest required is determined by deducting A from the expected, we see that the interest required can be calculated directly by adding one half A to the mean of the mean reserves and multiplying the sum by i; substituting then in this calculation the second expression for A referred to, we find that this is the equivalent of taking the mean reserve at the beginning of the year and adding to it one half the excess of the net premiums over the reserves released. The further adjustments referred to by the author would also be required under this method.

If we designate the cost of insurance by C and the interest required by I, so that A is equivalent to $C - I$, and if we take into account only the first adjustment, modifying it so as to use one half the terminal reserve released by death instead of the mean reserves released by death after the anniversary, we have the following equation, where $S^D V$ denotes the terminal reserves released on death claims,

$$I = \frac{i}{2}(C - I + S_1 M_1 + S_2 M_2 + S^D V).$$

It may also be expressed as follows:

$$I = \frac{i}{2+i}(C + S^D V + S_1 M_1 + S_2 M_2).$$

The cost of insurance plus the actual reserves released by death will approximately equal the expected death claims, so that if an estimate can be made of the expected death claims on the business valued at the different interest rates, we see that a closer approximation to the average rate of interest can be obtained by using the force of interest as a basis expressed approximately by $\frac{2i}{2+i}$ and using one half the expected death claims as a correction to the mean of mean reserves.

MR. DAVIS:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

Mr. Sheppard points out the advisability of an investigation of the mortality experience of each company subdivided by years of issue, etc. While, however, such an investigation would doubtless prove of considerable value to a large company, my suggestion of a method of determining the total expected mortality for any calendar year was written from the view point of a small company whose business would not be large enough to justify such subdivisions.

His objection, however, that the method presupposes exact accuracy in all the other items of the exhibit, and that errors of considerable magnitude may be allowed to remain undetected is well-founded. Yet, for a company of the nature described, I still believe that the method will work well.

The items whose accuracy is assumed are the mean reserves of the current and preceding year, the net premiums of the year, and the reserves released. Of these, the mean reserves may be checked against the State Insurance Department valuations; the net premiums may be carried forward from year to year, an independent check being made every three or four years; while the reserves released are usually calculated and checked month by month during the year.

The method suggested may easily be shown to be equivalent to the use of the following formulas.

(*a*) for the Expected Mortality,

the mean reserve of the previous statment with one year's interest together with the net premiums of the year with one half year's interest, and less the mean reserve of the statement and the reserves released with one half year's interest; or, in symbols

$$S_1 M_1(1+i) + S_2 P\left(1+\frac{i}{2}\right) - S_2 M_2 - (S_1 - S_2) V_1\left(1+\frac{i}{2}\right).$$

(*b*) for the Interest Required,
one year's interest on the previous year's mean reserve with one half year's interest on the net premiums of the year and less one half year's interest on the reserves released; or in symbols

$$S_1M_1 \cdot i - S_2P \cdot \frac{i}{2} - (S_1 - S_2)V_1 \cdot \frac{i}{2},$$

where S_1 = amount of business in force at beginning of calendar year,
S_1M_1, S_1V_1 = mean and terminal reserves for the then current policy year, and cost for that year,
and $S_2, M_2 \ldots$ are similar figures for the end of the calendar year.

To demonstrate the first of these formulas, we have

$$(V_0 + P)(1 + i) = K_1 + V_1,$$

$$(V_1 + V_0 + P)(1 + i) = K_1 + V_1(2 + i),$$

or,

$$M_1(1 + i) = \frac{K_1}{2} + V_1\left(1 + \frac{i}{2}\right)$$

Also,

$$(V_1 + P)(1 + i) = K_2 + V_2,$$

$$(V_1 + P)(2 + i) = K_2 + V_2 + V_1 + P,$$

or,

$$(V_1 + P)\left(1 + \frac{i}{2}\right) = \frac{K_2}{2} + M_2.$$

Hence,

$$S_1M_1(1 + i) + S_2P\left(1 + \frac{i}{2}\right) - S_2M_2 - (S_1 - S_2)V_1\left(1 + \frac{i}{2}\right)$$

$$= \frac{S_1K_1}{2} + S_1V_1\left(1 + \frac{i}{2}\right) + S_2P\left(1 + \frac{i}{2}\right) - S_2M_2 - (S_1 - S_2)V_1\left(1 + \frac{i}{2}\right)$$

$$= \frac{S_1K_1}{2} + S_2(V_1 + P)\left(1 + \frac{i}{2}\right) - S_2M_2$$

$$= \frac{S_1K_1}{2} + \frac{S_2K_2}{2}$$

= Expected Mortality.

While for the second we have

$$S_1 M_1 \cdot i + S_2 P \cdot \frac{i}{2} - (S_1 - S_2) V_1 \cdot \frac{i}{2}$$

$$= \frac{i}{2} S_1(2M_1 - V_1) + \frac{i}{2} S_2(P + V_1)$$

$$= \frac{i}{2} S_1 I_1 + \frac{i}{2} S_{22}$$

= Interest Required.

The formulas assume that all terminations take place on policy anniversaries during the year. The correction for S^D losses occurring during the year after their anniversaries may easily be shown to be $+ iS^D M_2$, which may be approximated with sufficient accuracy as $iS^D V_2$.

MORTALITY EXPERIENCE OF THE ÆTNA LIFE INSURANCE COMPANY UNDER ITS TEN YEAR RENEWABLE TERM POLICIES—MAXIMILIAN H. PEILER.

VOL. XIII, PAGE 30.

WRITTEN DISCUSSION.

MR. HENDERSON:

The experience presented by Mr. Peiler illustrates how effectively a rigid selection on the part of the company can be applied to a form of insurance which naturally lends itself to a selection adverse to the company. An examination of the experience analyzed by policy years, and the fact that the graduated rates of mortality on the aggregate experience, including all years of insurance, are substantially the same as when the first five years are excluded, would appear to indicate that this selection on the part of the company was not strong enough to overcome the adverse influence during the early years, but that it shows its effect during the later insurance years. It is doubtless assisted in this direction by the tontine feature, whereby the profits of a decennial period are not paid during the period but applied to the reduction of subsequent premiums. The fact that the premiums have continued up to an advanced age without any increase in the cost to the insured, and that no surrender values have been available in the meantime, must have operated to reduce to a minimum the rate of withdrawal, and thus to eliminate to a large extent that selection adverse to the company which is undoubtedly an influential factor in connection with lapses under term insurance.

It is rather difficult to get a comparison between the results shown by this experience and those of the experience which forms the basis for the rates of mortality adopted as a standard by the Medico-Actuarial Joint Committee. We notice, however, that the experience by policies, omitting the first five years, on the issues of 1885 to 1908 inclusive are 75 per cent. of the $O^{[M]}$ Table as compared with the total experience by amounts which is 86 per cent. of the same table. On the average, therefore, the experience by policies of the more recent issues would appear to be about 87 per cent. of the total experience by amounts. An examination of the table on pages 40 and 41, however, shows that the Ætna Life experience is a smaller percentage of the $O^{M(5)}$ Table at the younger ages than at the older ages, so that this percentage is somewhat affected by the younger average attained age among the

recent issues than among the older issues. I have therefore taken 90 per cent. of the graduated rates of mortality by amounts, excluding the first five years, as an approximation to the corresponding rates by policies on the issues of 1885 to 1908.

The following table shows the resulting rates compared with the M. A. table:

Age.	90 Per Cent. of Aetna.	M.A.
25	.0037	.0047
30	.0045	.0049
35	.0050	.0051
40	.0059	.0057
45	.0082	.0075
50	.0108	.0106
55	.0147	.0158
60	.0230	.0240

A noteworthy feature of this experience is the fact that the mortality by amounts is substantially higher than the mortality by policies in spite of the fact that the maximum amount insured is limited to $10,000.

I have assumed, although it is not clearly indicated, that the experience given on page 36 of whole life policies issued in exchange for 10 year renewable term policies at ages 70 to 79 is compared with the ultimate part of the O[M] Table and not with the select.

MR. GORE:

Mr. Peiler gives us the results of a very interesting experience, although under modern statutory conditions as to methods and periods of dividend distribution, its value as an illustration is somewhat doubtful.

While actually an experience of renewable term policies, it is evident that the rates of mortality exhibited are different from those normally experienced under this form of insurance. This difference is occasioned, as Mr. Peiler suggests, by two causes; first, the special care given in selecting the risks, an unusually high standard being required both as to physical condition and residence; and, second, the complete or almost complete absence of adverse selection at the conclusion of a term, resulting from the fact that policyholders were enabled to continue their insurances at the original premiums.

This last is undoubtedly a most important feature, and practically transfers the policies from the term to the non-participating whole life class. The resulting mortality rates, therefore, might be expected to show the effect of the adverse selection that a low rate of premium produces, offset by the special care exercised in selection.

To obtain an idea as to how far these contending forces have neutralized each other we may observe that the actual mortality by

policies of the issues of the years 1885–1908 was, for all policy years, 81 per cent. of the expected by the $O^{[M]}$ Table. On page 86 of the recently issued first volume of the Medico-Actuarial Investigation the experience by policies of forty-three companies on the issues of one month of each of the years 1885 to 1900, with observations carried to 1909, shows actual deaths to have been 80 per cent. of the $O^{[M]}$ expectation. By means of other tables in the same volume a rough estimate has been made, giving 78 per cent. as the proportion of actual to expected deaths under the issues of one month of each of the years 1885 to 1908—the precise period of the section of the Ætna experience in question. It thus appears that the unfavorable influence of a low premium rate was very nearly counterbalanced by special care in the selection of risks.

It is evident that no great amount of discontinuance took place at the close of the tenth year, but as the insured would no doubt be notified in some way of the conclusion of the first period, it is possible that some rise in the rate of discontinuance occurred at this point. If so, the effect on the mortality would probably have been most marked in the following year, the eleventh year of insurance, and it would perhaps have been interesting to have had the ratio for this year stated separately.

In Table II Mr. Peiler gives an aggregate mortality table compiled from the Ætna renewable term experience, and in view of his remark that "the ultimate object of the investigation was the construction of a mortality table on the basis of the monetary experience" it may be concluded that such aggregate table is based on the experience by amounts. The O^{M} (aggregate) is also given for comparison, and the conclusion that "the experience of the Ætna on its term policies has been much more favorable than the aggregate experience of the British companies on their whole life participating contracts" would at first sight appear to be amply justified, the values of q_x by the British table being uniformly in excess except at the relatively unimportant ages, 20–24 and 75–79, the maximum excess being nearly 30 per cent. of the O^{M} rate. Certain considerations suggest, however, that it is doubtful whether the actual experience has been nearly so much more favorable than that of the British Offices as the above figures indicate.

In the first place, a comparison between two aggregate tables depends for its validity upon the existence of similar proportions of new and old business. Now, the O^{M} Table is based upon the experience of British Offices from the policy anniversaries in 1863 to those in 1893, not only on issues of 1863 to 1892, but also on all issues in force at the policy anniversaries in 1863, while the Ætna experience is of issues commencing with 1868. The proportion of old business is very much greater in the case of the O^{M} Table, rendering a comparison between the aggregate tables of little value. A better comparison with the corresponding select table, the $O^{[M]}$, is given earlier in the paper, and from the second

of the two tables on page 35 we find that the Ætna experience, by amounts, was 90 per cent. of the expectation.

A further point is that the British table, being earlier in point of time, does not benefit by the improvement in mortality to the same extent as that of the Ætna. The first table on page 35 shows that the Ætna experience on the *issues* of 1868–1884 exhibited a mortality by amounts of 98 per cent. of the O[M] expectation, and the following table indicates that on a strictly step-by-step comparison this percentage would be increased.

Year of Insurance.	Corresponding Years of Experience	
	Aetna.	O[M]
1	1868–1884	1863–1892
2	1869–1885	1863–1892
3	1870–1886	1863–1892
4	1871–1887	1863–1892
5	1872–1888	1863–1892
10	1877–1893	1863–1892
20	1887–1903	1863–1892
30	1897–1908	1863–1892
40	1907–1908	1863–1892

Except for the first two or three insurance years the advantage gained from the general improvement in mortality is with the Ætna table, and very much so in the later years.

It should be noted also that the 98 per cent. referred to results from ratios of actual to expected of 121 per cent. in the first five insurance years, and 94 per cent. subsequently. It thus appears that a large excess mortality, as compared with the O[M] expectation, occurred when the reserves were lowest and the net amount at risk consequently at its maximum, while the compensatory savings occurred at a later period, so that the average of 98 per cent., if adjusted for the true financial effect, would have to be somewhat increased.

It is true that the experience by amounts is being considered, as that was apparently the basis of the aggregate table, but the difference between the results by policies and amounts is not in this case important. Taking all material factors into account the conclusion seems to be justified that the renewable term mortality experience reported by Mr. Peiler is little, if any, more favorable than that of the British participating whole life policies.

MR. PEILER:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

From certain points advanced in the remarks of Mr. Gore and Mr. Henderson it becomes evident that I have failed, in the description of the conditions of the experience, to lay sufficient emphasis on the high rate of withdrawal due to certain liberal options contained in the ten year renewable term contract. The

experience as first submitted for publication included a set of fundamental tables which, incidentally, would have served to indicate the withdrawal rate involved in the experience. When these tables had to be eliminated, for lack of space, the fact that all evidence of the withdrawal rate would thereby be obliterated was overlooked, and the necessary explanations of this feature were omitted.

The ten year renewable term contract contains several surrender options: a conditional cash value, available after the payment of a certain number of premiums, and provisions that after the first year the full value, reserve and surplus, existing at time of surrender may be applied in the purchase of paid-up life insurance or may be used toward the payment of new premiums on any of the company's life or endowment forms. From general observation of the material presently available it would seem that, by means of the latter option, the renewable term plan supplied other forms of insurance with the most desirable risks, originally selected for that plan, and that its mortality experience, taken by itself, was considerably weakened thereby. An analysis of these withdrawals would undoubtedly explain several of the points raised in the remarks of Mr. Gore and Mr. Henderson. Unfortunately the material which is available at present is only sufficient for a mere indication of the effects.

The following table shows the withdrawal rate, by policies, on that section of the renewable term issued from 1885 to 1908. The withdrawals include lapsed, surrendered for cash, surrendered for paid-up life insurance, and changed to other forms of insurance. On account of the labor involved, the separation of the lapsed policies, and the policies surrendered for cash from those changed to paid-up and to other forms could not be accomplished in time, but a cursory observation shows the number pertaining to the first two classes to have been comparatively small.

TEN YEAR RENEWABLE TERM POLICIES ISSUED FROM 1885 TO 1908.

Rate of Withdrawal Experienced by Policies to Anniversary in 1909, from the End of the First to the End of the Tenth Policy Year.

Ages at Entry.	1st Year, Per Cent.	2d Year, Per Cent.	3d Year, Per Cent.	4th Year, Per Cent.	5th Year, Per Cent.	6th to 10th Years, Per Cent.
15–29	41.0	16.5	13.3	9.8	9.6	4.5
30–39	36.7	15.3	11.2	8.2	9.3	4.0
40–49	32.6	11.7	9.1	7.5	7.2	3.1
50–60	25.6	9.6	7.7	4.6	4.5	3.7
15–60	36.3	14.3	10.9	8.1	8.6	3.9

In order to indicate in some way the comparative value of the risks remaining in the renewable term plan and of the risks changed to other forms, an aggregate table, the O^M, had to be

used for the expected losses, as it was impossible, within the given time, to trace the history of the latter class with regard to policy-years. The results were obtained by policy-years for remaining insurance, and in the aggregate for insurance changed to other plans. The material available pertained to the section issued from 1885 to 1908, and was based on amounts of insurance.

TEN YEAR RENEWABLE TERM INSURANCE ISSUED FROM 1885 TO 1908.

Actual Losses and Expected Losses, O^M Table, on Insurance remaining within the Plan and on Insurance changed to other Plans.

Policy Year.	Actual Losses.	Expected Losses.	Ratio.
2	$ 327,300	$ 375,885	.87
3	240,875	334,224	.72
4	270,700	302,974	.89
5	193,400	283,235	.68
6	203,200	262,970	.77
7 to 10	743,700	954,135	.78
2 to 10	1,979,175	2,513,423	.79
Total changed to other plans.....	$ 303,400	$ 472,862	.64

By adding the results for the insurance changed to those for the insurance remaining, the ratio pertaining to total insurance selected is made .76, three points lower than the ratio pertaining to the insurance remaining. In estimating an apportionment among policy years of the aggregate effect of selection as herein represented, two extremes may be assumed, as, on the one hand, the withdrawal rates for the earlier policy years are far in excess of those for the later years, and, on the other, the effect is not immediate but fades away gradually after the selection has been exercised. Roughly speaking the true result will therefore probably fall between a proportionate allotment and an allotment based on the weight of the withdrawal rate pertaining to the end of the preceding year.

With reference to the proportionately larger bulk of recently selected risks contained in the present experience, as compared with the experience of the British participating whole life policies, the following results based on amounts of insurance and the aggregate O^M Table will be interesting to observe:

TEN YEAR RENEWABLE TERM EXPERIENCE.

Policy Year.	Actual Losses.	Expected Losses.	Ratio.
Issued 1868 to 1884:			
1	$ 196,000	$ 197,873	.99
2 to 10	926,107	1,055,228	.88
11th and sub.	1,872,901	1,970,463	.95
Issued 1885 to 1908:			
1	$ 285,350	$ 535,255	.53
2 to 10	1,979,175	2,513,423	.79
11th and sub.	1,424,200	1,837,915	.77
Entire Issues:			
All Years	$6,683,733	$8,110,157	.82

SELECT RATES OF MORTALITY AMONGST IMPAIRED LIVES AND THE PROBABILITIES OF LIVES BECOMING IMPAIRED—

PERCY C. H. PAPPS.

VOL. XIII, PAGE 42.

WRITTEN DISCUSSION.

MR. HARDCASTLE:

Mr. Papps has done the Society a distinct service by directing attention again to the interesting subject of selection; a subject with which all actuaries are familiar in a general way, but which possesses interesting possibilities of development which are likely to be overlooked except by those who have to study them for some special purpose, unless brought to notice by a paper such as the author has given us.

In attempting to follow clearly the reasoning of Mr. Papps and Dr. Sprague on this subject, the first question that presents itself is what is meant by a "select" or an "impaired" life?

To form a definite conception of these ideas it is necessary to asume that impairment is a function susceptible of accurate, quantitative determination; and that there is some point in the scale of impairment, arbitrarily selected, on one side of which all risks shall be deemed select, and on the other side of which all risks shall be deemed impaired. For practical purposes this point is assumed to be located in such a way as to include as select such risks as life insurance companies would consider safe to insure at regular rates of premium. Thus "select" does not imply freedom from any impairment whatever, but merely freedom from impairment of sufficient gravity to exclude from acceptance at regular rates. In saying, therefore, that out of the survivors of a given number of select lives at age twenty so many are still select at age twenty-one, and so many have become impaired, it is implied that the latter number have in the interval crossed the arbitrary line of demarcation.

While, therefore, we may regard select lives as being such as are subject to the same rate of mortality as the freshly accepted risks of life insurance companies, it is not correct to assume that impaired lives correspond in respect to mortality with the rejected risks of life insurance companies, for among the former would be included lives so seriously impaired that they would not apply for insurance, and consequently, would not be subject to rejection.

According to the theory of select tables all lives becoming impaired must die within the period during which selection is effec-

tive after becoming impaired. This fact, in itself, would seem to indicate that the effects of selection cannot wear off in five years or ten, but rather that they never wear off.

We may obtain from any set of data, by varying the assumption as to the duration of the effect of selection, a series of curves of ultimate mortality running approximately parallel, but converging as the age advances, and coinciding after the assumed duration of selection has expired on the oldest entrants; and from the same data we can obtain a set of curves of select mortality which would in general intersect at some point the curves of ultimate mortality. Thus while it may be assumed that the rate of mortality among lives of a given age that have been insured exactly n years will be lower than that of lives that have been insured n years and more, yet we cannot assume anything as to the rate of mortality of those that have been insured exactly $2n$ years as compared with that among those insured n years and more, except that it is likely that each select curve will at some duration intersect successively the ultimate curves.

This is a very different situation from assuming a limited duration for the operation of selection, and merging the select tables in the ultimate table when this duration is reached

May it not be that the result of following select tables to the end, instead of merging them into an ultimate table, would be to produce considerably different values of the rate of becoming non-select, and considerably different rates of mortality among the non-select? It would at least spare us the necessity of assuming that all select lives must die within a specified period from the date of becoming non-select, which is a necessary corollary of the assumption of a limited period for the operation of selection, but which seems to conflict with everyday experience.

The dividing line between a select and an impaired risk is exceedingly fine. A death or two in the immediate family; a change of a few pounds in weight; a change in occupation, or any of a number of similar causes may be sufficient to transfer a risk from the select to the non-select class; and we know that these impairments are not necessarily fatal within a limited and comparatively brief period.

In Table G, Mr. Papps shows the rate of mortality among lives becoming impaired at age twenty-one as 41 per cent, in the first year; in the ninth year it rises from 41 per cent. to 67 per cent., and in the tenth year reaches 100 per cent. Surely such figures as these cast doubt on the soundness of the assumption of ten years as being the limiting period of selection.

If this assumption is a fallacy may it not be responsible for a wide variation from the truth in results predicated on its validity?

In Table A of Mr. Papps' paper we commence with 100,000 select lives at age twenty; by applying successively select probabili-

ties we arrive at the conclusion that 96,879 of these survive at age twenty-five. We assume that selection is no longer operative and that the ultimate rate of mortality prevails, and that, consequently, at age twenty-six there are 96,137 survivors. We then work backwards by select probabilities, and conclude that these 96,137 are the number surviving out of 99,264 select at age twenty-one. Hence we conclude that out of the original 100,000, 736 have either died or become impaired in a year, and, 420 having died, 314 must have become impaired.

In so indirect a process a slight inaccuracy in a fundamental assumption might well occasion a serious error in the final result.

These considerations would appear to confirm the surmise of Mr. Papps, that the incongruity disclosed by comparing the mortality of the impaired with that of the disabled, may be due to an error in the assumption that selection only operates for ten years, and suggest that it may not be necessary to look further for the explanation.

It may be, however, that the results are not quite so contradictory to what might be expected as might appear at first sight. The rate of mortality among the disabled after the first few years of disability is relatively low. It seems possible that the rate of mortality among the impaired may be in fact higher than that among the disabled. Impairment from disease may give rise to a higher rate of mortality than impairment from disability, especially if the insured should survive the first years after becoming disabled.

MR. MOIR:

The analysis and inquiry submitted to us by Mr. Papps is of great interest from a theoretical standpoint, as showing the possibilities of connecting invalidity with our ordinary select life tables. But there are underlying assumptions which seem to impair the value of the tables from a practical standpoint. In the first place it is perhaps desirable to make perfectly clear to students a point which was apparently in Mr. Papps' mind, viz.:—there is a clear distinction between *invalid* lives as understood in connection with disability insurance or health insurance, and *damaged* or *non-select* lives from a life insurance standpoint. We would not accept for life insurance a man with hardening of the arteries or a badly diseased heart, although he might go on working until the day of his death, and never be an "invalid" in the other sense.

The most serious assumption is that under select life tables all who become non-select die within a limited period. It is indeed a dismal outlook that none who are invalid should ever recover. On this subject I have been on record for about fifteen years as follows:

> "But the fault lies in the assumption made in the Mortality Table that the effect of selection passes away in five years. For this assumption to be accurate it is necessary that all those whose health deteriorates in any year must, as Dr.

> Sprague assumes in the investigation already referred to, die within five years of such deterioration, and this is quite inconsistent with fact. For such questions to be answered properly, Mortality Tables running strictly according to policy years, and tracing the entrants at each age separately to their destination, would be necessary."*

Mathematical rules lead to most dangerous results when there is any fallacy in the primary assumption. We all know how it is possible to follow mathematical rules and prove that 2 is equal to 1, the argument being based on the assumption that $0/0=1$. Now when Dr. Sprague first showed that his select tables involved the death of all the non-select lives in five years, he used the tables for calculations which would not be much affected by this error. But where the differences and ratios are used in the refinements of disability rates, the error is magnified and the results shown in the later years of Tables G and I submitted last May are simply absurd from any standpoint of common sense. The tables as published prove that these theoretical mortality rates depend more upon the assumed duration of selection than they do upon the rates of disability or of mortality. Accidental deaths are ignored and the rate of invalidity deduced from select life tables must be affected by such deaths as well as by the recovery of invalids. Again if we put $t=0$ in Mr. Papps' formula (4) we find that $d^{i}_{x} = 0$; this means another assumption that no one can become invalid and afterwards die within a year after selection.

But the force of selection is a very doubtful quantity varying between wide limits, not only as between one company and another, but also varying with reference to the class of policy taken. Mr. Papps quotes in the early part of the paper a section of the $O^{[NM]}$ Table, but he afterwards uses the O^{M} Table entirely for illustrative purposes. In the latter the effect of selection is much more noticeable than in the former, and net premiums for the option to take life insurance without medical examination as well as probabilities of becoming impaired must be entirely different when taken from one table or from the other.

Without making any very complete analysis of the question, I am inclined to think that the rates of mortality amongst disabled lives as published by Mr. Papps on page 52 are very seriously affected by the underlying assumptions to which I have above referred, and in consequence these mortality rates become less and less trustworthy as they extend away from the first year. If selection endured for three years only, the mortality rate in the third year (Table G) would be 100 per cent.

A similar problem is discussed by Messrs. Robinson & Ross in "Actuarial Theory," but from an entirely different standpoint (see pages 142–3). The system they follow brings about the following results for age 35:

* *Trans. Act. Soc. of Edinburgh*, Vol. IV, p. 283.

O[M] TABLE. AGE AT ENTRY 35.

Duration, n.	Mixed Lives, l_{x+n}	Select Lives and their Survivors, $l_{[x]+n}$	Difference being Damaged Lives.	Number of Damaged Lives Dying.	Rate of Mortality of Damaged.
(1)	(2)	(3)	(4)	(5)	(9)
0	85100	83586	1514	419	.2768
1	84379	83284	1095	264	.2411
2	83645	82814	831	206	.2479
3	82897	82272	625	171	.2736
4	82132	81678	454	142	.3128
5	81349	81037	312	114	.3654
6	80547	80349	198	88	.4444
7	79723	79613	110	62	.5636
8	78876	78828	48	36	.7500
9	78005	77993	12	12	1.0000
10	77105	77105	0		

In the above table the fourth column shows the number of non-select lives, the fifth column shows the differences of the fourth, being presumably those who either die or recover, and in the sixth column we have the combined rate of death and recovery. Of course this table is subject to the same fundamental error involved in assuming that selection endures for ten years only; and it is not comparable with the results of Mr. Papps. It deals with those who are damaged at the exact age 35 and traces these for 10 years thereafter, while Mr. Papps' Table G gives the first rate of mortality .35306 as taking place between ages 36 and 37. The additional table is given merely to illustrate the fallacy of building up a system of correct mathematical reasoning on an erroneous foundation. In such cases if two systems are used which may each be correct in themelves, they generally bring out different results.

MR. HENDERSON:

The close agreement shown in the rates of mortality for the first year after disability on page 52 is a striking illustration of the close agreement which sometimes exists between substantially unrelated functions. In making this statement I have no intention of calling into question the assumption which might be made that the mortality shown by the select table represents that of all lives insurable at the date of selection, each such life being followed in the experience until death occurs. Although it might be contended that the self selection exercised by those who refuse to be insured or who, having taken out a policy, subsequently terminate it, might interfere with this correspondence. Neither am I calling into question the assumption which might be made that the ultimate table represents the general mortality experience of the class from which the insured are selected.

The point which I desire to make is that, in order to apply the

method adopted in this paper to the case of totally and permanently disabled lives, it would be necessary to assume that the select table represents the mortality of those who, at the age at entry, were not so disabled, and I think it would be generally conceded that a select table based on the experience of lives insured in the ordinary class after medical examination would not represent such mortality. Even if a mortality table could be constructed to represent the future experience of lives not now totally disabled, the method adopted by Mr. Papps would labor under the difficulty that any errors or inaccuracies in such table would be magnified a thousand times in the resulting table on impaired lives. To illustrate my meaning in connection with this statement, take the method of determining the mortality among the 30 lives who, according to the assumption made in the paper, became disabled at age 21 last birthday and are still alive at age 30. This mortality is in effect determined by deducting the mortality on 88,501 lives surviving at age 30 who were insurable at age 22 from that on 88,531 lives at the same age who were insurable at age 21.

It is only fair to state that the author of this paper recognizes the fundamental objection to his method in the last sentence of the first paragraph of his paper which reads as follows:

> "If it were true that those who become non-select lives are really invalid lives, then the select tables would furnish us with the means of ascertaining the rates of invalidity and mortality amongst invalids."

MR. MOWBRAY:

Mr. Papps has rendered another service to our profession by again calling attention to the important subject of mortality rates as effected by medical selection, bringing the matter up in a new aspect, and calling upon us to review our foundations relative to this subject.

Medical selection does not tend to render more immune the selected lives. It therefore does not in reality affect the *mortality* of the *selected* group. Its only effect is to separate the group from which selection is made into two parts, the select group and the non-select, or impaired. Hence the true measure of the benefit of medical selection on the future mortality experienced by a company is the difference between the mortality of the select group and that of the original group from which selection was made, *i. e.*, the whole body of lives from which applicants are drawn, including those whose condition was such that the fear of inability to stand the test of examination prevented the making of an application. This difference is the mortality of the non-selects. From this it follows that the benefit of medical selection continues as long as the non-select group exists and is subject to a higher rate of mortality than the select group and no longer.

If this reasoning is correct, it should be possible mathematically to show "that the assumption that the effects of selection are exhausted in ten years, for example, results in all non-select lives dying of necessity within ten years after becoming non-select; unless it be assumed that a certain number regain their health and become once more select lives." This Mr. Papps has done and, working with the $O^{[M]}$ Table, has produced the "Rates of Mortality Amongst Disabled Lives" shown in Table I. Commenting on the fact shown in that table that the rate for disabled lives as derived from the $O^{[M]}$ Table by him is generally greater than the rate for totally and permanently disabled lives as derived by Mr. Hunter, he says: "This is so entirely at variance with what might be expected that it would be interesting to know the cause of the anomaly." He suggests "that it may be due partly to the fact that the period of selection really lasts much longer than the ten years assumed."

The foundation for this assumption of such a limit as ten years for the effect of medical selection is the observed fact that after the lapse of some such period of time we are unable to detect any appreciable relation between the mortality among selected lives and the elapsed time since selection. If this assumption is false what is the true explanation of the observed phenomenon?

The following relations point to a possible answer.

$$q_{[x]+t} = \frac{d_{[x]+t}}{l_{[x]+t}} = \frac{(d^i_{[x]+t} + d_{[x+t]})}{l_{[x]+t}} \tag{1}$$

$$= q^i_{[x]+t} \cdot \frac{l^i_{[x]+t}}{l_{[x]+t}} + q_{[x+t]} \cdot \frac{l_{[x+t]}}{l_{[x]+t}}. \tag{2}$$

Let

$$\frac{l_{[x+t]}}{l_{[x]+t}} = s_{[x]+t}$$

which we may term the index of resistance to impairment, then

$$\frac{l^i_{[x]+t}}{l_{[x]+t}} = 1 - s_{[x]+t}$$

and,

$$q_{[x]+t} = q_{[x+t]} \cdot s_{[x]+t} + q^i_{[x]+t}(1 - s_{[x]+t}). \tag{3}$$

a mathematical statement of the obvious fact that the rate of mortality for the group depends upon the proportion of select and impaired lives present in the group and the rate of mortality to which each class is subject. Of course, if $x + t$ is constant so is $q_{[x+t]}$.

At least for small values of t, $x+t$ being constant, $s_{[x]+t}$ becomes smaller as t increases because a larger proportion of non-select lives are included and consequently within this range $q_{[x]+t}$ increases with t, but for larger values of t this may not be the case.

If equation (3) be differenced as to t, $x+t$ being constant, it becomes

$$\Delta_t q_{[x]+t} = q_{[x+t]}\Delta_t s_{[x]+t} + \Delta_t\{q^i_{[x]+t}(1 - s_{[x]+t})\}. \qquad (4)$$

If $q_{[x]+t}$ is independent of t, then $\Delta_t q_{[x]+t} = 0$, and

$$q_{[x+t]} \cdot \Delta_t s_{[x]+t} = -\Delta_t\{q^i_{[x]+t}(1 - s_{[x]+t})\}, \qquad (5)$$

indicating that when the effect of selection is no longer observable the decrease in the proportion of select lives is such that the resultant decrease in the number of deaths from among such lives exactly balances the increase in the number of deaths among the non-selects.

Expanding the right hand member of (5) and collecting,

$$(q^i_{[x]+t} - q_{[x+t]}) \cdot \Delta_t s_{[x]+t} = (1 - s_{[x-1]+t+1})\Delta_t q^i_{[x]+t}$$

and by division, since

$$\Delta_t(q^i_{[x]+t} - q_{[x+t]}) = \Delta_t q^i_{[x]+t} \quad \text{and} \quad \Delta_t(1 - s_{[x]+t}) = -\Delta_t s_{[x]+t},$$

$$\frac{-\Delta_t(1 - s_{[x]+t})}{(1 - s_{[x-1]+t+1})} = \frac{\Delta_t(q^i_{[x]+t} - q_{[x+t]})}{(q^i_{[x]+t} - q_{[x+t]})}. \qquad (6)$$

Equation (6) expresses this condition of balance in a different and perhaps more useful form, viz., in terms of the rates of change in the proportion of impaired lives present and the excess of mortality rate for impaired lives over that for select lives.

A numerical example may make the matter a little clearer. Assume two groups of mixed lives composed respectively of 9,750 select and 250 impaired lives, and 9,700 select lives and 300 impaired lives. Let the rate of mortality among select lives at the age in question be 4 per thousand. Then if the rates of mortality among the impaired lives be respectively 244 and 204 per thousand the rate for the entire group in each case will be 10 per thousand.

The form in which select tables are now cast forces the assumption that $s_{[x]+t}$ is independent of t after the period of observable difference in mortality of the entire group is past, if $x+t$ does not change.

From Mr. Hunter's and Mr. Mead's tables of mortality rates among totally disabled lives it appears that for impaired lives of that class $\Delta_t(q^i_{[x]+t} - q_{[x+t]})$ is probably negative indicating that for impaired lives of that class at least the effect of selection does persist after the time when its observable effect upon $q_{[x]+t}$ is lost.

These considerations indicate that the rates of mortality shown in Table I for the longer durations are excessive.

The rates for the shorter durations may perhaps be better tested by approaching the subject anew and considering the nature of medical selection from a different point of view.

The knowledge of the requirement of examination prevents the presentation of applications from those suffering from acute and known chronic functional disorders and the more advanced stages of degenerative organic changes. The examination itself weeds out those in the less advanced stages and discovers previously unnoticed chronic functional disorders and latent possibilities of acute diseases. The weeding out of sufferers from organic lesions and chronic disorders may be expected to have an enduring beneficial effect. The effect of weeding out those suffering from acute disorders will hardly be so long noticed since recovery or early death is a more probable outcome than degeneration into chronic disorder or organic lesion, and no protection is furnished by examination against infection soon after it is passed. These acute cases impress one as about what was referred to in Gompertz' suggestion that "death may be the consequence of two generally coexisting causes," one being the existence of "a number of diseases to which young and old are equally liable" and which are "equally destructive whether the patient be young or old."

The O[M] Table is graduated according to Makeham's law in the form adapted to select tables and in the "Account of Principles and Methods" issued in connection with B. O. Tables at page 157, Mr. Hardy, the graduator, says:

> "Speaking generally, it may be said that the effect of selection upon the constant α is much the greater for the first two or three years after entry, but is somewhat rapidly exhausted, not being very important after the fifth year. The effect on the constant β is, however, much more durable, and has by no means worn off at the end of ten years."

It seems reasonable to suggest that the change in α may be due to the effect of selection in having temporarily reduced the number of lives present suffering from acute disorders, and the change in β to the reduction in the number suffering from chronic disorders and organic lesion. If this be the case a separation of the impaired lives into two groups along these lines ought to give different rates of mortality for the separate groups.

The separation may be approximately made as follows:

$$d_{[x]+t} = l_{[x]+t} \cdot q_{[x]+t} = \int_t^{t+1} l_{[x]+t} \mu_{[x]+t} dt, \qquad (7)$$

$$\left(\text{since } q_{[x]+t} = \frac{1}{l_{[x]+t}} \int_t^{t+1} l_{[x]+t}\mu_{[x]+t}dt\right)$$

$$= \int_t^{t+1} l_{[x]+t}(A_t + B_t c^{x+t})dt. \tag{8}$$

Since $l_{[x]+t}$ does not change much in the interval from t to $t+1$ this may be approximately written,

$$d_{[x]+t} = l_{[x]+t} \int_t^{t+1} A_t dt + l_{[x]+t} \int_t^{t+1} B_t c^{x+t} dt,$$

also

$$d_{[x+t]} = l_{[x+t]} \int_0^1 A_t dt + l_{[x+t]} \int_0^1 B_t c^{x+t} dt,$$

whence

$$d^i_{[x]+t} = l_{[x]+t} \int_t^{t+1} A_t dt - l_{[x+t]} \int_0^1 A_t dt + l_{[x]+t} \int_t^{t+1} B_t c^{x+t} dt - l_{[x+t]} \int_0^1 B_t c^{x+t} dt. \tag{9}$$

Again

$$l_{[x]+t} \int_0^1 A_t dt = l_{[x+t]} \int_0^1 A_t dt$$

approximately.

Then

$$A_{d^i_{[x]+t}} = d^i_{[x]+t} \cdot \frac{\int_t^{t+1} A_t dt - \int_0^1 A_t dt}{q_{[x]+t} - q_{[x+t]}},$$

and

$$B_{d^i_{[x]+t}} = d^i_{[x]+t} - A_{d^i_{[x]+t}}.$$

The computation work is simple, since

$$A_t = \frac{1}{M}\left(\alpha + \frac{d}{dt} f_t\right)$$

whence

$$\int_t^{t+1} A_t dt = \left(\frac{t\alpha}{M} + \frac{f_t}{M}\right)_t^{t+1},$$

and

$$f_t = m(10 - t)^2 + m'(c')^t$$

where

$$M = \log_{10} e = .43429448,$$

$$\alpha = .0026111, \quad m = .000040955, \quad m' = .00112 \quad \text{and} \quad c' = .24,$$

$$A_{d^{i}_{x+t}}^{i[x]} \quad \text{and} \quad B_{d^{i}_{x+t}}^{i[x]}$$

may be found by a formula identical with Mr. Papps' formula (10) and

$$A_{l^{i}_{x+t}}^{i[x]} \quad \text{and} \quad B_{l^{i}_{x+t}}^{i[x]}$$

determined by the summation of the d's.

From these

$$A_{q^{i}_{x+t}}^{i[x]} \quad \text{and} \quad B_{q^{i}_{x+t}}^{i[x]},$$

the rates of mortality corresponding to the two classes, determined.

I have tested this theory for age at entry 30 years with the following results.

t	$d^{i}_{[30]+t}$	$A_{d^{i}_{[30]+t}}$	$B_{d^{i}_{[30]+t}}$	$d^{i}_{[31]+t-1}$	$A_{d^{i}_{[31]+t-1}}$	$A_{d^{i}_{[31]+t-1}}$
1	158.95	147.19	11.76			
2	218.89	194.81	24.08	158.88	145.69	13.19
3	255.20	216.92	38.28	219.57	193.66	25.91
4	286.03	234.83	51.20	256.84	216.26	40.58
5	315.58	249.62	65.96	288.68	232.38	56.30
6	345.01	263.59	81.42	319.33	247.80	71.53
7	374.52	278.64	95.88	349.82	261.67	88.15
8	404.13	292.19	111.94	380.44	276.20	104.24
9	433.92	305.48	128.44	411.25	289.52	121.73
10	453.29	310.50	142.79	442.27	302.50	139.77
t	$A_{d^{i[30]}_{30+t}}$	$A_{l^{i[30]}_{30+t}}$	$A_{q^{i[30]}_{30+t}}$	$B_{d^{i[30]}_{30+t}}$	$B_{l^{i[30]}_{30+t}}$	$B_{q^{i[30]}_{30+t}}$
1	147.19	328.09	0.448	11.76	90.33	0.136
2	49.12	180.90	.262	10.87	78.57	.138
3	23.26	131.78	.176	12.37	67.70	.183
4	18.57	108.52	.171	10.62	55.33	.192
5	17.24	89.95	.192	9.66	44.71	.213
6	15.79	72.71	.217	9.89	35.05	.282
7	16.97	56.92	.298	7.73	25.16	.307
8	15.99	39.95	.400	7.70	17.43	.441
9	15.96	23.96	.666	6.71	9.73	.689
10	8.00	8.00	1.000	3.02	3.02	1.000

Commenting on these results it may be noted that, since the original observation is of rates of mortality only, there is nothing to bring into the account lives becoming impaired, whose subsequent recovery prevented that impairment showing its presence by a change in the mortality of the group. Hence the vast number of temporary impairments from acute diseases are unrecorded. In the light of this fact the heavy mortality shown the first two years

after impairment is perhaps not unexpected. The group of lives under class B probably more nearly resembles in its composition what we ordinarily think of when the term "impaired lives" is used than either the combined groups or group A alone.

The rates at the longer durations in both groups are undoubtedly excessive, due to the forced assumption that the effect of selection has worn off at the end of ten years. This difficulty in the way of determining $q_x^{i(x)}$ by the use of select tables seems to be insurmountable unless we can find some way of determining $s_{[x]+t}$.

The fundamental assumption which is at the root of the entire difficulty is that the rate of impairment among the select lives remaining in an originally select, but now mixed, group is independent of the time elapsed since the original selection. There may be ground to question the truth of this assumption.

As pointed out by Mr. Weber, as quoted by Mr. Mead (*T. A. S. A.*, XII, 76), the mortality among totally and permanently disabled lives is subject to a similar analysis, one element being due to the presence of "the blind, insane, those with both arms or legs amputated, and others whose chances of longevity would not be greatly lessened although incapacitated for active work" (Hunter, *T. A. S. A.*, XII, 49). This fact should also be borne in mind in comparing the mortality of impaired and disabled lives.

MR. LITTLE:

If in equation (1) of Mr. Papps' paper the symbol $(ul)_{[x]+t}$ be substituted for its equivalent $l^i_{[x]+t}$, the equation becomes

$$(ul)_{[x]+t} = l_{[x]+t} - l_{[x]+t},$$

as Dr. Sprague originally wrote it. It rests therefore upon very high authority, and upon the foundation it affords Mr. Papps has erected an interesting superstructure.

It will be noticed that the number of select lives out of $l_{[x]+t}$ is taken as $l_{[x+t]}$ and as this latter section will provide all the survivors after a further period of n years, where n is equal to the maximum period for which the effects of selection last, none of the non-select group can survive n years. Mr. Papps qualifies this by adding "unless it be assumed that a certain number regain their health and become once more select lives," but this qualification is inadmissible according to the formula, which irrevocably dooms the unfortunates to extinction within n years, the period for which selection is effective.

I submit, however, that formula and deduction are wholly wrong. If n years is the period at the end of which selection is exhausted, lives becoming non-select do not necessarily die within n years thereafter, $l_{[x+t]}$ is *not* the number of select lives included in $l_{[x]+t}$ survivors of $l_{[x]}$ lives select at age x, and $l_{[x]+t} - l_{[x+t]}$ has no con-

nection whatever with the number of non-select lives included in $l_{[x]+t}$.

That the assumption referred to is invalid will, I think, be admitted when it is remembered that it requires that the moment a life becomes non-select it must fail within a comparatively short period; in other words the insured lives, originally all "select," are either very good or very bad, with not a solitary life in any intermediate state of health. If the whole body of the existing insured of a life office were re-examined for insurance, most unquestionably some could be accepted select, and some must be rejected, but there would also be a number eligible for acceptance at a rating-up. Of these latter a majority would survive a further ten years, and even the rejected lives would be far from extinct in that period.

The assumption objected to would be exactly paralleled by the supposition that if one farm of 50 acres produced 1,000 bushels of wheat and another of 70 acres produced 1,100 bushels, the latter *must* consist of 50 acres of land of quality equal to that of the first farm, producing 20 bushels per acre, plus a further 20 acres producing only 5 bushels per acre.

If further proof of my contention were necessary, it would be supplied abundantly by Tables H and I of Mr. Papps' paper. In the former of these tables Mr. Papps produces rates of becoming non-select many times greater than the rate of total disablement. In view of this, and of the obvious fact that totally disabled lives form the worst section of non-select lives, Mr. Papps looks for lower rates of mortality among the non-select than among the totally disabled, but finds the reverse to be very markedly the case. Probably the true explanation is that the total extra mortality is distributed over too few lives, that the number of non-select lives in $l_{[x]+t}$ is greater than $l_{[x]+t} - l_{[x+t]}$ and the rate of mortality of these lives very much less than shown.

I have used the word "probably" in the preceding sentence advisedly, as I am inclined to think that it is not a full explanation. It is at least possible that, if the lives were re-examined after payment of the second year's premium, those then passed as still select would not be quite equal to newly-selected lives of the same age; in other words, there would be a smaller percentage of specially good lives, due to the adverse influence of withdrawals. Of this there is, of course, no evidence available and it must rest merely as a surmise. In any case its effect is likely to be trifling.

We may therefore conclude that select mortality tables as at present constituted give us no means of ascertaining the proportion of select lives among the survivors of those who have been insured for some time. Consequently, rates of invalidity, or of becoming impaired in health, and rates of mortality of impaired lives deduced from any such tables are purely factitious, and conclusions drawn therefrom are unreliable.

It may be well to mention in conclusion that although Dr. Sprague apparently makes use of the fallacious assumption in calculating "option" premiums, yet, as the total extra mortality and not its distribution is in question, his resulting premiums are correct. Thus his single premium for the option of effecting an insurance (whole life) at the end of n years is

$$\frac{(nD)_{x+n}}{D_{[x]}}(uV)_{x+n}$$

and though this formula treats the extra mortality as confined to $l_{[x]+n} - l_{[x+n]}$ out of $l_{[x]+n}$ survivors, as Mr. Moir has pointed out (*T. F. A.*, II, 242), it produces the same result as

$$\frac{P_{[x]+n} - P_{[x+n]}\,N_{[x]+n}}{D_{[x]}}$$

which makes no assumption at all as to the incidence of the extra mortality.

MR. MURPHY:

Mr. Papps' paper is based upon an assumption with regard to select mortality tables originally brought forward by Dr. Sprague in 1881 and which, I think, has not been questioned. It seems in the light of the recent investigations into the mortality among impaired lives that this assumption was erroneous. The number of select lives surviving at the end of t years from the group entering at age x has been assumed to be the number of persons entering at age $x+t$. This is not necessarily so. Among $l_{[x]+t}$ lives there cannot be more than $l_{[x+t]}$ select lives, for these two groups must produce the same number of mixed lives n years hence, where n is the period of selection, and $l_{[x+t]}$ select lives are sufficient to furnish the required number of mixed lives. We therefore have determined the maximum number of select lives that can exist among $l_{[x]+t}$ mixed lives. But the point which should be emphasized is that the exact number is not determined.

Dr. Sprague's hypothesis seems to be that of any group of mixed lives the select sub-group will show a certain number of death losses dependent only upon the size of the group and the attained age, which is true, and that the subgroup of impaired lives will also show a number of death losses dependent only upon its size and attained age, which is probably not true. With this hypothesis, however, it is clear that to fit the conditions of the ultimate table there must be $l_{[x+t]}$ select lives in every group of attained age $x+t$, so that there may be exactly the same number of select lives when they all reach the ultimate table, and consequently the same number of impaired lives. The next point was then obvious, that all impaired lives must die in the following n years after becoming impaired.

The above hypothesis seems inconclusive when it is remembered that, as a rule, the mortality among disabled lives decreases with the increase in duration since disability. If non-select or impaired lives should show the same trend, it is apparent that we may have, at the same attained age, two equal groups of mixed lives in which the first has fewer select lives and consequently more impaired lives, but whose impaired lives have on the average a longer duration since impairment, and yet have the two groups of mixed lives show the same aggregate mortality. Dr. Sprague states that "When we say the benefit of selection wears off in five years, we mean that after five years a body of lives originally select will contain the proportion of damaged lives that will give us the $H^{m(5)}$ rate of mortality." It was not apparently considered, however, that the proportion of damaged lives might vary with the age at entry. That the proportion of damaged lives among a group at age 60 who had entered at age 20 should be larger than among those who entered 10 years before at age 50 seems quite reasonable, and still have the ultimate mortality rate realized in each case.

For those who became damaged lives at age 21, for instance, and had survived 39 years, would very possibly show a lower mortality rate than any among the age 50 entrants who had become impaired in the last ten years. This also recalls the possibility that some damaged lives may later become select lives again, as Mr. Papps suggests, and upset his formulas.

If the above suggestions are followed out we arrive at the conclusion that Mr. Papps' rates of impairment are too low throughout and his rates of death among impaired lives too high. A select mortality table does not furnish us with definite data for the calculation of such rates, but we must have in addition either the rates of impairment or of mortality among impaired lives.

MR. E. B. FACKLER:

Mr. Papps' entire article is based upon acceptance of Dr. Sprague's theory that there is a direct relation between the survivors of select lives and the select lives entering observation at the advanced age. On this assumption his conclusions follow mathematically, subject to the considerations which he so clearly brings out regarding the limitation of the select period and the possibility that lives may regain their select character after once losing it. If, therefore, there is any doubt of the correctness of Dr. Sprague's theory as to the properties of select tables Mr. Papps' results are to that extent invalidated.

Select tables are based upon actual statistics of groups of lives entering at the several ages and traced through their successive years of observation. It, therefore, follows in a graded table that the group remaining select at, say, age 21 from those who were select at entry at age 20 does not, as a matter of statistics, repre-

sent the lives which correspond to the group entering as select lives at age 21, except as any individual represented in the group might have taken out policies in successive years. As the individuals making up the two groups are, as a matter of statistics, not identical, the statistics do not support us in assuming that the number of select lives is the same in both cases. Again, if we assume for the purposes of argument that the numbers are equal, it does not follow from the statistics that the groups are equal as regards their "selectness." Dr. Sprague's rule assumes, in effect, that the select lives remaining in a group after one year's exposure would be of precisely the same grade of selectness as lives of the same age newly examined.

Our conception of what a select life is needs careful definition. Having in mind the sources of the $O^{[M]}$ and similar tables, select lives may be defined as, lives considered acceptable for new insurance at ordinary rates at the time of application to regular companies. The expression "select life" is, therefore, not synonymous with either a "perfect life" or an "average life," and we have reason to believe that the standard of acceptability was not uniform throughout the period of observation.

When entering observation by medical examination the risks of a company taking only standard risks may be said to grade upward from those who are just barely acceptable to those whose acceptability is unquestioned, and considering the former as being 100 per cent. acceptable, the latter may be assumed in many cases to be 150 per cent. acceptable, or above that. We may also conceive of a gradual tendency to impairment in all lives, and this would operate differently on the two extremes of the select class. Many of those who were classed as 150 per cent. acceptable might suffer impairment for several years before being ruled out of the select class, while those just barely acceptable might be expected to suffer considerable losses into the non-select class within a short period. It is, of course, well understood that many causes of impairment would have the same effect on either class.

It would thus seem, from general reasoning, that the select survivors of any group of lives would gradually suffer a loss in grade, so that the rate of loss to the non-select class might be expected to increase with the duration of the exposure.

It is appreciated that this criticism of Mr. Papps' article is not in any way constructive, nor does it lead us any closer to the facts regarding rates of impairment and mortality. How any statistics can be obtained to prove mathematically the correct theory in this case is hard to imagine. An institution such as a college which requires annual physical examinations of all its members might throw some light on the subject, but this data would refer only to a limited class at the younger ages and would cover only four years for any considerable group. If statistics were taken in the same way by some railroad or industrial establishment, we might get

some sort of basis for calculations as to the death rate among healthy lives and the rate of impairment. The mortality statistics regarding the impaired lives would probably, however, be defective because of the dismissal of many employees on becoming impaired.

MR. MEAD:

While Mr. Papps' paper on "Select Rates of Mortality Amongst Impaired Lives and the Probabilities of Lives becoming Impaired" may not be of direct practical use, it is of especial value to the actuary as a study to improve his technique. Mr. Papps' paper brings squarely before us the whole question of the construction of select tables and their properties, as well as of the nature of medical selection, and what may be termed the movement of selection and impairment in a body of insured lives. This question is especially opportune at this time in connection with our study of specialized mortality experiences, and in view of the fact that in the near future the Society may construct a series of standard American mortality tables, one of which would doubtless be a select table for office, if not for general, use for the computations of premiums and reserves.

Toward the close of his paper Mr. Papps calls attention to the fact that the mortality amongst impaired lives is shown to be in excess of that amongst disabled lives after the first year and that this is so entirely at variance with what might be expected that it would be interesting to know the cause of the phenomenon. I do not believe that there can be any question that those impaired lives which are totally and permanently disabled will show a much heavier mortality than the lives which are merely impaired or non-select, for the totally and permanently disabled are not only impaired, but so badly impaired that they cannot work and are never expected to work again. Consequently, as the results are erroneous, some of the assumptions made in obtaining those results must be erroneous and it would be interesting to trace the source of these anomalous or erroneous results.

In the earlier part of his paper Mr. Papps also refers to the assumption which has been current for the past thirty years or more, namely, that since the effects of selection are exhausted in s years, all lives becoming non-select must of necessity die within s years after becoming non-select. In other words, in accordance with the latest British experience we find that those who took non-participating whole life insurance die within five years after becoming impaired, and those who took participating whole life insurance die within ten years after becoming impaired. A study of this question has led me to believe that some of the assumptions that have been made in the past as to the properties of select tables are erroneous and this I shall attempt to show.

Strictly speaking, it is a misnomer to say that the effects of medi-

cal selection wear off in five or ten years. It would be more proper to say that the *immediate* effects of selection do so, and that medical selection itself extends throughout the experience.

For instance, the benefit of the exclusion of those with heart murmur or light-weight dyspeptics is felt throughout the experience. Many other classes of risks more or less impaired might be cited such that, if there were a large enough group of them, there would be some surviving to extreme old age, although the mortality throughout would be heavier than that of standard risks. The exclusion of these by selection would therefore affect the mortality throughout. In connection with the graduation of the $O^{[M]}$ experience Mr. Hardy calls attention in his lectures on "The Theory of the Construction of Tables of Mortality" to the fact that the effect of selection as reflected in the values of the alpha constant disappears after about seven years, whereas the effect upon the values of beta constant probably continue throughout the whole of life, the explanation being that the alpha or A constants represent mortality from accidental causes and from non-constitutional diseases of short duration, whereas the beta or B constants represent mortality due to diseases of longer duration and to constitutional defects.

The select entrants at any age represent a body of lives which would be generally such that the consensus of opinion would be that each individual would be entitled to a standard policy, although embraced among these so-called select lives would be some borderline risks that certain individual approving officers would cast out as being too greatly impaired or non-select. The mortality of the select lives increases rapidly through the wearing off of the immediate benefits of selection, until after from about five to ten years the mortality of any group of entrants at any particular age approximately equals the mortality for the same attained age of entrants at other ages after the immediate effect of selection has disappeared. However, we are safe in saying that after the period of the immediate effects of selection, the mortality, owing to the *permanent* effects of selection, is more favorable than that among the general population.

For instance, the ultimate mortality of the new Medico-Actuarial Table at age 30 is only 59 per cent. of the graduated rate of mortality in the "registration states" according to the census of 1900; at age 40 it is only 52 per cent., and at age 60 the ultimate mortality of the M. A. Table is 82 per cent. of the census table. Owing to the fact that the ultimate mortality of the M. A. Table is so extremely low, it appears to me that we have one of the principal reasons why the period of the immediate effect of selection is so short. It would not be reasonable to consider for a moment that the entire effect of selection has worn off in from three to four years or that all becoming impaired die within three to four years. It is, in my judgment, only that the *immediate* effects of selection have worn off.

Now let us consider the section of the $O^{[NM]}$ Table which Mr. Papps gives on page 43. The relative numbers of mixed lives surviving in the last column are obtained by applying the graduated probabilities of surviving derived from data based upon the experience of lives which have been insured five years or more. The radix relating to select entrants at any age $[x]$ is obtained by working back from those living at age l_{x+5} by applying the reciprocal of the probabilities of surviving according to the select data. This is done as a matter of convenience to abbreviate the extent of the tables. The $l_{[25]}$ select lives in the first column have no direct relations with the l_{25} mixed lives in the last column. It is true that in the table these are reduced to the same number living at attained age thirty, but the probabilities of surviving which bring this about in the two instances are based upon entirely separate and distinct data, in the one case the actual data relates exclusively to new entrants at age 25, whereas in the other the data relates to entrants at ages 24 and under, combined according to attained age. The table merely shows that the mixed rates of mortality are so much greater than the select that we must have 96,879 mixed lives existing at age 25 to reduce to 93,124 at age 30, as compared with 96,316 select lives for entry at age 25 to reduce to the same number of mixed lives living at age 30. It, therefore, seems improper to me to assume that by subtracting $l_{[25]}$ from l_{25} we obtain the number of impaired lives existing amongst the l_{25} mixed lives. I do not consider that it is proper to separate in this way the l_{25} lives into so many select and so many impaired. If we do make this separation we reach the erroneous conclusions already referred to. We have here in the l_{25} mixed lives a general body representing all gradations from extra select to extremely impaired, and we have merely a mass subject to a certain general rate of mortality. Some of the lives are doubtless in an even better state of health than when originally selected.

Or, to approach the question from a slightly different angle, the 96,879 lives existing at age 25 might all be impaired slightly and to the same extent, and none of them might individually be considered deserving of rejection, *i. e.,* individually they might be acceptable border-line cases, and the rate of mortality amongst them be such that they would be reduced to 93,124 in number at age 30 as in the ultimate section of the $O^{[NM]}$ Select Table.

While, strictly speaking, the 99,580 mixed lives at age 21 resulting from 100,000 select entrants at age 20 may not be definitely separated into 99,264 select lives and 316 impaired, we may say that this is *in effect* an approximately correct assumption, particularly for the first year after selection; but to assume that these same 316 lives may be segregated and taken as impaired and traced until they are *all* dead within five years after becoming impaired appears to me erroneous and the basis of the error to which I have referred.

However, it does not appeal to me that the method of graduation affects the validity of the foregoing argument. It may be conceded that the $O^{[M]}$ and the $O^{[NM]}$ data might have been graduated by some experimental or natural method such as the graphic and approximately the same rates obtained, in which case the radix at each age of entry would be obtained by the original method of working back from the ultimate column.

Take any group of lives which become impaired, say on account of hemorrhage from the air passages, renal and hepatic calculus, appendicitis, stomach or liver trouble or rheumatism, these will not only be depleted by deaths, but some of them will become once more insurable. Some of them, which continue to be uninsurable, will live to extreme old age. We have all of us known of rejected risks like this. Then why should it be assumed that those which become impaired after being accepted for life insurance die within a certain number of years?

I believe that Mr. Papps has given one portion of the solution of this problem when toward the close of his paper he states that it may be that lives which have become impaired may later on once more become select and so upset the formulas on which his paper is based.

Every impairment manual recognizes the possibility of becoming insurable after being impaired. Those who have had urinary calculus will be considered for insurance after three years during which time they are considered impaired. Those who have had biliary calculus are considered impaired for five years, appendicitis from six months to a number of years according to whether an operation has been performed or the number of attacks, pneumonia for at least six months, rheumatism for one year or more, and so on. During the period of postponement as a whole those who have these or other similar impairments are subject to a heavy mortality—some survive for years as impaired risks while others become subsequently insurable.

The other chief factor of the solution seems to me to be that it is not proper to assume that the mixed lives of a select table may be separated into so many select and so many impaired lives. What we have is a relative number of survivors which in the mass are subject to a certain average rate of mortality. So many will become more or less impaired and all in the mass are subject to the general rate of mortality. As the lives move on from year to year some of them deteriorate more or less and a smaller proportion improve, but on the whole the quality of the mass deteriorates, the rate of deterioration being relatively high during the first years after selection. A select table is merely an instrument showing the relative numbers surviving according to age at entry and duration since selection. I do believe, however, that the assumption that the lives may be divided into so many select and so many impaired is more or less reasonable as an approximate calculation

for the first year of insurance, but for a longer duration disturbing influences come strongly into play which make the assumption untrustworthy.

ORAL DISCUSSION.

Mr. Strong: It seems to me that any conclusion about the number of impaired lives and the mortality among impaired lives reached directly from a graduated select table with a certain period of selection, is taking out of a select table a conclusion which the method of construction put into it. In other words the conclusion is from the method of formation of the table, not from the facts underlying the table. How extreme a result can be reached may be seen by making use of the section of the $O^{[NM]}$ table shown on page 43. If we compare $l_{[20]+2}$ with $l_{[22]}$ we find that the former shows 99,000 lives, the latter about 470 lives less. In other words, at the end of the second policy year, two years after the medical examination, there are only 470 impaired lives out of 99,000 lives. This is evidently not within the possibilities since the ordinary minor ailments, such as colds, would render more than that proportion of lives aged twenty-two uninsurable at the particular instant of time. Thus it seems evident that the number of non-select lives is not shown correctly by such comparisons under a select table. This is in addition to the evident impossibility that the conclusions as to the time within which lives becoming non-select can survive, can be correct. I consider, in view of such things and others that have been brought out in the discussion, that it is not possible to obtain any information as to the mortality among lives becoming non-select simply from numbers shown at the same ages but removed by different periods from entry in a select table, and from the mortality shown by that table.

Mr. Macaulay: I think that the point has been made quite clear to the most of us that a table of select statistics cannot be made the basis for a table of mortality on impaired lives. In particular I would like to emphasize what has been said on that subject by Mr. Little, Mr. Mead and Mr. Strong. It will perhaps help us to get a clearer grasp of this subject if we consider for a moment what it is that medical selection does. Medical selection merely eliminates, as far as we possibly can do so, all those who are suffering from any disease or have any peculiarity of constitution, or of environment, or any other peculiarity which would be likely to cause an abnormally heavy rate of mortality to prevail among them. Now these impairments can be divided into several groups. Medical selection, of course, shuts out all such acute cases as those of people suffering from smallpox, or typhoid fever, or diphtheria. Then it is supposed to shut out the chronic cases which run a rapid course, such as cancer. Then there are the cases that are of longer duration, Bright's disease, heart disease, and a good many other things. Then there are impairments that are not really diseases at

all, such as overweight, bad habits, residence in an unhealthy district, such as one of the swampy counties of Louisiana, or an occupation that is unfavorable, as for instance, a man working in a dynamite factory. So that we see that the cases which medical selection cuts out are not by any means cases all of which will necessarily cause death in five years or ten years or at any time. The cases of smallpox and typhoid fever will run a short duration, either favorable or unfavorable. Cases of Bright's disease or cancer will also run a comparatively rapid course. But cases of overweight, or of bad habits, or things of that kind are permanent in their effect upon mortality, and when we shut out cases of that kind, we are shutting out causes which would increase the mortality, not for five years or ten years, but for the whole course of life.

It is quite true that if you take one hundred thousand persons who pass a medical examination now, the survivors of those ten years from now will be unselect, because during that ten years a great many of those people will have developed disease or have changed their constitution, or habits, or environments so as to come into one or other of these groups which we shut out at the beginning. But it is very clear that in shutting out cases of impairment at the beginning we were not shutting out merely mortality for a few years, and it is equally true that those who became impaired during the next five or ten years are of the same kind. They, in the same way, cannot be expected to die off in five or ten years. Therefore, it seems to me, the point is perfectly clear that on the basis of a select table of mortality it is not possible to work out a table of mortality on diseased lives, on impaired lives, which will be of much, if any, practical value. Mr. Papps himself recognizes that fact at the close of his paper and I think he will be probably among the first to agree with me, but I would like to say this, that Mr. Papps has really rendered a great service to us because we have had one of the most instructive and interesting discussions of the nature of selection that we have had in the history of the Society.

MR. PAPPS:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

My object in presenting this paper was to draw attention to the theoretical properties assumed to attach to select tables, and without any reference to the state of health of the lives we actually insure, or the state of health into which such lives may eventually drift. A select table graduated by Makeham's Law is an arbitrary table, it is true, but according to the law of mortality assumed, the first column will show the course of a body of select lives who die before becoming non-select. The second column has been supposed to show the survivors of those who entered a year earlier as select lives. This has not been disputed, I believe, but the composition

of this surviving class is questioned. For the sake of argument, let it be assumed that the $O^{[M]}$ Table, no matter how obtained, does show the true law of mortality. Then, in the second column, the lives age 36 are the survivors of those entering at 35, and the lives age 36 in the first column are the survivors who remained select at age 36 out of those who entered at age 35. If this is so, then the basic formula on which my paper depends would appear to be correct.

As I understand it, the critics of the paper hold that the numbers living in the second column of the table are not the numbers who survive out of those entering in the first column, but merely such a number of lives, some select and some not, as will give a rate of mortality applicable to the second policy year. In other words, the table showing the select values of l_x must be considered as having no such properties as has generally been supposed.

In my paper I took a formula which has generally been supposed to be correct, developed that formula and the idea underlying it in a way which I believe was correct, and brought out results which were evidently absurd. The obvious explanation was that the basic formula or idea was incorrect. I was unaware that Mr. Moir had drawn attention to this matter in the *Transactions of the Faculty of Actuaries,* and it occurred to me that the presentation of the problem might result in some interesting discussion and a useful study of select tables. I am very glad that so many have been tempted to prepare discussions.

It has occurred to me that the smoothness of the graduation of the $O^{[M]}$ Table is very nicely shown in the values shown in Tables E, F and G, and it would seem that the process followed in my paper, while somewhat lengthy, would be a rigorous test of the smoothness of the graduation of any select tables.

WORKMEN'S COMPENSATION BENEFITS—
W. ARTHUR WATT.

VOL. XIII, PAGE 54.

WRITTEN DISCUSSION.

MR. FLYNN:

We are indebted to Mr. Watt for introducing to the attention of the Society the subject of workmen's compensation. It is probably a safe statement to make that the greatest problem before insurance men in America, at the present time, is that which deals with the compensation of the workingman for loss due to accidental injury while engaged at his occupation. Fourteen states and five provinces have enacted workmen's compensation acts during the past few years, and commissions of nine states are at this time studying the subject preparatory to the introduction of bills. All of these acts and proposed bills differ in important features, showing the result of various minds in grappling with the general problem. Some of the acts are operating, to a large extent, successfully, and others are manifestly not accomplishing the end for which they were created. To work out satisfactorily the many perplexing problems of this great question, the assistance is necessary of one having a knowledge of the theory of insurance and of the social, economic and political features of the movement. There is no subject which is worthier of the serious study of the actuary than this one.

Mr. Watt has given us a brief outline of the acts of foreign countries, and has touched upon many of the phases of the general problem which are engaging the attention of students of the subject in this country. It is a difficult matter to give a comprehensive idea of such a vast subject in a comparatively short paper, and we must compliment Mr. Watt for the able manner in which he has accomplished his purpose. Much has been written upon the experience of foreign countries with various plans of workmen's compensation, and in many respects this is an invaluable source of helpful information; still conditions in this country are peculiar to itself, and many new problems have arisen to which the solutions adopted in the older countries will not apply. They call for careful, original work. In my remarks which follow, I shall attempt to point out a few of these which are of particular interest to the actuary.

A serious obstacle to the enactment of compulsory workmen's compensation acts in this country has been the conflict with rights under the state constitutions. The Wainwright Act, which went

into effect July 4, 1910 in New York State, was declared unconstitutional by the Court of Appeals of that state because this statute did not preserve to the employer the "due process" of law guaranteed by the constitution, for it authorized the taking of the employer's property without his consent and without his fault. In other words, it made the employer liable notwithstanding he had faithfully observed every duty imposed upon him by law. California has removed this obstacle by amending its constitution to provide for legislation of this character, and Arizona provided in its constitution, when it was drawn a few years ago, for a workman's compulsory compensation law. In order to obtain a compulsory compensation act in other states, an ingenuous plan has been devised which accomplishes its purpose by forcing both the employer and the employe to accept workmen's compensation under penalty of the removal, or the retention, as the case may be, of the customary liability defenses, namely, "contributory negligence," the "fellow servant" rule and "assumption of risk." A good example of the successful use of this method is found in the compensation act of New Jersey. Under this law the employer is penalized by the loss of the three principal liability defenses unless he elects to provide compensation to his employes for accidents of occupation. No provision is made for the retention of these defenses by the employer in case he is willing but the employe elects not to come under the plan. No difficulty is experienced because of this, however, for the reason that the employer has a right to employ only those workmen who will come under the plan. In the acts of most other states, this part of the plan of compulsion is incorporated. Another part of the plan which conduces to successful operation is that both the employer and the employe are *presumed* to come under the compensation part of the act unless they take definite action to the contrary by written notice to each other. The result has been that, in New Jersey, all claims as a result of accidental injury at occupation are settled upon the basis of compensation—in fact, the act is, in effect, one of compulsory compensation. The constitutionality of the New Jersey law has as yet not been passed upon by the courts, but the weight of opinion of authorities upon the matter is that it is constitutional. The general plan of compelling the employer and the employe to accept workmen's compensation under penalty of removal of the common law defenses has been adopted in all states with the exception of one or two. An example of an unsuccessful plan of election is that of New Hampshire, in which the value of compulsion has been lost because of the choice which is given the employe after the accident of selecting the method under which he will be indemnified—liability or compensation.

Another phase of the problem which is being studied carefully at this time is the scale of benefits to be provided. Benefits, which are so liberal that they will encourage malingering upon the part

of the employe or which will be much more expensive for the employer to furnish than the cost of protection under the "penalty," will defeat the purpose of the compensation act. The scale of benefits of the Wisconsin Act is a good illustration of this. Under this compensation law, 65 per cent. of wages is paid for all injuries, beginning at the end of the first week, with a further provision that payments will be made for the first week of disability if the total period lasts more than four weeks. In addition, such medical and surgical treatment and surgical supplies as are necessary are provided without limit as to cost during the first ninety days of disability. If a nurse is required after the first ninety days of disability, 100 per cent. of wages is paid. This part of the scale of benefits is much more liberal than that of any foreign country and will, undoubtedly, lead to malingering upon the part of employes. It is in the numerous claims of comparatively short duration that the chief danger of malingering lies. When more than 50 per cent. of the wage is payable as compensation under foreign acts, the effect of this liberal treatment is generally offset by a long waiting period or by making the payments during the first few weeks, as, for instance, the first four in Austria and the first thirteen in Germany, from a separate fund to which the employes contribute heavily. The scale of compensation should be upon a reasonable basis so that the danger of malingering will be removed so far as possible.

A desirable feature of the scale of compensation is a schedule of specific benefits for particular accidents, which shall be as complete as practicable and on an equitable basis. It is true that certain extreme cases can be shown to prove that a specific injury, such as the loss of a hand, will produce a much greater loss of earning power to one workman than to another in a different occupation, but the importance of this consideration is overshadowed by the greater ease of adjustment which is obtained in the settlement of claims. Anything which tends to obviate friction and promote satisfaction in the handling of compensation claims is of utmost importance. The assignment of an equitable definite period of compensation for specific injuries is one of the problems to be worked out from experience.

The question of the proper premium rates to be charged for workmen's compensation in this country has been much discussed. It is the opinion of those who have studied the problem carefully that foreign statistics, such as those of Austria, which Mr. Watt has examined, while they may give indications of great value in the conduct of the business, can be used only as a rough guide in framing rates for workmen's compensation in this country. The difference in working conditions, in the methods of manufacture and, most of all, the difference in the industrial classifications of foreign countries as compared with those of America, even when such industrial classifications are similarly named, all tend to

lower the value of foreign experience. Further reasons for not following more closely indicated foreign rates are, first, that the scales of compensation differ greatly, and, second, that the plans of insurance of certain countries, as, for instance Germany and Austria, are upon an assessment basis and, therefore, not comparable with the one year term basis of the insurance companies in this country. Before deciding upon rates to be charged by the companies for compensation coverage in states such as New Jersey, all available experience data, which would throw light upon the problem, were examined. Workmen's collective experience, personal accident experience and the accident data of liability experience were utilized. A committee of underwriters, which represented the best knowledge upon the subject and which contained members who had had experience under the British Compensation Act, then decided upon rates which in its best judgment would apply to the conditions in this country. The work was done carefully and slowly, and rates were obtained only after giving due consideration to every factor which would have a bearing upon the subject. It must be remembered that the New Jersey Compensation Act has been in force now for over fifteen months and that experience is being accumulated which will before long begin to throw further light upon this problem. Workmen's compensation experience will mature much more rapidly than the old employers' liability experience, and it is reasonable to assume that within one or two years experience will be sufficiently matured to be used as a basis for the modification of existing rates. It is possible that the rates for some of the industrial classifications have been placed too high and that those for others have been too low. It is probable, however, that when all phases of the problem are considered, rates will be found to be within reasonable bounds of accuracy and that but few changes will be necessary.

The work of framing a practicable and reliable method for estimating the reserve liability under workmen's compensation claims is one which is going to engage the attention of the actuary during the next few years. The present employers' liability reserve law of New York State, which is similar to those of several other states, handles the reserve under employers' liability and workmen's compensation business together. A schedule is required of each company showing the number and the estimated cost of outstanding compensation claims resulting from fatal and non-fatal injuries, separately, incurred under business written in the ten calendar years preceding date of valuation. The reserve for each year of business earlier than the fifth year preceding date of valuation is made up of the estimated costs of outstanding cases. The reserve for each of the five years of business just preceding date of valuation is a certain percentage (derived from past experience) of the earned premiums of each year less claim payments under these contracts, with the further condition, however, that if the estimated

cost of outstanding cases for any one of the first three years of business of this period is larger than the figure obtained by the percentage method, it shall be used as the reserve for that year. At the present time, the number of outstanding cases under workmen's compensation is not too large to be estimated individually. In a year or two, however, when the number of outstanding compensation claims will be numbered by the thousands, it will be necessary to evolve some other plan of calculating the amount of outstanding obligations. The reserve for death claims, and cases of total and permanent disability, and other claims for which benefits for a definite term are provided, will probably be calculated upon the basis of a table of mortality and of interest. The less serious cases can possibly be cared for by the use of averages based upon experience.

The Assurance Companies' Act of Great Britain calls for the valuation of outstanding claims of five years' duration and upwards, upon the basis of 75 per cent. of the value of a life annuity purchased through the Post Office or upon some other approved actuarial basis. The reserve for compensation claims of shorter duration is estimated by the companies by years in which the claims have arisen. A statement is furnished by each company annually which compares the amount of reserve which was estimated one year earlier for outstanding claims incurred in a certain calendar year with the amount which was paid out on account of these claims during the twelve months which have elapsed plus the latest estimate of the future cost of thes outstanding claims. In this way, the accuracy of the companies' estimates of their outstanding obligations which have been made in the past is checked roughly. I am not familiar, however, with the methods used by the companies in obtaining these estimated costs of non-fatal cases of less than five years' duration. A paper read by Mr. William Penman, jr., before the Institute of Actuaries in December, 1910 (*J. I. A.*, XLV, 101), outlined a plan for calculating the future cost of outstanding claims of comparatively short duration. This method is interesting when viewed from a theoretical standpoint, but is of but little value as a practical solution of the problem.

MR. DAWSON:

Mr. Watt's paper is timely in the sense that the subject is one which is pressing for attention at this time and that every serious effort to cause actuaries to give heed to the social aspects of insurance—and, incidentally, to the special features of social insurance from an actuarial standpoint—is deserving of praise. At least two members of our Society, one Fellow and one Associate, have recently been called upon to deal, in a most practical way, with the actuarial problems of workmen's compensation; and it is well that others are being encouraged to take up their study and

that in the new advanced text-book, now being undertaken by the Society, the elements of such problems will be discussed.

Mr. Watt's paper is discursive and not technical, and should serve as a good introduction to the subject. What it contains may for the most part be found in books and documents published in the United States, such as the September 1910 Bulletin of the Bureau of Labor, the 24th Annual Report of the Commissioner of Labor, Frankel & Dawson's "Workingmen's Insurance in Europe," etc.; but what Mr. Watt presents is a good introduction to the subject.

He suggests no way, however, to handle it satisfactorily from an actuarial standpoint in practice, which is unfortunate, if he knows of any, for there is crying need for such service already, both in state insurance funds and in private companies; and a demand for something further and more elaborate may at any time come, here, as it did so recently to Mr. Hardy and Mr. Day, Fellows of the Institute, in Great Britain.

The tables which he republishes are chiefly from Austrian data. The results must, as Mr. Watt suggests, be accepted with caution, because of the great difference in conditions; but yet more so, perhaps, because the classifications of degrees of disablement are arbitrary, *i. e.*, have reference merely to physical disablement and not directly to impairment of earning power, as in Germany, for instance. The difference in the point of view between what have proved to be bureaucratic district funds, the managers of which are concerned chiefly to systematize settlements, and what have proved to be autonomous mutual associations of contributing employers, bent upon reducing and repairing actual impairments of earning power, could not be better illustrated.

The following passage on page 77 seems somewhat confused and in any event calls for comment:

> "The experience of the Austrian system gives emphasis to the argument that there is a decided tendency to charge too little at the outset and that steps taken to increase the rates are decidedly unpopular. This shows the necessity of charging adequate premiums at the outset, especially if reserves are to be accumulated. Further as the premium is merely an assessment, no account is taken of the ages of the assured workmen, and, consequently, the younger men pay more than their share, the older men less than their share of the cost. Whether or when an attempt will be made to secure adequate data on which may be based graded premiums it is not easy to predict. Against the cost and trouble of such a step is urged the frequency with which men change from one establishment to another."

The confusion is in "the younger men pay more than their share, the older men pay less than their share of the cost." The

premiums are paid by the employer, save as regards 10 per cent., and that will surely be paid by them after the next revision of the Austrian laws. To have resulted in analyzing the elements of workmen's compensation premiums so sharply in some cases as to cause discrimination against older workmen, is one of the indictments against the system of insuring in private stock insurance companies as in Great Britain; Austria, which has, despite all its defects as there practiced, a compulsory public insurance system, is most unlikly to attempt this, and, fortunately, in Germany the employers' associations have devoted themselves to economics by means of prevention and cure, which are social, instead of discouragement of the employment of all German citizens who can work, which would be most seriously anti-social.

Mr. Watt correctly attributes the insufficiency of Austrian rates to provide the "capitalized values," contemplated by the law, "to the opposition of the employers," who have had a preponderating voice in the insurance bureaus without the responsibility of being members of a mutual association which must pay its own debts. Still, in no country, as yet, have the managers of an insurance institution, whether state, mutual or stock, succeeded in discovering and establishing adequate rates, on a "capitalized value" basis. Of course the rates of the German associations of employers are "adequate," because they collect enough each year to cover the sums actually paid out by the Government Savings Banks for their account the previous year; their only actuarial problem is fairly to apportion this cost. But all others must fix the contribution *in advance;* to do which, the actuary must know many things besides how to apportion the aggregate of an outlay already made, such as what the risk *will be;* what the terms during which payment must be made to the temporarily disabled, the permanently disabled, widows and orphans; how disabilities will fall, as to degree as well as duration; what proportion of those who are killed by accident will leave widows and at what ages; what proportion will leave minor children, how many and at what ages; what proportion will leave dependent parents, grandparents or grandchildren and at what ages; what will be the death-rates among these, and what the marriage rates of widows. All this but suggests how complex the problem is and how great its difficulties. Norway, in its state department, is known to have come near to solving it; Sweden, in its state department, is thought by some to have solved it, though that is far from demonstrated. Virtually every stock and every mutual company the world over has failed to solve it and has lost money trying to do so.

It is my opinion that it is insoluble for them, and that seeking to do this business involves certain loss for them, as well as great social mischief, in consequence of which this field should be abandoned by them to public insurance. The grounds for this view are, in brief, as follow: in a given industry, the average cost will be

realized every year pretty well by the employer of several thousand employes, by his carrying his own risk. Nearly every employer of a very large force realizes this and carries his own risk, keeping careful account of the cost. Somewhat smaller employers, some carrying their own risks and some taking insurance, also keep such accounts, whether insured or not. The proportion who insure increases as the number employed diminishes, but not without limit, for a very large number of those who employ but one or two do not insure—of itself a serious defect in such a system. The insurance companies have not merely to meet the competition of other companies—which is always severe when so many uncertainties enter into the rate problem—but also the well-informed competition of those employers whose forces of employes are large enough to enable them to figure closely on the cost of carrying their risks. The struggle becomes to name rates which they, or at least some of them, will pay. Such rates are very near to what it would actually cost such employers to carry their own risks; and these rates become standards for others, though not necessarily adhered to closely. Yet the companies must pay commissions and other expenses, averaging in this country half the premiums. To make a profit, they must, then, settle the claims for but little, if any, more than half what it would cost the employer, which is a most serious social evil in that it defeats the provision for these unfortunates, and which is most bitterly complained of in Great Britain. They cannot, even when paring claims down by compromising them by commutation or otherwise, accomplish this at all; and, therefore, they lose money and in the nature of things must do so. Indeed, they surely would not be in the field, were it not that they found themselves there when workmen's compensation was introduced, and now have a large business organization, with many of those concerned, as employes and agents, profiting greatly.

These things, when compared with the great and indisputable success of workmen's compensation mutual insurance in Germany under compulsion, have impelled me to favor the latter here in my paper read before the American Academy of Social Science in April, 1911, referred to by Mr. Watt on page 78, and in argument and briefs before the Federal Commission in June of that year; but Mr. Watt is in error in saying that I "affirmed that the cost of this insurance, if conducted by the state, must be levied as a tax upon all the people." It was, instead, proposed by me that the cost be collected by a tax, apportioned by representatives of employers among themselves according to the hazards and the payrolls, precisely as in Germany. Mr. Watt was perhaps misled by the use of the word "tax"; an excise tax upon such employers was meant.

MR. WATT:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

The discussions of Mr. Dawson and Mr. Flynn were well designed as a commentary on my paper. Mr. Dawson in criticising the paper on its merits has drawn attention to points which should be considered, and which many are considering with varying results. He says truly that I made no suggestion as to how the problem could in practice be handled satisfactorily from an actuarial standpoint. The politician and the socialist in effect say to the actuary "We have undertaken this task and we want you to show us how to perform it. The wisdom of it or otherwise does not concern you." As a matter of fact the problem has two phases, a social and a scientific, and the difficulty is in deciding which to solve. If the latter alone may be considered, there is only scope for clever guessing. The scientific problem is to deal with data which must first be obtained and regarding which no amount of prophecy can shape the results. With reference to the passage from page 77 of my paper, which Mr. Dawson quotes, I had thought the context would indicate the correct interpretation. It was stated elsewhere in the paper that the employer paid all but 10 per cent. of the premiums, and it was my intention to demonstrate that the Austrian system discriminated against the factory or workshop in which there was a preponderance of young workers.

If, as the statistics prove, there is a steady increase with age in the proportion of cases of accident resulting in permanent disability and death, then the employers of labor should pay more in the case of older workers. This may be undesired by the politician and the socialist, but the actuary must state the facts. If the business is to be conducted on a scientific basis, the actuary must have the deciding voice.

The question everybody is asking is, what rates should be changed? or, in other words, are the present rates adequate? But, as the benefits are different in each state, it is evident that no standard rates can be named, and it would be injudicious to even attempt to do so. No actuary, I fancy, would be willing to lend his name to any proposed solution of the problem on the present basis. A comparison of the various compensation laws in force or under consideration leads one to think that, ere long, we shall have samples of each of the fifty-seven varieties of socialism, each with its distinctive frill and all alike in a delightful disregard of consequences.

Mr. Dawson says truly enough, that under present conditions the companies "surely would not be in the field were it not that they found themselves there when workman's compensation was introduced and now have a large business organization." That I conceive to be no reproach to the companies, but to those who, in effect, legislate to put the companies out of business. And to

what purpose? To provide a more expensive kind of insurance. Under similar conditions the life companies would doubtless also be mentioned with a sneer.

Speaking of the German system, Mr. Dawson says "their only actuarial problem is fairly to apportion this cost" among the employers. It is the chief problem of the whole subject, and has hitherto not been solved. The problem will be complex and difficult until the question is considered on a scientific basis. The socialist may desire to give compensation to a man of fifty on the same basis as to a man of thirty but the actuary is bound to say it is discrimination. As well return to the old system of life insurance whereby five per cent. per annum on the sum insured was the premium at all ages. By merely assessing each year's cost on the employers a point is bound to be reached, as the German experience seems to indicate has been reached in that country, where the outlay is enormous and a very heavy burden on industrial corporations. If compensation to workmen is "the greatest problem before insurance men in America at the present time," when may we expect it to be less so at the hands of politicians and socialists? The remarks of Mr. Flynn were valuable as establishing a link between the question in Europe and America. To both gentlemen I am grateful for the trouble taken to bring the subject before the Society in such a manner as to show its importance and its difficulties.

GILL'S MORTALITY TABLE—
S. A. JOFFE.

VOL. XIII, PAGE 80.

WRITTEN DISCUSSION.

MR. McCLINTOCK:

This additional contribution from Mr. Joffe strikes me personally with much force. I confess to not having given credit to Mr. Gill to an extent which is due, although I have been long acquainted with the chief facts. The discussion contained in the same issue of the *Transactions* (page 159) on the previous paper by Mr. Joffe is enriched by Mr. Fackler, who was Assistant under Mr. Homans, Mr. Gill's immediate successor, and also by Mr. Joffe himself in summing up the discussion. Mr. Gill was widely known as an adept in mathematics, and was consequently chosen as the first Actuary of the Mutual Life. I confess that I have, until now, been ignorant of the personal history of Charles Gill, the first of my own predecessors. If I live until the next meeting, that of May 1913, I hope to present the results of the researches on this interesting subject. It is fitting that the work should be attempted by me, since I happen to be the person who was for the longest time the holder of the office which Mr. Gill was the first to occupy. Though I am mortified that I should not earlier have recognized the importance of this work, it is gratifying that one so thoroughly conversant as Mr. Joffe with the office of the Mutual Life should have been the first to call the attention of the Society to his position and to his chief work.

Book Notices.

Medico-Actuarial Mortality Investigation—Report of the Joint Committee. Volume 1. New York, compiled and published by the Association of Life Insurance Medical Directors and the Actuarial Society of America, 1912. Pp. 131.

This is the first volume of the report of the Joint Committee of the Association of Life Insurance Medical Directors and the Actuarial Society of America on the results of the investigation of the experience of the life insurance companies on business issued during the years 1885 to 1908 inclusive, the experience being terminated on the policy anniversary in 1909. This volume contains an analysis of the statistics of height and weight of insured persons, tables being given of the graduated average weight, both for men and women, for each year of age from 15 to 55 and for intervals of one inch in height. It also contains the standard table which was constructed by the Committee as a basis for the comparison of the mortality in the different special classes. This standard table was constructed from the experience of the various companies on the issues of January of the odd years and July of the even years—1885 to 1908 inclusive, observed to the policy anniversary in 1909, thus covering the same period as is covered by each of the special classes.

The following table shows the extent of the statistics, the issues being divided into three periods of eight years each.

Years of Issue.	Number of Entrants.	Total Exposures.	Deaths.
1885–1892	80,976	781,852	7,180
1893–1900	148,995	1,106,216	8,000
1901–1908	270,404	926,108	5,042

The resulting table is shown in the form of analyzed rates of mortality for the first four years with an ultimate table for the 5th and subsequent years. The following table shows a comparison of rates of mortality per 1,000 in this ultimate table with those shown by the American Experience Table and by the ultimate part of the O[M] Table:

Attained Age.	M. A.	Am. Exp.	O[M]
25	4.7	8.06	7.00
35	5.1	8.95	8.46
45	7.5	11.16	12.05
55	15.8	18.57	20.79
65	39.0	40.13	41.92

Mortality of Government Life Annuitants. Report of the Actuary of the National Debt Office on the Mortality of Government Life Annuitants, London. Published by his Majesty's Stationery Office, 1912. Pp. 65.

This report gives the result of an investigation of the mortality experience of the government life annuitants in Great Britain during the years 1875 to 1904 inclusive. It covers not only the issues of that period but the experience beginning with the policy anniversary in 1875 on annuities then in force. The experience is terminated with the anniversary in 1904. Only the experience on the first annuity purchased on a given life is taken into account, so that duplicates are entirely eliminated.

The following table shows the summary of the data:

Government Annuity Experience, 1875–1904. Summary of Data.

	Males.	Females.	Total.
Number of lives under observation	5,504	13,863	19,367
Number of years of risk	57,652	163,378	221,030
Number of deaths	4,168	9,333	13,501
Number of lives existing at the close of the observation	1,336	4,530	5,866

This experience shows a mortality considerably lower than that recorded in the previous experience for the period 1808–1875 and in the case of female lives the mortality is much lower than was observed in the British Offices' Annuity Experience covering the years 1863 to 1893. In the Appendix graduated rates of mortality are shown which were constructed by Hardy's modification of Makeham's formula.

The following table shows a comparison of the values of annuities on selected lives at 3 per cent. interest according to the new Government Annuitants' Experience and according to the British Offices' Annuity Tables:

VALUE OF ANNUITY OF 1.

Age. (1)	Males.			Females.		
	New Government Table. (2)	British Offices Annuity Table. (3)	Col. (2) Minus Col. (3). (4)	New Government Table. (5)	British Offices Annuity Table. (6)	Col. (5) Minus Col. (6). (7)
40	17.34	17.60	−.26	18.69	18.26	.43
45	15.85	16.06	−.21	17.31	16.93	.38
50	14.24	14.40	−.16	15.78	15.51	.27
55	12.55	12.66	−.11	14.09	13.96	.13
60	10.82	10.88	−.06	12.29	12.23	.06
65	9.10	9.12	−.02	10.43	10.33	.10
70	7.45	7.44	.01	8.59	8.41	.18
75	5.93	5.90	.03	6.84	6.61	.23
80	4.59	4.54	.05	5.27	5.05	.22

The results of this experience appear to indicate that the British Offices Annuity Table is not too rigid a basis for annuity rates in Great Britain. It, in fact, indicates that possibly a margin for contingencies is necessary, especially in connection with female lives.

Abstract from the Minutes of the Semi-annual Meeting of the Actuarial Society of America, held in Toronto on Thursday and Friday, October 17 and 18, 1912.

King Edward Hotel, Toronto.
October 17, 1912.

FIRST DAY.

The meeting was called to order by the President, Colonel Macdonald, at 11:30 A. M. The following members were present:

FELLOWS.

Alsop,
Blehl,
Carpenter,
Cooper,
Craig, J. M.,
Dow,
Fackler, E. B.,
Ferguson,
File,
Fitzgerald, C. R.,
Gore, J. K.,
Hallman,
Hardcastle,
Henderson,
Hunter, A.,
Hunter, R. G.,
Ireland,
Johnston,
Kilgour,
Laing,
Laird,
Little,
McKechnie,
Macaulay,
Macdonald,
Marshall, E. P.,
Mead,
Morris, E. B.,
Murphy,
Papps,
Pipe,
Portch,
Rhodes,
Richter,
Rose,
St. John,
Strong, W. M.,
Wood, A. B.,
Wood, W. A. P.

ASSOCIATES.

Allstrom,
Angell,
Armstrong,
Baber,
Bain,
Brough,
Brown,
Cook,
Dark,
Davenport, J. S.,
Dickenson,
Fairlie,
FitzGerald, W. G.,
Gibb,
Grigg,
Hall, A. F.,
Homans,
Hope,
Howe,
Kime,
King,
Lithgow,
Linton,
Marshall, E. W.,
Matheson,
Moodie,
Moore,
Morris, W. O.,
Muckle,
Parker,
Pattison,
Pequegnat,
Reid,
Sinclair,
Smith, V. R.,
Somerville,
Speers,
Stephenson,
Strong, A. W.,
Vineberg,
Walker,
Washburne, A. C.,
White.

After the calling of the roll, the President read his address to the Society.

The minutes of the Annual Meeting held in New York, May 16 and 17, 1912, were approved as printed in the Transactions.

The Secretary presented the following report of the proceedings of Council:

The only meeting of the Council since the last meeting of the Society was the meeting held this morning. The Examination Committee presented a report showing that at the examinations held last June one candidate had passed both parts of the Fellowship Examination, six had passed Part II and five had passed Part I. Of the Associate Examination Section B, five had passed both parts, one had passed Part II and three had passed Part I. Of the Associate Section A, ten had passed both parts of the examination, five had passed Part II and nine had passed Part I. The Examination Committee made some recommendations which were referred to a special committee of the Council for action.

Mr. Hunter has proposed the name of George F. Hardy for election as a Fellow of the Society without examination, under the last sentence of Article 10 of the Constitution, and his proposal has been unanimously approved by the Council and recommended to the Society.

A communication was received from the Association of Life Insurance Medical Directors which will be later laid before the Society for action.

On motion the proceedings of the Council as reported were approved.

On motion of Mr. Hunter, Mr. G. F. Hardy was unanimously elected by ballot a Fellow of the Society without examination.

The following communication was received from the Association of Life Insurance Medical Directors.

"Resolved (if the Actuarial Society concur) that the Sub-Committee in charge of the Medico-Actuarial Mortality Investigation be requested to prepare—if they find it feasible to do so—a table of heights and weights which will accord with the lowest mortality.

"And further resolved that the said table be published, if prepared, in that part of the report of the Medico-Actuarial Mortality Investigation which incorporates the study of the special groups."

On motion the resolution was approved by the Society and referred to our own Committee for action along the lines indicated if they find it feasible to do so.

An oral report was submitted by the Chairman of the Ceneral Bureau of the Medico-Actuarial Mortality Investigation.

The Society adjourned at 12:45.

Afternoon Session.

When the Society reassembled at 2 P. M. the reading of papers prepared for this meeting was proceeded with. After the reading of the papers was completed the Society took up the discussion of the papers presented at the meeting in May, 1912.

At 5 o'clock the Society adjourned to meet at dinner at 7:30 o'clock and to reassemble for business on the following morning at 10 o'clock.

SECOND DAY.

Friday, *Oct. 18,* 1912.

The meeting was called to order by the President. The discussion of papers read at the meeting in May was resumed and completed.

On motion it was unanimously resolved that the thanks of the Society be given to President Macdonald and the other Ontario members for the generous hospitality they have shown the Society and their very successful efforts to make the fall meeting an Actuarial success and a most enjoyable social reunion.

At 12:45 the Society finally adjourned.

RESULTS OF EXAMINATIONS.

The following is a list of those who passed the examinations held by this Society on June 5 and 6, 1912.

FELLOWSHIP.

Parts I and II.

Little, J. F.

Part II.

Blehl, E. M.,
Cooper, J. J.,
Fitzgerald, C. R.,
Laing, J. M.,
Langstaff, M. P.,
Murphy, R. D.

Part I.

Johnson, M. L.,
Kime, V. M.,
Maclean, J. B.,
Moodie, J. R.,
Vineberg, H. E.

ASSOCIATESHIP.

Section B—Parts I and II.

Fetsch, H. C.,
Olifiers, E.,
Marshall, E. W.,
Mather, S. P.,
Warwick, R. W.

Section B—Part II.

Speers, A. A.

Section B—Part I.

McLean, P. S., Rydgren, A. A., Shaw, D. W.

Section A—Parts I and II.

Birkenshaw, J. H.,
Cameron, W. J.,
Campbell, J. E.,
Goring, R. B.,
Grant, Isabel M.,
Holmes, H.,
Hurd, H. G.,
McConaghy, C. A.,
Tebbetts, J. W.,
Trimble, J. R.

SECTION A—PART II.

Flynn, J. D.,
Little, R.,
MacCharles, F. D.,
Williamson, J. D.
Wheeler, R. P.

SECTION A—PART I.

Buchanan, J. D.,
Green, W. W.,
Keyes, M.,
Leslie, W.,
McConney, E. M.,
Riley, A. J.,
Shepperd, C. O.,
West, C. J.,
Wilson, S. L.

WILLIAM HUGHES.

By the death of William Hughes, F.I.A., which occurred at his residence, Tulse Hill, London, England, on April 16, 1912, the actuarial profession, and life assurance generally, has lost one of its most prominent members. For thirty-one years, until his retirement in 1904, he was Manager of the Ordinary Branch of the Prudential Assurance Company (of Great Britain).

Mr. Hughes entered the service of the Prudential in 1858, and rapidly rose to prominence, eventually being appointed in 1873 to the important post which he held for so many years. He became a member of the Institute of Actuaries in 1864, and, after passing the necessary examinations, was admitted as a Fellow in 1871.

The value of his services was appreciated not only by the company he served so well, but by the actuarial profession in general. On October 6, 1898, our own Society, in recognition of his services to the profession, unanimously admitted him to full membership. In the year 1902 his confrères of the Institute of Actuaries bestowed upon him the highest honor in their gift, by electing him to the presidential chair for the customary period of two years. During his term of office the Fourth International Congress of Actuaries was held in New York. Mr. Hughes attended in his official capacity, and as the presiding representative of the large delegation of distinguished British actuaries, he became well known to our members, who learned to deeply appreciate his sterling character, and that genial, kindly disposition, which endeared him to all who had the privilege of entering the circle of his friendship. He died in his seventy-third year.

EDWARD JAMES SARTELLE.

Edward James Sartelle, for six years Actuary of the State Mutual Life Assurance Company, died at his home in Worcester, Mass., September 15, 1912. He was one of the older Fellows of the Actuarial Society of America, having been elected to membership April 28, 1892, while he was Assistant Actuary of his company and associated with Mr. William E. Starr, one of the Society's charter members.

Mr. Sartelle was born in Pepperell, Mass., May 9, 1857, the son of James Quincy and Mary (Fessenden) Sartelle. His family was of old colonial stock, two of his ancestors in the direct line having served as Revolutionary soldiers.

He received his education in the public schools of Townsend, Mass., and at Lawrence Academy, Groton, Mass., and entered Harvard College in 1879.

After leaving college he taught school at Pepperell and at Lawrence Academy and for a time was Superintendent of Schools at Townsend. He left the teaching profession February 1, 1887, to accept the General Agency of the State Mutual Life Assurance Company at Lowell, Mass. In May, 1889, he gave up the field work to enter the home office of the company, where he at once directed his attention to the mathematical side of the life insurance business.

On January 30, 1903, after the death of Mr. Starr, Mr. Sartelle was elected Actuary and a Director of the State Mutual, and held the position until his health completely failed and he was obliged to resign in 1908.

As long as his health permitted, he was untiring in his devotion to his business, and he also found time to interest himself actively in the social life of his city. He was particularly prominent in the Masonic bodies of Massachusetts and would have been a 33d degree Mason in September, 1907, if the illness which totally incapacitated him during the latter days of his life had not come upon him in that year.

To those of us who knew him intimately, his pleasing personality and good judgment and business ability were strongly evident, and by his death the Society has lost a genial friend, a kindly gentleman, and an able actuary.

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